

# THE ELECTROWEAK THEORY

CHRIS QUIGG

*Theoretical Physics Department  
Fermi National Accelerator Laboratory  
Batavia, IL 60510 USA  
E-mail: quigg@fnal.gov*

After a short essay on the current state of particle physics, I review the antecedents of the modern picture of the weak and electromagnetic interactions and then undertake a brief survey of the  $SU(2)_L \otimes U(1)_Y$  electroweak theory. I review the features of electroweak phenomenology at tree level and beyond, present an introduction to the Higgs boson and the 1-TeV scale, and examine arguments for enlarging the electroweak theory. I conclude with a brief look at low-scale gravity.

## 1 Introduction

### 1.1 Our picture of matter

At the turn of the third millennium, we base our understanding of physical phenomena on the identification of a few constituents that seem elementary at the current limits of resolution of about  $10^{-18}$  m, and a few fundamental forces. The constituents are the pointlike quarks

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad (1)$$

and leptons

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \quad (2)$$

with strong, weak, and electromagnetic interactions specified by  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  gauge symmetries.

This concise statement of the standard model invites us to consider the agenda of particle physics today under four themes. *Elementarity.* Are the quarks and leptons structureless, or will we find that they are composite particles with internal structures that help us understand the properties of the individual quarks and leptons? *Symmetry.* One of the most powerful lessons of the modern synthesis of particle physics is that symmetries prescribe interactions. Our investigation of symmetry must address the question of which gauge symmetries exist (and, eventually, why). We must also under-

stand how the electroweak symmetry<sup>a</sup> is hidden. The most urgent problem in particle physics is to complete our understanding of electroweak symmetry breaking by exploring the 1-TeV scale. *Unity.* We have the fascinating possibility of gauge coupling unification, the idea that all the interactions we encounter have a common origin—and thus a common strength—at suitably high energy. Next comes the imperative of anomaly freedom in the electroweak theory, which urges us to treat quarks and leptons together, not as completely independent species. Both ideas are embodied in unified theories of the strong, weak, and electromagnetic interactions, which imply the existence of still other forces—to complete the grander gauge group of the unified theory—including interactions that change quarks into leptons. Supersymmetry and the self-interacting quanta of non-Abelian theories both hint that the traditional distinction between force particles and constituents might give way to a unified understanding of all the particles. *Identity.* We do not understand the physics that sets quark masses and mixings. Although experiments are testing the idea that the phase in the quark-mixing matrix lies behind the observed CP violation, we do not know what determines that phase. The accumulating evidence for neutrino oscillations presents us with a new embodiment of these puzzles in the lepton sector. At bottom, the question of identity is very simple to state: What makes an electron an electron, a neutrino a neutrino, and a top quark a top quark?

### 1.2 QCD is part of the standard model

The quark model of hadron structure and the parton model of hard-scattering processes have such pervasive influence on the way we conceptualize particle physics that quantum chromodynamics, the theory of strong interactions that underlies both, often fades into the background when the standard model is discussed. I want to begin these lectures on the electroweak theory with a clear statement that QCD is indeed part of the standard model, and with the belief that understanding QCD may be indispensable for deepening our understanding of the electroweak theory. Other lecturers will explore the application of QCD to flavor physics.

Quantum chromodynamics is a remarkably simple, successful, and rich theory of the strong interactions.<sup>b</sup> The perturbative regime of QCD exists,

---

<sup>a</sup>and, no doubt, others—including the symmetry that brings together the strong, weak, and electromagnetic interactions.

<sup>b</sup>For a passionate elaboration of this statement, see Frank Wilczek’s keynote address at PANIC ’99, Ref. <sup>1</sup>. An authoritative portrait of QCD and its many applications appears in the monograph by Ellis, Stirling, and Webber, Ref. <sup>2</sup>.

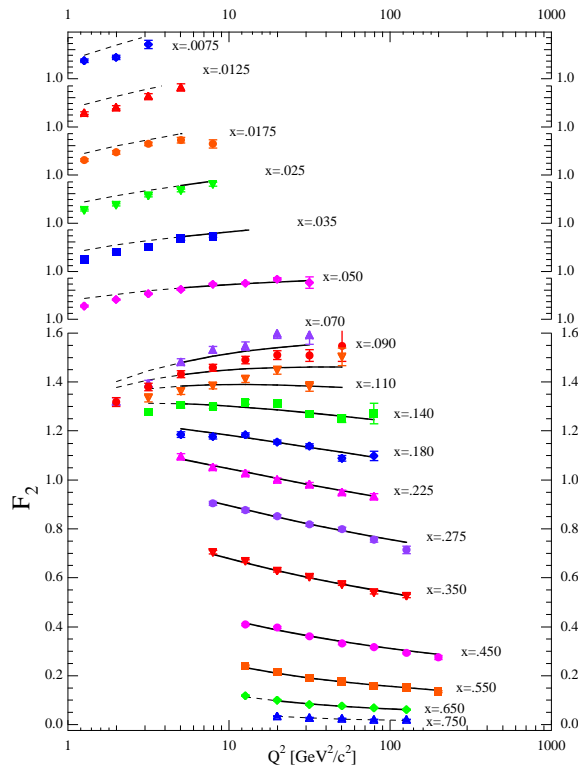


Figure 1. The structure function  $F_2$  measured in  $\nu N$  interactions, from Ref. <sup>3</sup>.

thanks to the crucial property of asymptotic freedom, and describes many phenomena in quantitative detail. The strong-coupling regime controls hadron structure and gives us our best information about quark masses.

The classic test of perturbative QCD is the prediction of subtle violations of Bjorken scaling in deeply inelastic lepton scattering. As an illustration of the current state of the comparison between theory and experiment, I show in Figure 1 the singlet structure function  $F_2(x, Q^2)$  measured in  $\nu N$  charged-current interactions by the CCFR Collaboration at Fermilab. The solid lines for  $Q^2 \gtrsim (5 \text{ GeV}/c)^2$  represent QCD fits; the dashed lines extrapolate to smaller values of  $Q^2$ . As we see in this example, modern data are so precise that one can search for small departures from the QCD expectation.

CDF Preliminary

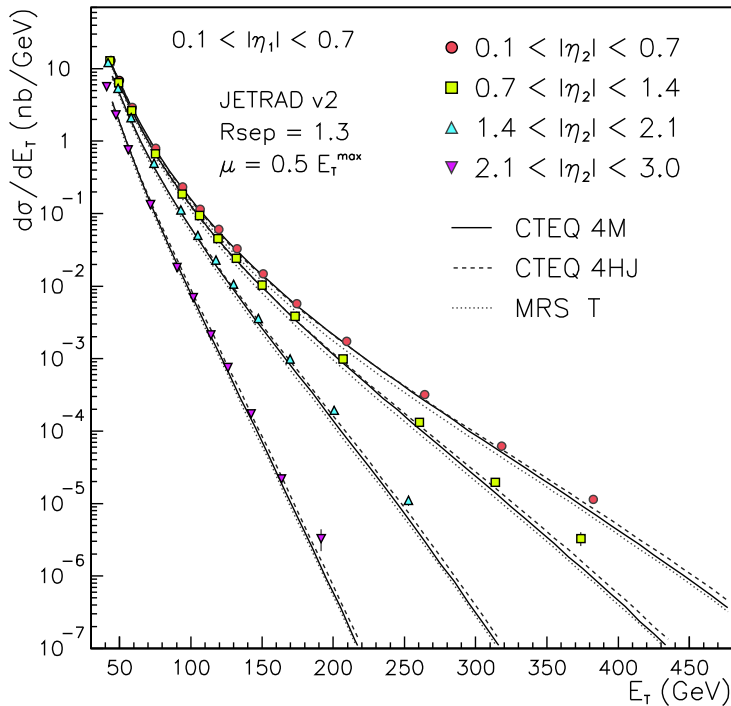


Figure 2. Cross sections measured at  $\sqrt{s} = 1.8$  TeV by the CDF Collaboration for central jets (defined by  $0.1 < |\eta_1| < 0.7$ ), with the second jet confined to specified intervals in the pseudorapidity  $\eta_2$ .<sup>4</sup> The curves show next-to-leading-order QCD predictions based on the CTEQ4M (solid line), CTEQ4HJ (dashed line), and MRST (dotted line) parton distributions.

Perturbative QCD also makes spectacularly successful predictions for hadronic processes. I show in Figure 2 that pQCD, evaluated at next-to-leading order using the program JETRAD, accounts for the transverse-energy spectrum of central jets produced in the reaction

$$\bar{p}p \rightarrow \text{jet}_1 + \text{jet}_2 + \text{anything} \quad (3)$$

over at least six orders of magnitude, at  $\sqrt{s} = 1.8$  TeV.<sup>c</sup>

<sup>c</sup>For a systematic review of high- $E_T$  jet production, see Blazey and Flaugh, Ref. <sup>5</sup>.

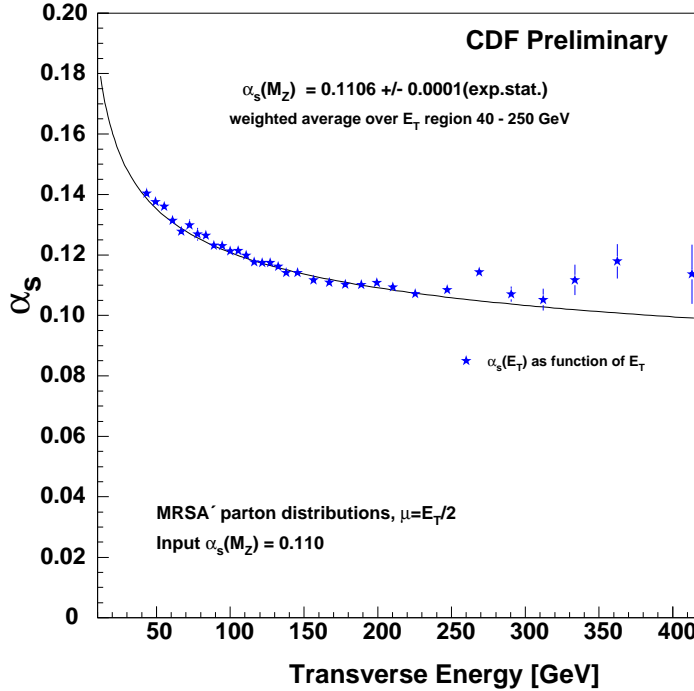


Figure 3. Determinations of  $\alpha_s$  inferred from the comparison of measured inclusive jet cross sections with the JETRAD NLO Monte-Carlo program. Source of this figure is [http://www-cdf.fnal.gov/physics/new/qcd/qcd99\\_pub\\_blessed.html](http://www-cdf.fnal.gov/physics/new/qcd/qcd99_pub_blessed.html).

The  $Q^2$ -evolution of the strong coupling constant predicted by QCD, which in lowest order is

$$1/\alpha_s(Q^2) = 1/\alpha_s(\mu^2) + \frac{33 - 2n_f}{12\pi} \log(Q^2/\mu^2), \quad (4)$$

where  $n_f$  is the number of active quark flavors, has been observed within individual experiments<sup>6,7</sup> and by comparing determinations made in different experiments at different scales.<sup>d</sup> Figure 3, from the CDF Collaboration, shows the values of  $\alpha_s(E_T)$  inferred from jet production cross sections in 1.8-TeV  $\bar{p}p$  collisions. The curve shows the expected running of the strong coupling constant.

<sup>d</sup>For a review, see Hinchliffe and Manohar, Ref. <sup>8</sup>.

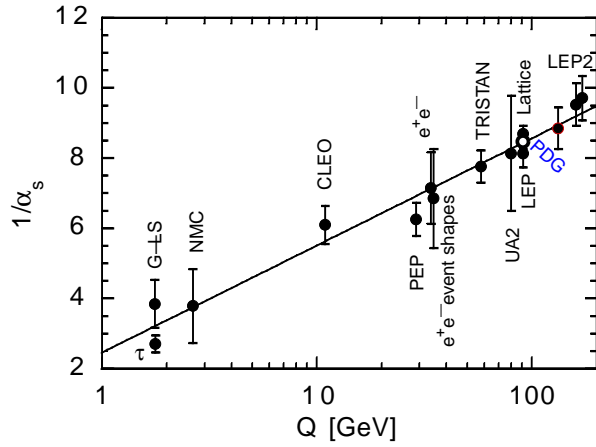


Figure 4. Determinations of  $1/\alpha_s$ , plotted at the scale  $\mu$  at which the measurements were made. The line shows the expected evolution (4).

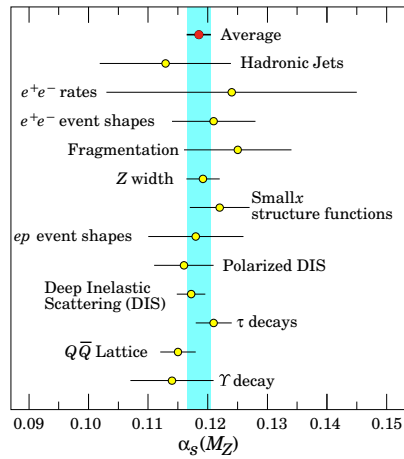


Figure 5. Determinations of  $\alpha_s(M_Z)$  from several processes. In most cases, the value measured at a scale  $\mu$  has been evolved to  $\mu = M_Z$ . Error bars include the theoretical uncertainties. From the *Review of Particle Physics*, Ref. <sup>9</sup>.

A compilation of  $1/\alpha_s$  determinations from many experiments, shown in Figure 4, exhibits the expected behavior.<sup>e</sup> Evolved to a common scale  $\mu = M_Z$ , the various determinations of  $\alpha_s$  lead to consistent values shown in Figure 5.

### 1.3 Sources of mass

We sometimes hear the statement that the discovery of the Higgs boson will reveal the origin of all mass. I am the first to say that unraveling the origin of electroweak symmetry breaking—for which “the discovery of the Higgs boson” is a common shorthand—will be a spectacular achievement in the history of science, but we are physicists, and we should say what we mean. There are, in fact, several sources of mass, and we can imagine soon understanding them all. At a level we now find so commonplace as to seem trivial, we understand the mass of any atom or molecule in terms of the masses of the atomic nuclei, the mass of the electron, and quantum electrodynamics.<sup>f</sup> And in precise and practical—if not quite “first-principle”—terms, we understand the masses of all the nuclides in terms of the proton mass, the neutron mass, and our knowledge of nuclear forces.

What about the proton and neutron masses?<sup>g</sup> Do we require the Higgs mechanism to understand them?<sup>h</sup> Thanks to QCD, we have learned that the dominant contribution to the light-hadron masses is not the masses of the quarks of which they are constituted, but the energy stored up in confining the quarks in a tiny volume. Our most useful tool in the strong-coupling regime is lattice QCD. Calculating the light-hadron spectrum from first principles has been one of the main objectives of the lattice program, and important strides have been made recently. In 1994, the GF11 Collaboration<sup>13</sup> carried out a quenched calculation of the spectrum (no dynamical fermions) that yielded masses that agree with experiment within 5–10%, with good understanding of the residual systematic uncertainties. The CP-PACS Collaboration centered in Tsukuba has embarked on an ambitious program that will soon lead to a full (unquenched) calculation. Their quenched results, along with those of

---

<sup>e</sup>A useful plot of  $\alpha_s$  vs.  $Q^2$  appears as Fig. 9.2 of the *Review of Particle Physics*, Ref. <sup>9</sup>.

<sup>f</sup>This is the sense in which quantum theory explains all of chemistry. The calculations are hard enough that we leave them to chemists!

<sup>g</sup>An accessible essay on our understanding of hadron mass appears in Ref. <sup>10</sup>.

<sup>h</sup>The standard model of particle physics has taught us many fascinating interrelations, including the effect of heavy-quark masses on the low-energy value of  $\alpha_s$ , which sets the scale of the light-hadron masses. For a quick tour, see my *Physics Today* article on the top quark<sup>11</sup>; Bob Cahn’s *RMP* Colloquium<sup>12</sup> is a more expansive tour of connections in the standard model.

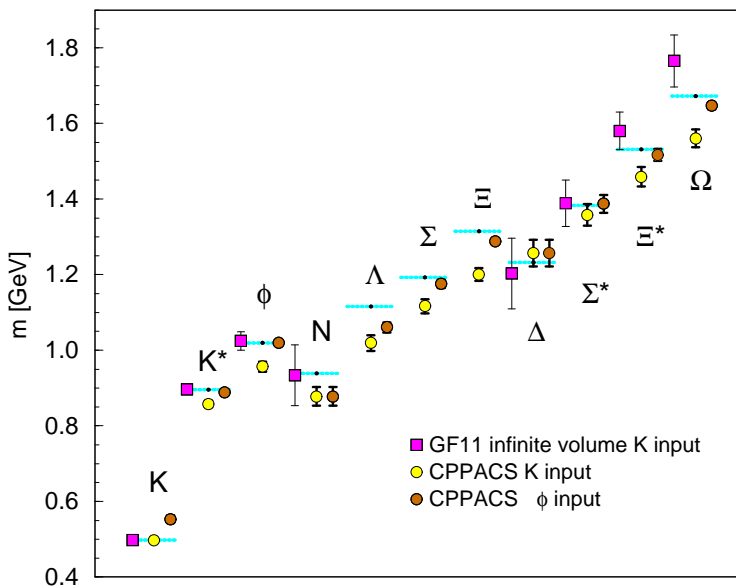


Figure 6. Final results of the CP-PACS Collaboration's quenched light hadron spectrum in the continuum limit.<sup>14</sup> Experimental values (horizontal lines) and earlier results from the GF11 Collaboration<sup>13</sup> are plotted for comparison.

the GF11 Collaboration, are presented in Figure 6.<sup>14</sup> The gross features of the light-hadron spectrum are reproduced very well, but if you look with a critical eye (as the CP-PACS collaborators do), you will notice that the quenched light hadron spectrum systematically deviates from experiment. The  $K$ - $K^*$  mass splitting is underestimated by about 10%, and the results differ depending on whether the strange-quark mass is fixed from the  $K$  mass or the  $\phi$  mass. The forthcoming unquenched results should improve the situation further, and give us new insights into how well—and why!—the simple quark model works.

We also have a reasonably good understanding of the masses of the electroweak gauge bosons, as we will develop in §3.2.<sup>15</sup> Gauge-boson masses are predicted in terms of the gauge coupling  $g$  and the weak mixing parameter  $\sin^2 \theta_W$ :

$$M_W^2 = \frac{g^2 v^2}{2} = \frac{\pi \alpha}{G_F \sqrt{2} \sin^2 \theta_W}, \quad (5)$$



$$M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W}, \quad (6)$$

where  $v = (G_F \sqrt{2})^{-1/2} = 246$  GeV sets the electroweak scale. Completing our understanding of the mechanism that endows the gauge bosons with mass is what we expect to accomplish by exploring the 1-TeV scale; that is what we can promise that the discovery of the Higgs boson—broadly understood—will deliver. While we don't yet have a complete understanding of the electroweak scale or the value of the weak mixing parameter, we can imagine how those two quantities might arise in unified theories or from new (strong) dynamics.

The masses of the elementary fermions are a more mysterious story: Each fermion mass involves a new, so far incalculable, Yukawa coupling. For example, the term in the electroweak Lagrangian that gives rise to the electron mass is

$$\mathcal{L}_{\text{Yuk}} = -\zeta_e [\bar{\mathbf{R}}(\varphi^\dagger \mathbf{L}) + (\bar{\mathbf{L}}\varphi)\mathbf{R}], \quad (7)$$

where  $\varphi$  is the (complex) Higgs field and the left-handed and right-handed fermions are specified as

$$\mathbf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \mathbf{R} = e_R \quad (8)$$

When the electroweak symmetry is spontaneously broken, the electron mass emerges as

$$m_e = \zeta_e v / \sqrt{2}. \quad (9)$$

The Yukawa couplings that reproduce the observed quark and lepton masses range over many orders of magnitude, from  $\zeta_e \approx 3 \times 10^{-6}$  for the electron to  $\zeta_t \approx 1$  for the top quark. Their origin is unknown.

In one sense, therefore, *all fermion masses involve physics beyond the standard model.*<sup>16</sup> We cannot be sure that finding the Higgs boson, or understanding electroweak symmetry breaking, will bring enlightenment about the fermion masses. Neutrino masses may have a different origin than the masses of the quarks and charged leptons: alone among the known fermions, the neutral neutrino can be its own antiparticle.<sup>17</sup> This fact opens the possibility of several varieties of neutrino masses.

It is worth remarking on another manifestation of the logical separation between the origin of gauge-boson masses and the origin of fermion masses. The observation that a fermion mass is different from zero ( $m_f \neq 0$ ) implies that the electroweak gauge symmetry  $SU(2)_L \otimes U(1)_Y$  is broken, but electroweak symmetry breaking is only a necessary, not a sufficient, condition for the generation of fermion mass. The separation is complete in simple

technicolor,<sup>18</sup> the theory of dynamical symmetry breaking modeled on the Bardeen–Cooper–Schrieffer theory of the superconducting phase transition.

Finally, the electroweak theory we are about to describe does not predict the mass of the Higgs boson, and there is no assurance that finding the Higgs boson will tell us the origin of this mass.

Will the discovery of the Higgs boson be stupendously important? Beyond any doubt, as the rest of these lectures will begin to show. But will it explain *the origin of all mass*? Be careful what you promise!

## 2 Antecedents of the Electroweak Theory

In *The Odd Quantum*, Sam Treiman<sup>19</sup> quotes from the 1898–99 University of Chicago catalogue: “While it is never safe to affirm that the future of the Physical Sciences has no marvels in store even more astonishing than those of the past, it seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice . . . . An eminent physicist has remarked that the future truths of Physical Science are to be looked for in the sixth place of decimals.” These confident words were written, we now know, just as the classical world of determinism and uncuttable atoms and continuous distributions of energy was beginning to come apart. Much crucial progress did come from precise measurements—not always in the sixth place of the decimals, but precise nonetheless. At the same time, the century we are leaving has repeatedly shown that Nature’s marvels are not limited by our imagination, and that exploration can yield surprises that completely change the way we think.

Before we leap into a discussion of the modern electroweak theory, it will be useful to spend a few moments recalling the soil in which the electroweak theory grew. We shall not attempt anything resembling a full intellectual history,<sup>20,21,22,23</sup> but only hit a few of the high spots.

### 2.1 Radioactivity, $\beta$ decay, and the neutrino

Becquerel’s discovery of radioactivity in 1896 is one of the wellsprings of modern physics. In a short time, physicists learned to distinguish several sorts of radioactivity, classified by Rutherford according to the energetic projectile emitted in the spontaneous disintegration. Natural and artificial radioactivity includes nuclear  $\beta$  decay, observed as

$${}^A_Z \rightarrow {}^A_{(Z+1)} + \beta^- , \quad (10)$$

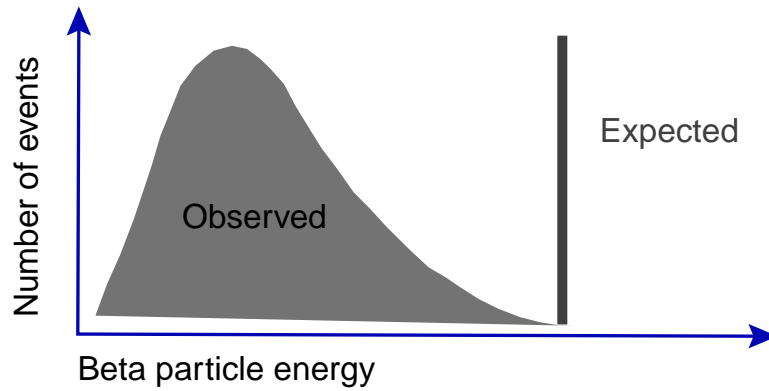


Figure 7. Expectations and reality for the beta decay spectrum.

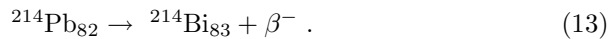
where  $\beta^-$  is Rutherford's name for what was soon identified as the electron and  ${}^AZ$  stands for the nucleus with  $Z$  protons and  $A - Z$  neutrons. Examples<sup>i</sup> are tritium  $\beta$  decay,



neutron  $\beta$  decay,



and  $\beta$  decay of Lead-214,



For two-body decays, the Principle of Conservation of Energy & Momentum says that the  $\beta$  particle should have a definite energy, indicated by the spike in Figure 7. What was observed was very different: in 1914, James Chadwick (later to discover the neutron) showed conclusively<sup>24</sup> that in the decay of Radium B and C ( ${}^{214}\text{Pb}$  and  ${}^{214}\text{Bi}$ ), the  $\beta$  energy follows a continuous spectrum, as shown in Figure 7.

<sup>i</sup>It is a curious fact that  $\beta^+$ -emitters,  ${}^AZ \rightarrow {}^A(Z-1) + \beta^+$ , are rare among the naturally occurring isotopes. The first example, radio-phosphorus produced in  $\alpha\text{Al}$  collisions, was found by Irène and Frédéric Joliot-Curie in 1934, after the discovery of the positron in cosmic rays. In our time, the decay  ${}^{19}\text{Ne} \rightarrow {}^{19}\text{F} + \beta^+$  has been a favorite for the study of right-handed charged currents and time reversal invariance. And, of course, the positron emitters form the technological basis for positron-emission tomography.

What could be the meaning of this completely unexpected behavior? Niels Bohr was willing to consider the possibility that energy and momentum are not uniformly conserved in subatomic events. The  $\beta$ -decay energy crisis tormented physicists for years. On December 4, 1930, Wolfgang Pauli addressed an open letter<sup>j</sup> to a meeting on radioactivity in Tübingen. Pauli could not attend in person because his presence at a student ball in Zürich was “indispensable.” In his letter, Pauli advanced the outlandish idea of a new, very penetrating, neutral particle of vanishingly small mass. Because Pauli’s new particle interacted very feebly with matter, it would escape undetected from any known apparatus, taking with it some energy, which would seemingly be lost. The balance of energy and momentum would be restored by the particle we now know as the electron’s antineutrino. The proper scheme for beta decay is thus

$${}^A_Z X \rightarrow {}^A_{(Z+1)} Y + \beta^- + \bar{\nu}. \quad (14)$$

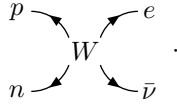
Pauli’s new particle was indeed a “desperate remedy,” but it was, in its way, very conservative, for it preserved the principle of energy and momentum conservation and with it the notion that the laws of physics are invariant under translations in space and time. The hypothesis fit the facts.<sup>k</sup> After Chadwick’s discovery of the neutron in 1932, Fermi named Pauli’s hypothetical particle the neutrino, to distinguish it from the neutron, and constructed his four-fermion theory of the weak interaction. Experimental confirmation of Pauli’s neutrino had to wait for dramatic advances in technology.<sup>l</sup>

<sup>j</sup>Pauli’s letter (in the original German) is reproduced in Ref. <sup>25</sup>. For an English translation, see pp. 127-8 of Ref. <sup>20</sup>. It begins, “Dear Radioactive Ladies and Gentlemen, I have hit upon a desperate remedy regarding . . . the continuous  $\beta$ -spectrum . . .” Pauli concluded, “For the moment I dare not publish anything about this idea and address myself confidentially first to you. . . I admit that my way out may seem rather improbable *a priori* . . . Nevertheless, if you don’t play you can’t win . . . Therefore, Dear Radioactives, test and judge.” Pauli’s neutrino, together with the discovery of the neutron, also resolved a vexing nuclear spin-and-statistic problem.

<sup>k</sup>As you TASI students continue in physics, you will be amazed and delighted to find how quickly we learn—or how little we knew just a short time ago. It is easy to assume that anything we read in textbooks has been known forever, so it is sometimes stunning to learn that something we take for granted actually had to be discovered. An example that has both scientific and touristic interest for students in Boulder is Jack Steinberger’s discovery on Mount Evans of the continuous electron spectrum in muon decay.<sup>26</sup>

<sup>l</sup>Detecting a particle as penetrating as the neutrino required a large target and a copious source of neutrinos. In 1953, Clyde Cowan and Fred Reines<sup>27</sup> used the intense beam of antineutrinos from a fission reactor  ${}^A_Z X \rightarrow {}^A_{(Z+1)} Y + \beta^- + \bar{\nu}$ , and a heavy target (10.7 ft<sup>3</sup> of liquid scintillator) containing about  $10^{28}$  protons to detect the reaction  $\bar{\nu} + p \rightarrow e^+ + n$ . Initial runs at the Hanford Engineering Works were suggestive but inconclusive. Moving their apparatus to the stronger fission neutrino source at the Savannah River nuclear plant,

We can now recognize  $\beta$  decay as the first hint for *flavor*, the theme of this summer school. Indeed, neutron beta decay is the prototype charged-current, flavor-changing interaction:



## 2.2 The neutron and flavor symmetry

The discovery of the neutron made manifest the case for flavor, with two species of nucleon nearly degenerate in mass:

$$\begin{aligned} M(n) &= 939.565\,63 \pm 0.000\,28 \text{ MeV}/c^2 \\ M(p) &= 938.272\,31 \pm 0.000\,28 \text{ MeV}/c^2 \\ \Delta M &= 1.293318 \pm 0.000\,009 \text{ MeV}/c^2 \end{aligned} \quad (15)$$

so that  $\Delta M/M \approx 1.4 \times 10^{-3}$ . The similarity of the neutron and proton masses makes it plausible to inquire into the charge independence of nuclear forces. The two-nucleon system is not particularly informative: among  $NN$  states,  $pp$  and  $nn$  are unbound, and only the isocalar  $np$  state, the deuteron, is very lightly bound. Hints for the charge independence of nuclear forces come from many light nuclei. We may compare the binding energy of

$$\begin{aligned} {}^3\text{H}(ppn) &= 8.481\,855 \pm 0.000\,013 \text{ MeV} \\ {}^3\text{He}(pnn) &= 7.718\,109 \pm 0.000\,013 \text{ MeV} \\ \Delta(\text{B.E.}) &= 0.763\,46 \text{ MeV} . \end{aligned} \quad (16)$$

The difference in binding energy is very close to a primitive estimate for the Coulomb repulsion in  ${}^3\text{He}$ : taking the measured charge radius of  $r = 1.97 \pm 0.015$  fm, we estimate the Coulomb energy as  $\alpha/r \approx 0.731$  MeV.

More detailed evidence that nuclear forces are the same for protons and neutrons come from the level structures in mirror nuclei. I show in Figures 8 and 9 the kinship between the  ${}^7\text{Li}(4p + 3n)$  and  ${}^7\text{Be}(3p + 4n)$  level schemes, and between the  ${}^{11}\text{B}(5p + 6n)$  and  ${}^{11}\text{C}(6p + 5n)$  level schemes. In both cases, isospin  $I = 3/2$  isobaric analogue levels are present in the  ${}^7\text{He}(2p + 5n)$  and  ${}^7\text{B}(5p + 2n)$  ground states, and the  ${}^{11}\text{Be}(4p + 7n)$  and  ${}^{11}\text{N}(7p + 4n)$  ground states.

---

Cowan and Reines and their team made the definitive observation of inverse  $\beta$  decay in 1956.<sup>28</sup>

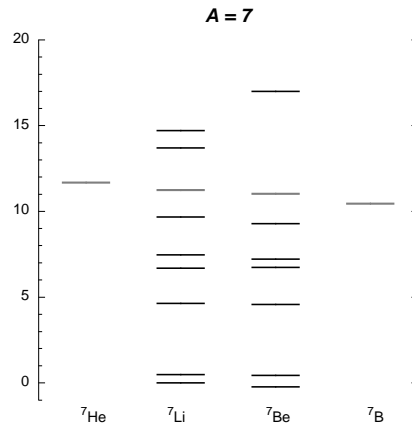


Figure 8. Simplified isobar diagram for the  $A = 7$  nuclei. The presumed  $I = 3/2$  isobaric analogue levels are shown in grey. Following the usual practice, the diagrams for individual isobars are shifted vertically to eliminate the  $n-p$  mass difference and the Coulomb energy. Data taken from Ref. <sup>29</sup>.

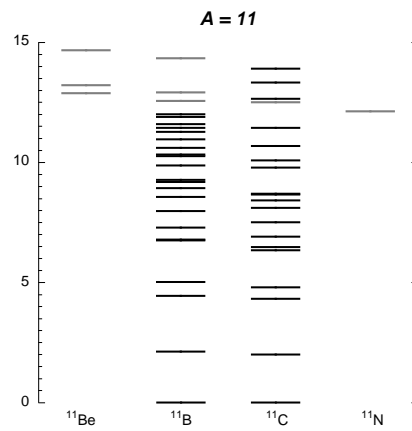


Figure 9. Simplified isobar diagram for the  $A = 11$  nuclei. The presumed  $I = 3/2$  isobaric analogue levels are shown in grey. Following the usual practice, the diagrams for individual isobars are shifted vertically to eliminate the  $n-p$  mass difference and the Coulomb energy. Data taken from Ref. <sup>30</sup>.

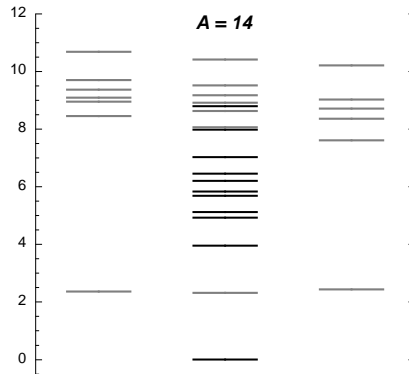


Figure 10. Simplified isobar diagram for the  $A = 14$  nuclei. The presumed  $I = 1$  isobaric analogue levels are shown in grey. Following the usual practice, the diagrams for individual isobars are shifted vertically to eliminate the  $n$ - $p$  mass difference and the Coulomb energy. Data taken from Ref. <sup>31</sup>.

Extremely compelling evidence for the charge independence of nuclear forces comes from the systematic study of two-nucleon states in the  $A = 14$  nuclei, which consist of two nucleons outside a closed core:

$$\begin{aligned}
 {}^{14}\text{O} &: {}^{12}\text{C} + (pp) \quad I_3 = +1 \\
 {}^{14}\text{N} &: {}^{12}\text{C} + (pn) \quad I_3 = 0 \\
 {}^{14}\text{C} &: {}^{12}\text{C} + (nn) \quad I_3 = -1
 \end{aligned}
 \tag{17}$$

We see in Figure 10 that the  $I = 1$  levels are common to all three elements. Nitrogen-14 has many additional  $I = 0$  levels.

The charge independence of nuclear forces led to the first flavor symmetry, isospin invariance. We regard the proton and neutron as two states of the nucleon doublet,

$$\begin{pmatrix} p \\ n \end{pmatrix},$$

and consider the nuclear interaction to be invariant under rotations in isospin space. In the absence of electromagnetism, which supplies the distinguishing information that the proton carries an electric charge but the neutron does not, it is *a matter of convention* which state—or which superposition of states—we name the proton or neutron.<sup>m</sup> Once we have the notion of

<sup>m</sup>If we lived in a world without electromagnetism, how could you determine that there are,

isospin invariance in mind, it is a small step to the idea of weak isospin, which we infer from the fact that the charged-current weak interaction transforms one member of the nucleon doublet into another. The notion of weak-isospin families is a cornerstone of our modern electroweak theory.

### 2.3 Parity violation in weak decays

A series of observations and analyses through the 1950s led to the suggestion that the weak interactions did not respect reflection symmetry, or parity. In 1956, C. S. Wu and collaborators detected a correlation between the spin vector  $\vec{J}$  of a polarized  $^{60}\text{Co}$  nucleus and the direction  $\hat{p}_e$  of the outgoing  $\beta$  particle.<sup>32</sup> Since parity inversion leaves spin, an axial vector, unchanged:

$$\mathcal{P} : \vec{J} \rightarrow \vec{J}, \quad (18)$$

while reversing the electron direction:

$$\mathcal{P} : \hat{p}_e \rightarrow -\hat{p}_e, \quad (19)$$

the correlation  $\vec{J} \cdot \hat{p}_e$  is *parity violating*. Detailed analysis of the  $^{60}\text{Co}$  result and others that came out in quick succession established that the charged-current weak interactions are left-handed. Since parity links a left-handed neutrino with a right-handed neutrino,

$$\nu_L \xleftrightarrow{\leftarrow} \mathcal{P} \xleftrightarrow{\leftarrow} \cancel{\nu_R}$$

we build a manifestly parity-violating theory with only  $\nu_L$ .

How can we establish that the known neutrino is left-handed? The simplest experiment to describe—though not the earliest to measure neutrino helicity—is to measure the helicity of the outgoing  $\mu^+$  in the decay of a spin-zero  $\pi^+ \rightarrow \mu^+ \nu_\mu$ .<sup>33,34</sup>

$$\nu_\mu \xleftrightarrow{\Rightarrow} (\pi^+) \xleftrightarrow{\Leftarrow} \mu^+$$

By angular momentum conservation, the spin projections of the muon and neutrino must sum to zero, so the helicity of the neutrino is equal to that of the muon:  $h(\nu_\mu) = h(\mu^+)$ . Note that because the massless neutrino must be left-handed, the  $\mu^+$  is forced to have the “wrong helicity” in pion decay: the antilepton  $\mu^+$  is naturally right-handed, and can only have a left-handed helicity because it is massive. This “helicity suppression” inhibits the decay  $\pi^+ \rightarrow \mu^+ \nu_\mu$ , and it inhibits the analogue decay  $\pi^+ \rightarrow e^+ \nu_e$  still more. The

---

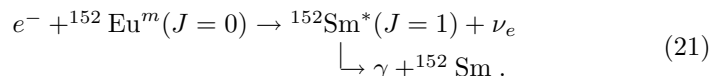
in fact, two species of nucleons?



decay amplitude in each case is proportional to the charged-lepton mass, and this accounts for the dramatic ratio

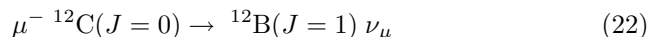
$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = 1.23 \times 10^{-4} . \quad (20)$$

The classic determination of the electron neutrino's helicity was made by M. Goldhaber and collaborators, who inferred  $h(\nu_e)$  from the longitudinal polarization of the recoil nucleus in the electron-capture reaction<sup>35</sup>



Obviously, this achievement required not only impressive experimental technique, but also a remarkable knowledge of the characteristics of nuclear levels!

Recently a group in Zürich has emulated the original achievement in a muon-capture reaction,



to determine  $h(\nu_\mu)$  by angular momentum conservation.<sup>36</sup> See the *Review of Particle Physics* for the most recent determinations of  $h(\nu_\tau)$  in tau decays.<sup>9</sup>

The fact that only left-handed neutrinos and right-handed antineutrinos are observed also means that charge-conjugation invariance is violated in the weak interactions. Charge conjugation takes a  $\nu_L$  into a nonexistent  $\bar{\nu}_L$ :

$$\nu_L \xrightarrow{\mathcal{C}} \bar{\nu}_L \quad \mathcal{C} \xrightarrow{\nu_L} \bar{\nu}_L$$

The consequence of the  $\mathcal{C}$  violation is very dramatic for muon decay, as the decay angular distributions of the outgoing  $e^\pm$  in  $\mu^\pm$  decay are reversed,

$$\frac{dN(\mu^\pm \rightarrow e^\pm + \dots)}{dx dz} = x^2(3-2x) \left[ 1 \pm z \frac{(2x-1)}{(3-2x)} \right] , \quad (23)$$

where  $x \equiv p_e/p_e^{\text{max}}$  and  $z \equiv \hat{s}_\mu \cdot \hat{p}_e$ . The positron follows the spin direction of the  $\mu^+$ , but the electron avoids the spin direction of the  $\mu^-$ .

#### 2.4 An effective Lagrangian for the weak interactions

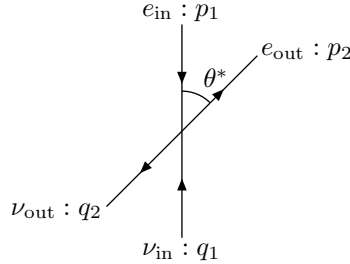
After the observation of maximal parity violation in the late 1950s, a serviceable effective Lagrangian for the weak interactions of electrons and neutrinos could be written as the product of charged leptonic currents,

$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) e \bar{e} \gamma^\mu (1 - \gamma_5) \nu + \text{h.c.} , \quad (24)$$

where Fermi's coupling constant is  $G_F = 1.16632 \times 10^{-5} \text{ GeV}^{-2}$ . This is often called the  $V - A$  (vector minus axial vector) interaction. It is straightforward to compute the Feynman amplitude for antineutrino-electron scattering,<sup>n</sup>

$$\mathcal{M} = -\frac{iG_F}{\sqrt{2}} \bar{v}(\nu, q_1) \gamma_\mu (1 - \gamma_5) u(e, p_1) \bar{u}(e, p_2) \gamma^\mu (1 - \gamma_5) v(\nu, q_2), \quad (25)$$

where the c.m. kinematical definitions are indicated in the sketch.



The differential cross section is related to the absolute square of the amplitude, averaged over initial spins and summed over final spins. It is

$$\frac{d\sigma_{V-A}(\bar{\nu}e \rightarrow \bar{\nu}e)}{d\Omega_{\text{cm}}} = \frac{|\mathcal{M}|^2}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu (1-z)^2}{16\pi^2}, \quad (26)$$

where  $z = \cos\theta^*$ . The total cross section is simply

$$\sigma_{V-A}(\bar{\nu}e \rightarrow \bar{\nu}e) = \frac{G_F^2 \cdot 2mE_\nu}{3\pi} \approx 0.574 \times 10^{-41} \text{ cm}^2 \left( \frac{E_\nu}{1 \text{ GeV}} \right); \quad (27)$$

it is *small!* for small energies.

Repeating the calculation for neutrino-electron scattering, we find

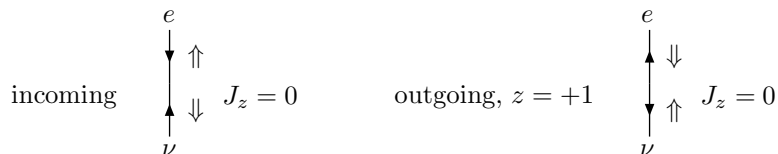
$$\frac{d\sigma_{V-A}(\nu e \rightarrow \nu e)}{d\Omega_{\text{cm}}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}, \quad (28)$$

and

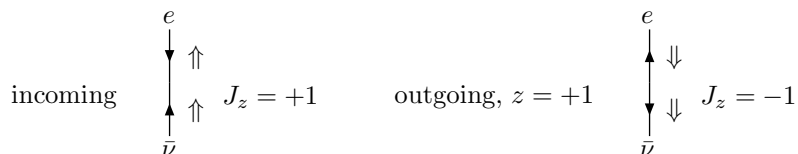
$$\sigma_{V-A}(\nu e \rightarrow \nu e) = \frac{G_F^2 \cdot 2mE_\nu}{\pi} \approx 1.72 \times 10^{-41} \text{ cm}^2 \left( \frac{E_\nu}{1 \text{ GeV}} \right). \quad (29)$$

It is interesting to trace the origin of the factor-of-three difference between the  $\nu e$  and  $\bar{\nu}e$  cross sections, which arises from the left-handed nature of the charged current. In neutrino-electron scattering, the initial state has spin projection  $J_z = 0$ , because the incoming neutrino and electron are both left-handed. They can emerge in any direction—in particular, in the backward direction denoted by  $z = +1$ —and still satisfy the constraint that  $J_z = 0$ .

<sup>n</sup>See Ref. <sup>37</sup> for conventions and tricks for calculating amplitudes.



In antineutrino-electron scattering, the situation is different, because the antineutrino is *right-handed*. The initial angular momentum has spin projection  $J_z = 1$ ; for backward scattering, the outgoing electron and antineutrino combine to give  $J_z = -1$ , so scattering at  $z = +1$  is forbidden by angular momentum conservation.



## 2.5 Lepton families and universality

The muon is distinct from the electron; what is the nature of the neutrino emitted in pion decay,  $\pi^+ \rightarrow \mu^+ \nu$ ? In 1962, Lederman, Schwartz, Steinberger, and collaborators carried out a *two-neutrino experiment* using neutrinos created in the decay of high-energy pions from the new Alternating Gradient Synchrotron at Brookhaven.<sup>38</sup> They observed numerous examples of the reaction  $\nu N \rightarrow \mu + X$ , but found no evidence for the production of electrons. Their study established that the muon produced in pion decay is a distinct particle,  $\nu_\mu$ , that is different from either  $\nu_e$  or  $\bar{\nu}_e$ . This observation suggests that the weak (charged-current) interactions of the leptons display a family structure,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L . \quad (30)$$

We are led to generalize the effective Lagrangian (24) to include the terms

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e + \text{h.c.} , \quad (31)$$

in the familiar current-current form. With this interaction, we easily compute the muon decay rate as

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192 \pi^3} . \quad (32)$$

With the value of the Fermi constant inferred from  $\beta$  decay, (32) accounts for the 2.2- $\mu$ s lifetime of the muon.

The resulting cross section for inverse muon decay,

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \sigma_{V-A}(\nu_e e \rightarrow \nu_e e) \left[ 1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu} \right]^2, \quad (33)$$

is in good agreement with high-energy data in measurements up to  $E_\nu \approx 600$  GeV. However, partial-wave unitarity constrains the modulus of an inelastic amplitude to be  $|\mathcal{M}_J| < 1$ . According to the  $V - A$  theory, the  $J = 0$  partial-wave amplitude is

$$\mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi\sqrt{2}} \left[ 1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu} \right], \quad (34)$$

which satisfies the unitarity constraint for  $E_\nu < \pi/G_F m_e \sqrt{2} \approx 3.7 \times 10^8$  GeV. These conditions aren't threatened anytime soon at an accelerator laboratory (though they do occur in interactions of cosmic neutrinos). Nevertheless, we encounter here an important point of principle: although the  $V - A$  theory may be a reliable guide over a broad range of energies, the theory cannot be complete: physics must change before we reach a c.m. energy  $\sqrt{s} \approx 600$  GeV.

A few weeks after my TASI00 lectures, members of the DONUT (Direct Observation of NU Tau) experiment at Fermilab announced the first observation of charged-current interactions of the tau neutrino in a hybrid-emulsion detector situated in a “prompt” neutrino beam.<sup>39</sup> The  $\nu_\tau$  beam was created in the production and decay of the charmed-strange meson

$$\begin{array}{l} D_s^+ \rightarrow \tau^+ \nu_\tau \\ \quad \quad \quad \downarrow \\ \quad \quad \quad \bar{\nu}_\tau + \text{anything.} \end{array} \quad (35)$$

Their “three-neutrino” experiment was modeled on the two-neutrino classic: a beam of neutral, penetrating particles (the tau neutrinos) interacted in the hybrid target to produce tau leptons through the reaction

$$\nu_\tau N \rightarrow \tau + \text{anything.} \quad (36)$$

Although extensive studies of  $\tau$  decays had given us a rather complete portrait of the interactions of  $\nu_\tau$ , the observation of the last of the known standard-model fermions gives a nice sense of closure, as well as a very impressive demonstration of the experimenter's art.

We have a great deal of precise information about the properties of the leptons, because the leptons are free particles readily studied in isolation. All of them are spin- $\frac{1}{2}$ , pointlike particles—down to a resolution of a few  $\times 10^{-17}$  cm.

Table 1. Some properties of the leptons.

Lepton	Mass	Lifetime
$e^-$	$0.510\,999\,07 \pm 0.000\,000\,15 \text{ MeV}/c^2$	$> 4.3 \times 10^{23} \text{ y (68\% CL)}$
$\nu_e$	$< 10 - 15 \text{ eV}/c^2$	
$\mu^-$	$105.658\,389 \pm 0.000\,034 \text{ MeV}/c^2$	$2.197\,03 \pm 0.000\,04 \times 10^{-6} \text{ s}$
$\nu_\mu$	$< 0.19 \text{ MeV}/c^2 \text{ (90\% CL)}$	
$\tau^-$	$1777.06^{+0.29}_{-0.26} \text{ MeV}/c^2$	$290.2 \pm 1.2 \times 10^{-15} \text{ s}$
$\nu_\tau$	$< 18.2 \text{ MeV}/c^2 \text{ (95\% CL)}$	

The kinematically determined neutrino masses are all consistent with zero, though the evidence for neutrino oscillations argues that the neutrinos must have nonzero masses. A brief digest of lepton properties is given in Table 1.

An important characteristic of the charged-current weak interactions is their *universal strength*, which has been established in great detail. We'll content ourselves here with the most obvious check for the lepton sector. Using the generic formula (32) for muon decay, we can use the measured lifetime of the muon to estimate the Fermi constant determined in muon decay as

$$G_\mu = \left( \frac{192\pi^3\hbar}{\tau_\mu m_\mu^5} \right)^{\frac{1}{2}} = 1.1638 \times 10^{-5} \text{ GeV}^{-2}. \quad (37)$$

Similarly, we can evaluate the Fermi constant from the tau lifetime, taking into account the measured branching fraction for the leptonic decay. We find

$$G_\tau = \left( \frac{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \rightarrow \text{all})} \cdot \frac{192\pi^3\hbar}{\tau_\tau m_\tau^5} \right)^{\frac{1}{2}} = 1.1642 \times 10^{-5} \text{ GeV}^{-2}. \quad (38)$$

Both are in excellent agreement with the best value of the Fermi constant determined from nuclear  $\beta$  decay,<sup>o</sup>

$$G_\beta = 1.16639(2) \times 10^{-5} \text{ GeV}^{-2}. \quad (39)$$

The overall conclusion is that the charged currents acting in the leptonic and semileptonic interactions are of universal strength; we take this to imply a universality of the current-current form, or whatever lies behind it.

<sup>o</sup>In this discussion, but not in the number quoted, I'm glossing over the complication that the strangeness-preserving transition is not quite full (universal) strength. We'll encounter "Cabibbo universality" in §3.5.

### 3 The $SU(2)_L \otimes U(1)_Y$ Electroweak Theory

Let us review the essential elements of the  $SU(2)_L \otimes U(1)_Y$  electroweak theory.<sup>37,40,41</sup> The electroweak theory takes three crucial clues from experiment:

- The existence of left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- The universal strength of the (charged-current) weak interactions;
- The idealization that neutrinos are massless.

#### 3.1 A theory of leptons

To save writing, we shall construct the electroweak theory as it applies to a single generation of leptons. In this form, it is neither complete nor consistent: anomaly cancellation requires that a doublet of color-triplet quarks accompany each doublet of color-singlet leptons. However, the needed generalizations are simple enough to make that we need not write them out.

To incorporate electromagnetism into a theory of the weak interactions, we add to the  $SU(2)_L$  family symmetry suggested by the first two experimental clues a  $U(1)_Y$  weak-hypercharge phase symmetry.<sup>p</sup> We begin by specifying the fermions: a left-handed weak isospin doublet

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \tag{40}$$

with weak hypercharge  $Y_L = -1$ , and a right-handed weak isospin singlet

$$R \equiv e_R \tag{41}$$

with weak hypercharge  $Y_R = -2$ .

The electroweak gauge group,  $SU(2)_L \otimes U(1)_Y$ , implies two sets of gauge fields: a weak isovector  $\vec{b}_\mu$ , with coupling constant  $g$ , and a weak isoscalar

---

<sup>p</sup>We define the weak hypercharge  $Y$  through the Gell-Mann–Nishijima connection,  $Q = I_3 + \frac{1}{2}Y$ , to electric charge and (weak) isospin.

$\mathcal{A}_\mu$ , with coupling constant  $g'$ . Corresponding to these gauge fields are the field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g\varepsilon_{jkl} b_\mu^j b_\nu^k, \quad (42)$$

for the weak-isospin symmetry, and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu, \quad (43)$$

for the weak-hypercharge symmetry. We may summarize the interactions by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}, \quad (44)$$

with

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^\ell F^{\ell\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}, \quad (45)$$

and

$$\begin{aligned} \mathcal{L}_{\text{leptons}} = & \bar{\mathbf{R}} i\gamma^\mu \left( \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y \right) \mathbf{R} \\ & + \bar{\mathbf{L}} i\gamma^\mu \left( \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu \right) \mathbf{L}. \end{aligned} \quad (46)$$

The  $SU(2)_L \otimes U(1)_Y$  gauge symmetry forbids a mass term for the electron in the matter piece (46). Moreover, the theory we have described contains four massless electroweak gauge bosons, namely  $\mathcal{A}_\mu$ ,  $b_\mu^1$ ,  $b_\mu^2$ , and  $b_\mu^3$ , whereas Nature has but one: the photon. To give masses to the gauge bosons and constituent fermions, we must hide the electroweak symmetry.

The most apt analogy for the hiding of the electroweak gauge symmetry is found in superconductivity. In the Ginzburg-Landau description<sup>42</sup> of the superconducting phase transition, a superconducting material is regarded as a collection of two kinds of charge carriers: normal, resistive carriers, and superconducting, resistanceless carriers.

In the absence of a magnetic field, the free energy of the superconductor is related to the free energy in the normal state through

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4, \quad (47)$$

where  $\alpha$  and  $\beta$  are phenomenological parameters and  $|\psi|^2$  is an order parameter that measures the density of superconducting charge carriers. The parameter  $\beta$  is non-negative, so that the free energy is bounded from below.

Above the critical temperature for the onset of superconductivity, the parameter  $\alpha$  is positive and the free energy of the substance is supposed

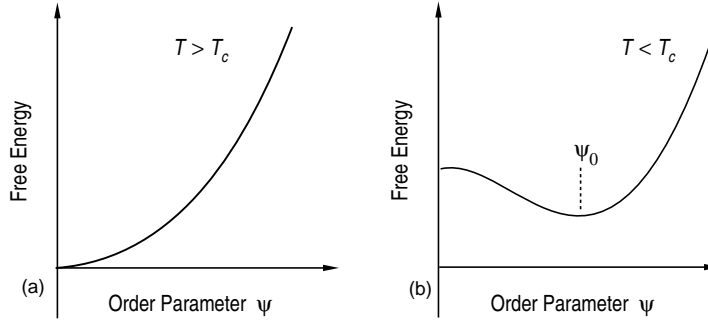


Figure 11. Ginzburg-Landau description of the superconducting phase transition.

to be an increasing function of the density of superconducting carriers, as shown in Figure 11(a). The state of minimum energy, the vacuum state, then corresponds to a purely resistive flow, with no superconducting carriers active. Below the critical temperature, the parameter  $\alpha$  becomes negative and the free energy is minimized when  $\psi = \psi_0 = \sqrt{-\alpha/\beta} \neq 0$ , as illustrated in Figure 11(b).

This is a nice cartoon description of the superconducting phase transition, but there is more. In an applied magnetic field  $\vec{H}$ , the free energy is

$$G_{\text{super}}(\vec{H}) = G_{\text{super}}(0) + \frac{\vec{H}^2}{8\pi} + \frac{1}{2m^*} | -i\hbar\nabla\psi - (e^*/c)\vec{A}\psi |^2 , \quad (48)$$

where  $e^*$  and  $m^*$  are the charge ( $-2$  units) and effective mass of the superconducting carriers. In a weak, slowly varying field  $\vec{H} \approx 0$ , when we can approximate  $\psi \approx \psi_0$  and  $\nabla\psi \approx 0$ , the usual variational analysis leads to the equation of motion,

$$\nabla^2 \vec{A} - \frac{4\pi e^*}{m^* c^2} |\psi_0|^2 \vec{A} = 0 , \quad (49)$$

the wave equation of a massive photon. In other words, the photon acquires a mass within the superconductor. This is the origin of the Meissner effect, the exclusion of a magnetic field from a superconductor. More to the point for our purposes, it shows how a symmetry-hiding phase transition can lead to a massive gauge boson.

To give masses to the intermediate bosons of the weak interaction, we take advantage of a relativistic generalization of the Ginzburg-Landau phase



transition known as the Higgs mechanism.<sup>43</sup> We introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (50)$$

with weak hypercharge  $Y_\phi = +1$ . Next, we add to the Lagrangian new (gauge-invariant) terms for the interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi), \quad (51)$$

where the gauge-covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu, \quad (52)$$

and the potential interaction has the form

$$V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|(\phi^\dagger \phi)^2. \quad (53)$$

We are also free to add a Yukawa interaction between the scalar fields and the leptons,

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e [\bar{\mathbf{R}}(\phi^\dagger \mathbf{L}) + (\bar{\mathbf{L}}\phi)\mathbf{R}]. \quad (54)$$

We then arrange their self-interactions so that the vacuum state corresponds to a broken-symmetry solution. The electroweak symmetry is spontaneously broken if the parameter  $\mu^2 < 0$ . The minimum energy, or vacuum state, may then be chosen to correspond to the vacuum expectation value

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad (55)$$

where  $v = \sqrt{-\mu^2/|\lambda|}$ .

Let us verify that the vacuum (55) indeed breaks the gauge symmetry. The vacuum state  $\langle \phi \rangle_0$  is invariant under a symmetry operation  $\exp(i\alpha\mathcal{G})$  corresponding to the generator  $\mathcal{G}$  provided that  $\exp(i\alpha\mathcal{G})\langle \phi \rangle_0 = \langle \phi \rangle_0$ , *i.e.*, if  $\mathcal{G}\langle \phi \rangle_0 = 0$ . We easily compute that

$$\begin{aligned} \tau_1 \langle \phi \rangle_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!} \\ \tau_2 \langle \phi \rangle_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!} \\ \tau_3 \langle \phi \rangle_0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!} \\ Y \langle \phi \rangle_0 &= Y_\phi \langle \phi \rangle_0 = +1 \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!} \end{aligned} \quad (56)$$

However, if we examine the effect of the electric charge operator  $Q$  on the (electrically neutral) vacuum state, we find that

$$\begin{aligned} Q\langle\phi\rangle_0 &= \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 = \frac{1}{2} \begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix} \langle\phi\rangle_0 \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{unbroken!} \end{aligned} \quad (57)$$

The original four generators are all broken, but electric charge is not. It appears that we have accomplished our goal of breaking  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$ . We expect the photon to remain massless, and expect the gauge bosons that correspond to the generators  $\tau_1$ ,  $\tau_2$ , and  $\kappa \equiv \frac{1}{2}(\tau_3 - Y)$  to acquire masses.

As a result of spontaneous symmetry breaking, the weak bosons acquire masses, as auxiliary scalars assume the role of the third (longitudinal) degrees of freedom of what had been massless gauge bosons. Specifically, the mediator of the charged-current weak interaction,  $W^\pm = (b_1 \mp ib_2)/\sqrt{2}$ , acquires a mass characterized by

$$M_W = \frac{gv}{2}. \quad (58)$$

With the definition  $g' = g \tan \theta_W$ , where  $\theta_W$  is the weak mixing angle, the mediator of the neutral-current weak interaction,  $Z = b_3 \cos \theta_W - \mathcal{A} \sin \theta_W$ , acquires a mass characterized by  $M_Z^2 = M_W^2 / \cos^2 \theta_W$ . After spontaneous symmetry breaking, there remains an unbroken  $U(1)_{\text{em}}$  phase symmetry, so that electromagnetism, a vector interaction, is mediated by a massless photon,  $A = \mathcal{A} \cos \theta_W + b_3 \sin \theta_W$ , coupled to the electric charge  $e = gg' / \sqrt{g^2 + g'^2}$ . As a vestige of the spontaneous breaking of the symmetry, there remains a massive, spin-zero particle, the Higgs boson. The mass of the Higgs scalar is given symbolically as  $M_H^2 = -2\mu^2 > 0$ , but we have no prediction for its value. Though what we take to be the work of the Higgs boson is all around us, the Higgs particle itself has not yet been observed.

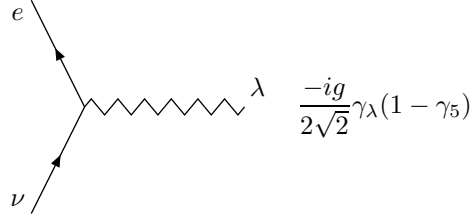
The fermions (the electron in our abbreviated treatment) acquire masses as well; these are determined not only by the scale of electroweak symmetry breaking,  $v$ , but also by their Yukawa interactions with the scalars. The mass of the electron is set by the dimensionless coupling constant  $\zeta_e = m_e \sqrt{2}/v$ , which is—so far as we now know—arbitrary.

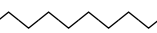
### 3.2 The $W$ boson

The interactions of the  $W$ -boson with the leptons are given by

$$\mathcal{L}_{W\text{-lep}} = \frac{-g}{2\sqrt{2}} [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e W_\mu^+ + \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e W_\mu^-] , \text{ etc.}, \quad (59)$$

so the Feynman rule for the  $\nu_e e W$  vertex is



The  $W$ -boson propagator is   $= \frac{-i(g_{\mu\nu} - k_\mu k_\nu / M_W^2)}{k^2 - M_W^2}$ .

Let us compute the cross section for inverse muon decay in the new theory. We find

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu / M_W^2)}, \quad (60)$$

which coincides with the four-fermion result (33) at low energies, provided we identify

$$\frac{g^4}{16M_W^2} = 2G_F^2, \quad (61)$$

which implies that

$$\frac{g}{2\sqrt{2}} = \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}}. \quad (62)$$

With the aid of (58) for the  $W$ -boson mass, we determine the numerical value,

$$v = \left( G_F \sqrt{2} \right)^{-\frac{1}{2}} \approx 246 \text{ GeV}. \quad (63)$$

The high-energy limit of the cross section (60) is

$$\lim_{E_\nu \rightarrow \infty} \sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F M_W^2}{\pi}, \quad (64)$$

independent of energy. The benign high-energy behavior means that partial-wave unitarity is now respected for

$$s < M_W^2 \left[ \exp \left( \frac{\pi\sqrt{2}}{G_F M_W^2} \right) - 1 \right], \quad (65)$$

an immense improvement over the four-fermion theory.

Let us now investigate the properties of the  $W$ -boson in terms of its mass,  $M_W$ . Consider first the leptonic disintegration of the  $W^-$ , with decay kinematics specified thus:

$$\begin{array}{c} \uparrow e(p) \quad p \approx \left( \frac{M_W}{2}; \frac{M_W \sin \theta}{2}, 0, \frac{M_W \cos \theta}{2} \right) \\ | \\ W^- \bullet \\ | \\ \downarrow \bar{\nu}_e(q) \quad q \approx \left( \frac{M_W}{2}; -\frac{M_W \sin \theta}{2}, 0, -\frac{M_W \cos \theta}{2} \right) \end{array}$$

The Feynman amplitude for the decay is

$$\mathcal{M} = -i \left( \frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e, p) \gamma_\mu (1 - \gamma_5) v(\nu, q) \varepsilon^\mu, \quad (66)$$

where  $\varepsilon^\mu = (0; \hat{\varepsilon})$  is the polarization vector of the  $W$ -boson in its rest frame. The square of the amplitude is

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}} \text{tr} [\not{\varepsilon} (1 - \gamma_5) \not{q} (1 + \gamma_5) \not{\varepsilon}^* \not{p}] \\ &= \frac{8G_F M_W^2}{\sqrt{2}} [\varepsilon \cdot q \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* q \cdot p + \varepsilon \cdot p \varepsilon^* \cdot q + i \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]. \end{aligned} \quad (67)$$

The *decay rate* is independent of the  $W$  polarization, so let us look first at the case of longitudinal polarization  $\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{*\mu}$ , to eliminate the last term. For this case, we find

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}} \sin^2 \theta, \quad (68)$$

so the differential decay rate is

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2} \frac{\mathcal{S}_{12}}{M_W^3}, \quad (69)$$

where  $S_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$ , so that

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2 \sqrt{2}} \sin^2 \theta, \quad (70)$$

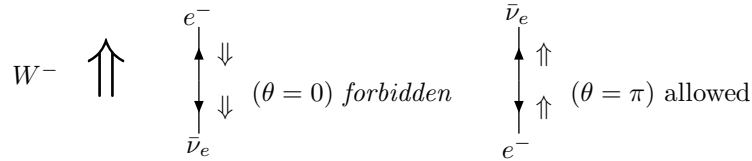
and

$$\Gamma(W \rightarrow e\nu) = \frac{G_F M_W^3}{6\pi \sqrt{2}}. \quad (71)$$

For the other helicities,  $\varepsilon_{\pm 1}^\mu = (0; -1, \mp i, 0)/\sqrt{2}$ , arithmetic that is only slightly more tedious leads us to

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2 \sqrt{2}} (1 \mp \cos \theta)^2. \quad (72)$$

The extinctions at  $\cos \theta = \pm 1$  are, as we have come to expect, consequences of angular momentum conservation:



The situation is reversed for the decay of  $W^+ \rightarrow e^+ \nu_e$ . Overall, the  $e^+$  follows the polarization direction of  $W^+$ , while the  $e^-$  avoids the polarization direction of  $W^-$ . This charge asymmetry was important for establishing the discovery of the  $W$ -boson in  $\bar{p}p$  ( $\bar{q}q$ ) collisions.

### 3.3 Neutral Currents

The interactions of the  $Z$ -boson with leptons are given by

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu \quad (73)$$

and

$$\mathcal{L}_{Z-e} = \frac{-g}{4 \cos \theta_W} \bar{e} [L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5)] e Z_\mu, \quad (74)$$

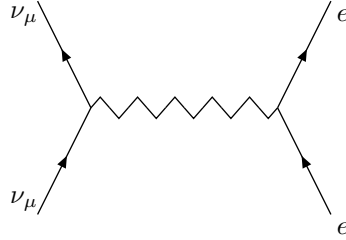
where the chiral couplings are

$$\begin{aligned} L_e &= 2 \sin^2 \theta_W - 1 = 2x_W + \tau_3, \\ R_e &= 2 \sin^2 \theta_W. \end{aligned} \quad (75)$$

By analogy with the calculation of the  $W$ -boson total width (71), we easily compute that

$$\begin{aligned}\Gamma(Z \rightarrow \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \rightarrow e^+e^-) &= \Gamma(Z \rightarrow \nu\bar{\nu}) [L_e^2 + R_e^2] .\end{aligned}\quad (76)$$

The neutral weak current mediates a reaction that did not arise in the  $V - A$  theory,  $\nu_\mu e \rightarrow \nu_\mu e$ , which proceeds entirely by  $Z$ -boson exchange:



This was, in fact, the reaction in which the first evidence for the weak neutral current was seen by the Gargamelle collaboration in 1973.<sup>44</sup> It's an easy exercise to compute all the cross sections for neutrino-electron elastic scattering. We find

$$\begin{aligned}\sigma(\nu_\mu e \rightarrow \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2 + R_e^2/3] , \\ \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2/3 + R_e^2] , \\ \sigma(\nu_e e \rightarrow \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2 + R_e^2/3] , \\ \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2/3 + R_e^2] .\end{aligned}\quad (77)$$

By measuring all the cross sections, one may undertake a “model-independent” determination<sup>q</sup> of the chiral couplings  $L_e$  and  $R_e$ , or the traditional vector and axial-vector couplings  $v$  and  $a$ , which are related through

$$\begin{aligned}a &= \frac{1}{2}(L_e - R_e) & v &= \frac{1}{2}(L_e + R_e) \\ L_e &= v + a & R_e &= v - a\end{aligned}\quad (78)$$

By inspecting (77), you can see that even after measuring all four cross sections, there remains a two-fold ambiguity: the same cross sections result

<sup>q</sup>It is model-independent within the framework of vector and axial-vector couplings only, so in the context of gauge theories.

if we interchange  $R_e \leftrightarrow -R_e$ , or, equivalently,  $v \leftrightarrow a$ . The ambiguity is resolved by measuring the forward-backward asymmetry in a reaction like  $e^+e^- \rightarrow \mu^+\mu^-$  at energies well below the  $Z^0$  mass. The asymmetry is proportional to  $(L_e - R_e)(L_\mu - R_\mu)$ , or to  $a_e a_\mu$ , and so resolves the sign ambiguity for  $R$  or the  $v$ - $a$  ambiguity.

### 3.4 Electroweak interactions of quarks

To extend our theory to include the electroweak interactions of quarks, we observe that each generation consists of a left-handed doublet

$$L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{matrix} I_3 & Q & Y = 2(Q - I_3) \\ \frac{1}{2} & +\frac{2}{3} & \\ -\frac{1}{2} & -\frac{1}{3} & \frac{1}{3} \end{matrix}, \quad (79)$$

and two right-handed singlets,

$$\begin{matrix} R_u = u_R & I_3 & Q & Y = 2(Q - I_3) \\ & 0 & +\frac{2}{3} & +\frac{4}{3} \\ R_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3} \end{matrix}, \quad (80)$$

Proceeding as before, we find the Lagrangian terms for the  $W$ -quark charged-current interaction,

$$\mathcal{L}_{W\text{-quark}} = \frac{-g}{2\sqrt{2}} [\bar{u}_e \gamma^\mu (1 - \gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u W_\mu^-], \quad (81)$$

which is identical in form to the leptonic charged-current interaction (59). Universality is ensured by the fact that the charged-current interaction is determined by the weak isospin of the fermions, and that both quarks and leptons come in doublets.

The neutral-current interaction is also equivalent in form to its leptonic counterpart, (73) and (74). We may write it compactly as

$$\mathcal{L}_{Z\text{-quark}} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu [L_i (1 - \gamma_5) + R_i (1 + \gamma_5)] q_i Z_\mu, \quad (82)$$

where the chiral couplings are

$$\begin{aligned} L_i &= \tau_3 - 2Q_i \sin^2 \theta_W, \\ R_i &= -2Q_i \sin^2 \theta_W. \end{aligned} \quad (83)$$

Again we find a quark-lepton universality in the form—but not the values—of the chiral couplings.

### 3.5 Trouble in Paradise

Until now, we have based our construction on the idealization that the  $u \leftrightarrow d$  transition is of universal strength. The unmixed doublet

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

does not quite describe our world. We attain a better description by replacing

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L ,$$

where

$$d_\theta \equiv d \cos \theta_C + s \sin \theta_C , \quad (84)$$

with  $\cos \theta_C = 0.9736 \pm 0.0010$ .<sup>r</sup> The change to the ‘‘Cabibbo-rotated’’ doublet perfects the charged-current interaction—at least up to small third-generation effects that we could easily incorporate—but leads to serious trouble in the neutral-current sector, for which the interaction now becomes

$$\begin{aligned} \mathcal{L}_{Z\text{-quark}} = & \frac{-g}{4 \cos \theta_W} Z_\mu \{ \bar{u} \gamma^\mu [L_u(1 - \gamma_5) + R_u(1 + \gamma_5)] u \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \cos^2 \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin^2 \theta_C \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin \theta_C \cos \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \sin \theta_C \cos \theta_C \} , \quad (85) \end{aligned}$$

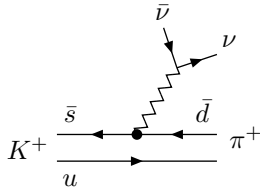
Until the discovery and systematic study of the weak neutral current, culminating in the heroic measurements made at LEP and the SLC, there was not enough knowledge to challenge the first three terms. The last two *strangeness-changing* terms were known to be poisonous, because many of the early experimental searches for neutral currents were fruitless searches for precisely this sort of interaction. Strangeness-changing neutral-current interactions are not seen at an impressively low level.<sup>s</sup>

Only recently has Brookhaven Experiment 787<sup>47</sup> detected a single candidate for the decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,

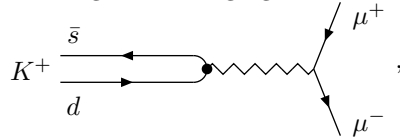
<sup>r</sup>The arbitrary Yukawa couplings that give masses to the quarks can easily be chosen to yield this result.

<sup>s</sup>For more on rare kaon decays, see the TASI 2000 lectures by Tony Barker<sup>45</sup> and Gerhard Buchalla.<sup>46</sup>

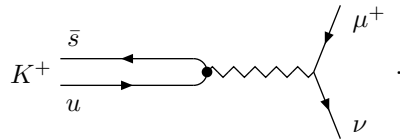




and inferred a branching ratio  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.5_{-1.3}^{+3.5} \times 10^{-10}$ . The good agreement between the standard-model prediction,  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-) = 0.8 \pm 0.3 \times 10^{-10}$  (through the process  $K_L \rightarrow \gamma \gamma \rightarrow \mu^+ \mu^-$ ), and experiment<sup>48</sup> leaves little room for a strangeness-changing neutral-current contribution:



that is easily normalized to the normal charged-current leptonic decay of the  $K^+$ :



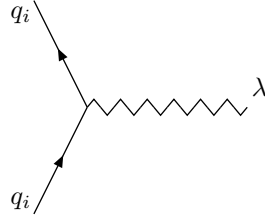
The resolution to this fatal problem was put forward by Glashow, Iliopoulos, and Maiani.<sup>49</sup> Expand the model of quarks to include two left-handed doublets,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L, \quad (86)$$

where

$$s_\theta = s \cos \theta_C - d \sin \theta_C, \quad (87)$$

plus the corresponding right-handed singlets,  $e_R$ ,  $\mu_R$ ,  $u_R$ ,  $d_R$ ,  $c_R$ , and  $s_R$ . This required the introduction of the charmed quark,  $c$ , which had not yet been observed. By the addition of the second quark generation, the flavor-changing cross terms vanish in the  $Z$ -quark interaction, and we are left with:



$$\frac{-ig}{4 \cos \theta_W} \gamma_\lambda [(1 - \gamma_5)L_i + (1 + \gamma_5)R_i] \quad ,$$

which is flavor diagonal!

The generalization to  $n$  quark doublets is straightforward. Let the charged-current interaction be

$$\mathcal{L}_{W\text{-quark}} = \frac{-g}{2\sqrt{2}} [\bar{\Psi} \gamma^\mu (1 - \gamma_5) \mathcal{O} \Psi W_\mu^+ + \text{h.c.}] \quad , \quad (88)$$

where the composite quark spinor is

$$\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix} \quad (89)$$

and the flavor structure is contained in

$$\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix} \quad , \quad (90)$$

where  $U$  is the unitary quark-mixing matrix. The weak-isospin contribution to the neutral-current interaction has the form

$$\mathcal{L}_{Z\text{-quark}}^{\text{iso}} = \frac{-g}{4 \cos \theta_W} \bar{\Psi} \gamma^\mu (1 - \gamma_5) [\mathcal{O}, \mathcal{O}^\dagger] \Psi \quad . \quad (91)$$

Since the commutator

$$[\mathcal{O}, \mathcal{O}^\dagger] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (92)$$

the neutral-current interaction is flavor diagonal, and the weak-isospin piece is, as expected, proportional to  $\tau_3$ .

In general, the  $n \times n$  quark-mixing matrix  $U$  can be parametrized in terms of  $n(n-1)/2$  real mixing angles and  $(n-1)(n-2)/2$  complex phases, after

exhausting the freedom to redefine the phases of quark fields. The  $3 \times 3$  case, of three mixing angles and one phase, often called the Cabibbo–Kobayashi–Maskawa matrix, presaged the discovery of the third generation of quarks and leptons.<sup>50</sup>

## 4 Precision Tests of the Electroweak Theory<sup>51</sup>

### 4.1 Measurements on the $Z^0$ pole

In its simplest form, with the electroweak gauge symmetry broken by the Higgs mechanism, the  $SU(2)_L \otimes U(1)_Y$  theory has scored many qualitative successes: the prediction of neutral-current interactions, the necessity of charm, the prediction of the existence and properties of the weak bosons  $W^\pm$  and  $Z^0$ . Over the past ten years, in great measure due to the beautiful experiments carried out at the  $Z$  factories at CERN and SLAC, precision measurements have tested the electroweak theory as a quantum field theory,<sup>52,53</sup> at the one-per-mille level, as indicated in Table 2. A classic achievement of the  $Z$  factories is the determination of the number of light neutrino species. If we define the invisible width of the  $Z^0$  as

$$\Gamma_{\text{invisible}} = \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}} , \quad (93)$$

then we can compute the number of light neutrino species as

$$N_\nu = \Gamma_{\text{invisible}} / \Gamma^{\text{SM}}(Z \rightarrow \nu_i \bar{\nu}_i) . \quad (94)$$

A typical current value is  $N_\nu = 2.984 \pm 0.008$ , in excellent agreement with the observation of light  $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$ .

As an example of the insights precision measurements have brought us (one that mightily impressed the Royal Swedish Academy of Sciences in 1999),

Table 2. Precision measurements at the  $Z^0$  pole. (For sources of the data, see the Review of Particle Physics, Ref. <sup>9</sup> and the LEP Electroweak Working Group web server, <http://www.cern.ch/LEPEWWG/>.)

$M_Z$	$91\,188 \pm 2.2 \text{ MeV}/c^2$
$\Gamma_Z$	$2495.2 \pm 2.6 \text{ MeV}$
$\sigma_{\text{hadronic}}^0$	$41.541 \pm 0.037 \text{ nb}$
$\Gamma_{\text{hadronic}}$	$1743.8 \pm 2.2 \text{ MeV}$
$\Gamma_{\text{leptonic}}$	$84.057 \pm 0.088 \text{ MeV}$
$\Gamma_{\text{invisible}}$	$499.4 \pm 1.7 \text{ MeV}$

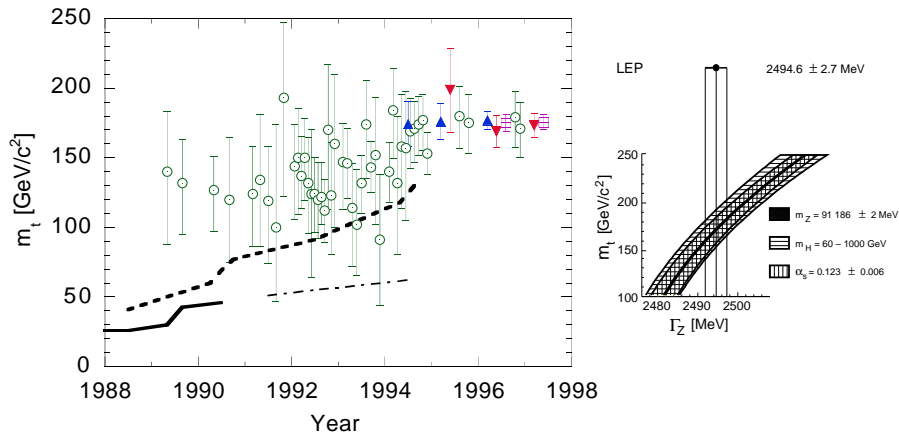


Figure 12. Indirect determinations of the top-quark mass from fits to electroweak observables (open circles) and 95% confidence-level lower bounds on the top-quark mass inferred from direct searches in  $e^+e^-$  annihilations (solid line) and in  $p\bar{p}$  collisions, assuming that standard decay modes dominate (broken line). An indirect lower bound, derived from the  $W$ -boson width inferred from  $p\bar{p} \rightarrow (W \text{ or } Z) + \text{anything}$ , is shown as the dot-dashed line. Direct measurements of  $m_t$  by the CDF (triangles) and  $D\bar{O}$  (inverted triangles) Collaborations are shown at the time of initial evidence, discovery claim, and at the conclusion of Run 1. The world average from direct observations is shown as the crossed box. For sources of data, see Ref. <sup>9</sup>. *Inset*: Electroweak theory predictions for the width of the  $Z^0$  boson as a function of the top-quark mass, compared with the width measured in LEP experiments. (From Ref. <sup>11</sup>.)

I show in Figure 12 the time evolution of the top-quark mass favored by simultaneous fits to many electroweak observables. Higher-order processes involving virtual top quarks are an important element in quantum corrections to the predictions the electroweak theory makes for many observables. A case in point is the total decay rate, or width, of the  $Z^0$  boson: the comparison of experiment and theory shown in the inset to Figure 12 favors a top mass in the neighborhood of 180  $\text{GeV}/c^2$ .

The comparison between the electroweak theory and a considerable universe of data is shown in Figure 13,<sup>54</sup> where the pull, or difference between the global fit and measured value in units of standard deviations, is shown for some twenty observables. The distribution of pulls for this fit, due to the LEP Electroweak Working Group, is not noticeably different from a normal distribution, and only a couple of observables differ from the fit by as much as about two standard deviations. This is the case for any of the recent fits. From fits of the kind represented here, we learn that the standard-model

### Osaka 2000

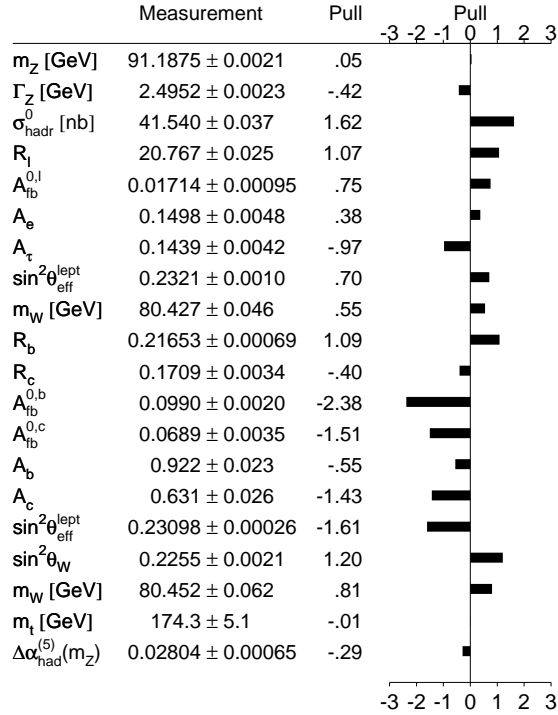


Figure 13. Precision electroweak measurements and the pulls they exert on a global fit to the standard model, from Ref. <sup>54</sup>.

interpretation of the data favors a light Higgs boson.<sup>55</sup> We will revisit this conclusion in §5.4.

The beautiful agreement between the electroweak theory and a vast array of data from neutrino interactions, hadron collisions, and electron-positron annihilations at the  $Z^0$  pole and beyond means that electroweak studies have become a modern arena in which we can look for new physics “in the sixth place of decimals.”

## 4.2 Parity violation in atomic physics

Later in TASI00, you will hear from Carl Wieman<sup>56</sup> about the beautiful table-top experiments that probe the structure of the weak neutral current. I want to spend a few minutes laying the foundation for the interpretation of those measurements.<sup>t</sup>

At very low momentum transfers, as in atomic physics applications, the nucleon appears elementary, and so we can write an effective Lagrangian for nucleon  $\beta$  decay in the limit of zero momentum transfer as

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu\bar{p}\gamma^\lambda(1-g_A\gamma_5)n, \quad (95)$$

where  $g_A \approx 1.26$  is the axial charge of the nucleon. Accordingly, the neutral-current interactions are

$$\begin{aligned} \mathcal{L}_{ep} &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\bar{p}\gamma^\lambda(1-4x_W-\gamma_5)p, \\ \mathcal{L}_{en} &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\bar{n}\gamma^\lambda(1-\gamma_5)n, \end{aligned} \quad (96)$$

where  $x_W \equiv \sin^2\theta_W$ .

For our purposes, we may regard the nucleus as a noninteracting collection of  $Z$  protons and  $N$  neutrons. We perform a nonrelativistic reduction of the interaction; nucleons contribute coherently to the axial-electron–vector-nucleon ( $A_e V_N$ ) coupling, so the dominant parity-violating contribution to the  $eN$  amplitude is

$$\mathcal{M}_{\text{p.v.}} = \frac{-iG_F}{2\sqrt{2}}Q^W\bar{e}\rho_N(\mathbf{r})\gamma_5e, \quad (97)$$

where  $\rho_N(\mathbf{r})$  is the nucleon density at the electron coordinate  $\mathbf{r}$ , and  $Q^W \equiv Z(1-4x_W) - N$  is the weak charge.

Bennett and Wieman (Boulder) have reported a new determination of the weak charge of Cesium by measuring the transition polarizability for the 6S-7S transition.<sup>57</sup> The new value,

$$Q_W(\text{Cs}) = -72.06 \pm 0.28 \text{ (expt)} \pm 0.34 \text{ (theory)}, \quad (98)$$

represents a sevenfold improvement in the experimental error and a significant reduction in the theoretical uncertainty. It lies about 2.5 standard deviations above the prediction of the standard model. We are left with the traditional

<sup>t</sup>The book by Commins and Bucksbaum, Ref. 22, is an excellent reference for many “real-world” problems at low  $Q^2$ .

situation in which elegant measurements of parity nonconservation in atoms are on the edge of incompatibility with the standard model.

A number of authors<sup>58</sup> have noted that the discrepancy in the weak charge  $Q_W$  and a  $2\text{-}\sigma$  anomaly<sup>u</sup> in the total width of the  $Z^0$  can be reduced by introducing a  $Z'$  boson with a mass of about  $800 \text{ GeV}/c^2$ . The additional neutral gauge boson resembles the  $Z_\chi$  familiar from unified theories based on the group  $E_6$ .<sup>59</sup>

#### 4.3 The vacuum energy problem

I want to spend a moment to revisit a longstanding, but usually unspoken, challenge to the completeness of the electroweak theory as we have defined it: the vacuum energy problem.<sup>60</sup> I do so not only for its intrinsic interest, but also to raise the question, “Which problems of completeness and consistency do we worry about at a given moment?” It is perfectly acceptable science—indeed, it is often essential—to put certain problems aside, in the expectation that we will return to them at the right moment. What is important is never to forget that the problems are there, even if we do not allow them to paralyze us.

For the usual Higgs potential,  $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|(\varphi^\dagger\varphi)^2$ , the value of the potential at the minimum is

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0. \quad (99)$$

Identifying  $M_H^2 = -2\mu^2$ , we see that the Higgs potential contributes a field-independent constant term,

$$\varrho_H \equiv \frac{M_H^2 v^2}{8}. \quad (100)$$

I have chosen the notation  $\varrho_H$  because the constant term in the Lagrangian plays the role of a vacuum energy density. When we consider gravitation, adding a vacuum energy density  $\varrho_{\text{vac}}$  is equivalent to adding a cosmological constant term to Einstein’s equation. Although recent observations<sup>v</sup> raise the intriguing possibility that the cosmological constant may be different from zero, the essential observational fact is that the vacuum energy density must

<sup>u</sup>In Erler & Langacker’s fit, for example, the number of light neutrino species inferred from the invisible width of the  $Z^0$  is  $N_\nu = 2.985 \pm 0.008 = 3 - 2\sigma$ .

<sup>v</sup>For a cogent summary of current knowledge of the cosmological parameters, including evidence for a cosmological constant, see Ref. <sup>61</sup>.

be very tiny indeed,<sup>w</sup>

$$\varrho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4. \quad (101)$$

Therein lies the puzzle: if we take  $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}$  and insert the current experimental lower bound<sup>62</sup>  $M_H \gtrsim 113.5 \text{ GeV}/c^2$  into (100), we find that the contribution of the Higgs field to the vacuum energy density is

$$\varrho_H \gtrsim 10^8 \text{ GeV}^4, \quad (102)$$

some 54 orders of magnitude larger than the upper bound inferred from the cosmological constant.

What are we to make of this mismatch? The fact that  $\varrho_H \gg \varrho_{\text{vac}}$  means that the smallness of the cosmological constant needs to be explained. In a unified theory of the strong, weak, and electromagnetic interactions, other (heavy!) Higgs fields have nonzero vacuum expectation values that may give rise to still greater mismatches. At a fundamental level, we can therefore conclude that a spontaneously broken gauge theory of the strong, weak, and electromagnetic interactions—or merely of the electroweak interactions—cannot be complete. Either we must find a separate principle to zero the vacuum energy density of the Higgs field, or we may suppose that a proper quantum theory of gravity, in combination with the other interactions, will resolve the puzzle of the cosmological constant. The vacuum energy problem must be an important clue. But to what?

## 5 The Higgs Boson

### 5.1 Why the Higgs boson must exist

How can we be sure that a Higgs boson, or something very like it, will be found? One path to the *theoretical* discovery of the Higgs boson involves its role in the cancellation of high-energy divergences. An illuminating example is provided by the reaction

$$e^+e^- \rightarrow W^+W^-, \quad (103)$$

which is described in lowest order by the four Feynman graphs in Figure 14. The contributions of the direct-channel  $\gamma$ - and  $Z^0$ -exchange diagrams of Figs. 14(a) and (b) cancel the leading divergence in the  $J = 1$  partial-wave amplitude of the neutrino-exchange diagram in Figure 14(c). This is the famous “gauge cancellation” observed in experiments at LEP 2 and the Tevatron. The LEP measurements in Figure 15 agree well with the predictions

<sup>w</sup>For a useful summary of gravitational theory, see the essay by T. d’Amour in §14 of the 2000 *Review of Particle Physics*, Ref. <sup>9</sup>.



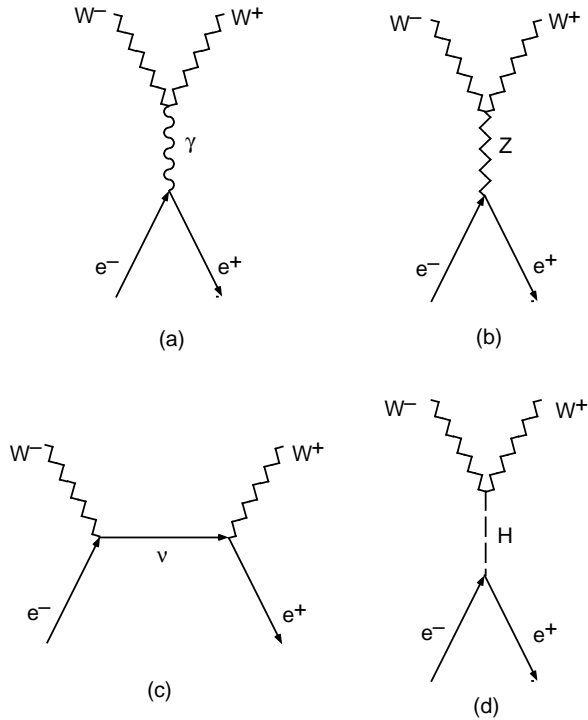


Figure 14. Lowest-order contributions to the  $e^+e^- \rightarrow W^+W^-$  scattering amplitude.

of electroweak-theory Monte Carlo generators, which predict a benign high-energy behavior. If the  $Z$ -exchange contribution is omitted (dashed line) or if both the  $\gamma$ - and  $Z$ -exchange contributions are omitted (dot-dashed line), the calculated cross section grows unacceptably with energy—and disagrees with the measurements. The gauge cancellation in the  $J = 1$  partial-wave amplitude is thus observed.

However, this is not the end of the high-energy story: the  $J = 0$  partial-wave amplitude, which exists in this case because the electrons are massive and may therefore be found in the “wrong” helicity state, grows as  $s^{1/2}$  for the production of longitudinally polarized gauge bosons. The resulting divergence is precisely cancelled by the Higgs boson graph of Figure 14(d). If the Higgs boson did not exist, something else would have to play this role. From the point of view of  $S$ -matrix analysis, the Higgs-electron-electron coupling must

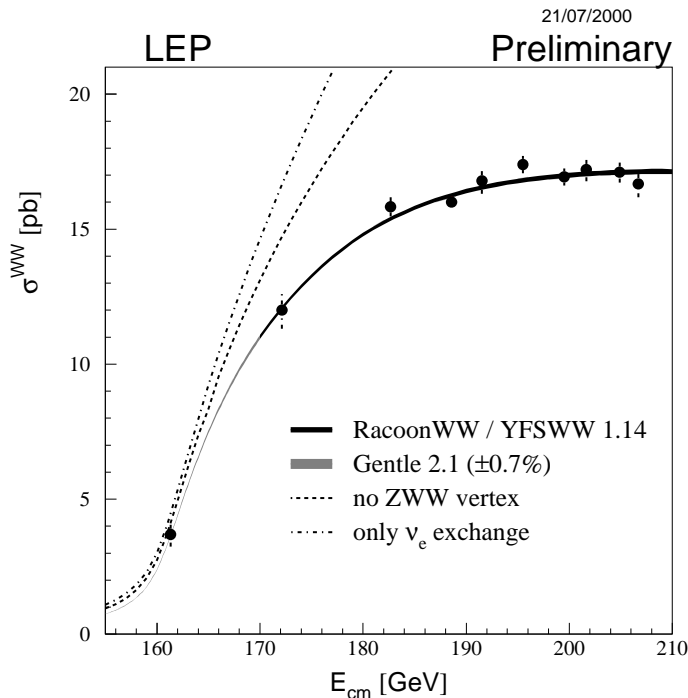


Figure 15. Cross section for the reaction  $e^+e^- \rightarrow W^+W^-$  measured by the four LEP experiments, together with the full electroweak-theory simulation and the cross sections that would result from  $\nu$ -exchange alone and from  $(\nu + \gamma)$ -exchange (from <http://www.cern.ch/LEPEWWG/>).

be proportional to the electron mass, because “wrong-helicity” amplitudes are always proportional to the fermion mass.

Let us underline this result. If the gauge symmetry were unbroken, there would be no Higgs boson, no longitudinal gauge bosons, and no extreme divergence difficulties. But there would be no viable low-energy phenomenology of the weak interactions. The most severe divergences of individual diagrams are eliminated by the gauge structure of the couplings among gauge bosons and leptons. A lesser, but still potentially fatal, divergence arises because the electron has acquired mass—because of the Higgs mechanism. Spontaneous symmetry breaking provides its own cure by supplying a Higgs boson to remove the last divergence. A similar interplay and compensation must exist in

any satisfactory theory.

## 5.2 Bounds on $M_H$

The Standard Model does not give a precise prediction for the mass of the Higgs boson. We can, however, use arguments of self-consistency to place plausible lower and upper bounds on the mass of the Higgs particle in the minimal model. Unitarity arguments<sup>63</sup> lead to a conditional upper bound on the Higgs boson mass. It is straightforward to compute the amplitudes  $\mathcal{M}$  for gauge boson scattering at high energies, and to make a partial-wave decomposition, according to

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta). \quad (104)$$

Most channels “decouple,” in the sense that partial-wave amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies), for any value of the Higgs boson mass  $M_H$ . Four channels are interesting:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad HH / \sqrt{2} \quad H Z_L^0, \quad (105)$$

where the subscript  $L$  denotes the longitudinal polarization states, and the factors of  $\sqrt{2}$  account for identical particle statistics. For these, the  $s$ -wave amplitudes are all asymptotically constant (*i.e.*, well-behaved) and proportional to  $G_F M_H^2$ . In the high-energy limit,<sup>x</sup>

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}. \quad (106)$$

Requiring that the largest eigenvalue respect the partial-wave unitarity condition  $|a_0| \leq 1$  yields

$$M_H \leq \left( \frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2 \quad (107)$$

as a condition for perturbative unitarity.

<sup>x</sup>It is convenient to calculate these amplitudes by means of the Goldstone-boson equivalence theorem,<sup>64</sup> which reduces the dynamics of longitudinally polarized gauge bosons to a scalar field theory with interaction Lagrangian given by  $\mathcal{L}_{\text{int}} = -\lambda v h (2w^+ w^- + z^2 + h^2) - (\lambda/4)(2w^+ w^- + z^2 + h^2)^2$ , with  $1/v^2 = G_F \sqrt{2}$  and  $\lambda = G_F M_H^2 / \sqrt{2}$ .

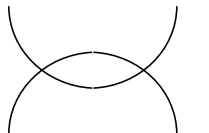
If the bound is respected, weak interactions remain weak at all energies, and perturbation theory is everywhere reliable. If the bound is violated, perturbation theory breaks down, and weak interactions among  $W^\pm$ ,  $Z$ , and  $H$  become strong on the 1-TeV scale. This means that the features of strong interactions at GeV energies will come to characterize electroweak gauge boson interactions at TeV energies. We interpret this to mean that new phenomena are to be found in the electroweak interactions at energies not much larger than 1 TeV.

It is worthwhile to note in passing that the threshold behavior of the partial-wave amplitudes for gauge-boson scattering follows generally from chiral symmetry.<sup>65</sup> The partial-wave amplitudes  $a_{IJ}$  of definite isospin  $I$  and angular momentum  $J$  are given by

$$\begin{aligned} a_{00} &\approx G_{Fs}/8\pi\sqrt{2} \text{ attractive,} \\ a_{11} &\approx G_{Fs}/48\pi\sqrt{2} \text{ attractive,} \\ a_{20} &\approx -G_{Fs}/16\pi\sqrt{2} \text{ repulsive.} \end{aligned} \tag{108}$$

The electroweak theory itself provides another reason to expect that discoveries will not end with the Higgs boson. Scalar field theories make sense on all energy scales only if they are noninteracting, or “trivial.”<sup>66</sup> The vacuum of quantum field theory is a dielectric medium that screens charge. Accordingly, the effective charge is a function of the distance or, equivalently, of the energy scale. This is the famous phenomenon of the running coupling constant.

In  $\lambda\phi^4$  theory (compare the interaction term in the Higgs potential), it is easy to calculate the variation of the coupling constant  $\lambda$  in perturbation theory by summing bubble graphs like this one:



The coupling constant  $\lambda(\mu)$  on a physical scale  $\mu$  is related to the coupling constant on a higher scale  $\Lambda$  by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log(\Lambda/\mu) . \tag{110}$$

This perturbation-theory result is reliable only when  $\lambda$  is small, but lattice field theory allows us to treat the strong-coupling regime.

In order for the Higgs potential to be stable (*i.e.*, for the energy of the vacuum state not to race off to  $-\infty$ ),  $\lambda(\Lambda)$  must not be negative. Therefore

we can rewrite (110) as an inequality,

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log(\Lambda/\mu) . \quad (111)$$

This gives us an *upper bound*,

$$\lambda(\mu) \leq 2\pi^2/3 \log(\Lambda/\mu) , \quad (112)$$

on the coupling strength at the physical scale  $\mu$ . If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit  $\Lambda \rightarrow \infty$  while holding  $\mu$  fixed at some reasonable physical scale. In this limit, the bound (112) forces  $\lambda(\mu)$  to zero. The scalar field theory has become free field theory; in theorist’s jargon, it is trivial.

We can rewrite the inequality (112) as a bound on the Higgs-boson mass. Rearranging and exponentiating both sides gives the condition

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right) . \quad (113)$$

Choosing the physical scale as  $\mu = M_H$ , and remembering that, before quantum corrections,

$$M_H^2 = 2\lambda(M_H)v^2 , \quad (114)$$

where  $v = (G_F\sqrt{2})^{-1/2} \approx 246$  GeV is the vacuum expectation value of the Higgs field times  $\sqrt{2}$ , we find that

$$\Lambda \leq M_H \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right) . \quad (115)$$

For any given Higgs-boson mass, there is a maximum energy scale  $\Lambda^*$  at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

This perturbative analysis breaks down when the Higgs-boson mass approaches  $1 \text{ TeV}/c^2$  and the interactions become strong. Lattice analyses<sup>67</sup> indicate that, for the theory to describe physics to an accuracy of a few percent up to a few TeV, the mass of the Higgs boson can be no more than about  $710 \pm 60 \text{ GeV}/c^2$ . Another way of putting this result is that, if the elementary Higgs boson takes on the largest mass allowed by perturbative unitarity arguments, the electroweak theory will be living on the brink of instability.

A lower bound is obtained by computing<sup>68</sup> the first quantum corrections to the classical potential (53). Requiring that  $\langle\phi\rangle_0 \neq 0$  be an absolute minimum of the one-loop potential up to a scale  $\Lambda$  yields the vacuum-stability

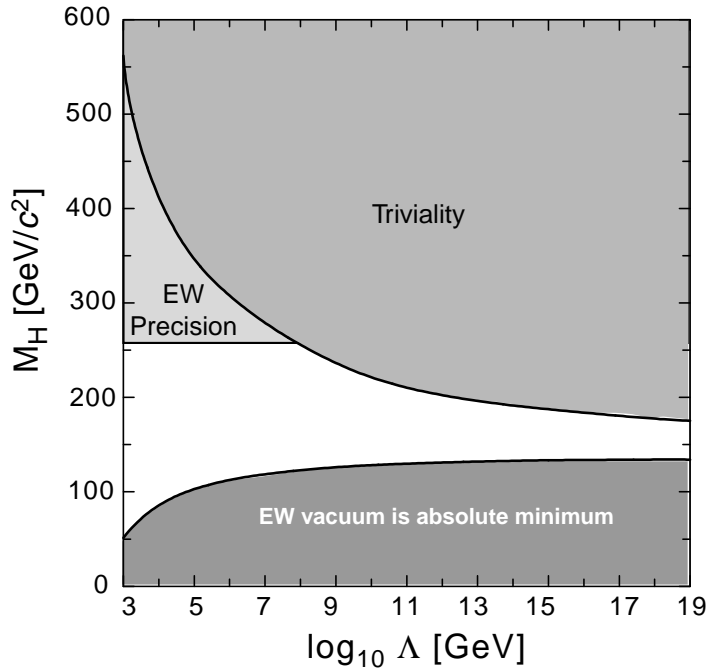


Figure 16. Bounds on the Higgs-boson mass that follow from requirements that the electroweak theory be consistent up to the energy scale  $\Lambda$ . The upper bound follows from triviality conditions; the lower bound follows from the requirement that  $V(v) < V(0)$ . Also shown is the range of masses permitted at the 95% confidence level by precision measurements.

condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2). \quad (116)$$

The upper and lower bounds plotted in Figure 16 are the results of full two-loop calculations.<sup>69</sup> There I have also indicated the upper bound on  $M_H$  derived from precision electroweak measurements *in the framework of the standard electroweak theory*. If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies. If the electroweak theory is to make sense all the way up to a unification scale  $\Lambda^* = 10^{16}$  GeV, then the Higgs-boson mass must lie in the interval  $134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2$ .<sup>70</sup>

### 5.3 Higgs-Boson Properties

Once we assume a value for the Higgs-boson mass, it is a simple matter to compute the rates for Higgs-boson decay into pairs of fermions or weak bosons.<sup>71</sup> For a fermion with color  $N_c$ , the partial width is

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}, \quad (117)$$

which is proportional to  $M_H$  in the limit of large Higgs mass. The partial width for decay into a  $W^+W^-$  pair is

$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2), \quad (118)$$

where  $x \equiv 4M_W^2/M_H^2$ . Similarly, the partial width for decay into a pair of  $Z^0$  bosons is

$$\Gamma(H \rightarrow Z^0 Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2), \quad (119)$$

where  $x' \equiv 4M_Z^2/M_H^2$ . The rates for decays into weak-boson pairs are asymptotically proportional to  $M_H^3$  and  $\frac{1}{2}M_H^3$ , respectively, the factor  $\frac{1}{2}$  arising from weak isospin. In the final factors of (118) and (119),  $2x^2$  and  $2x'^2$ , respectively, arise from decays into transversely polarized gauge bosons. The dominant decays for large  $M_H$  are into pairs of longitudinally polarized weak bosons.

Branching fractions for decay modes that hold promise for the detection of a light Higgs boson are displayed in Figure 17. In addition to the  $f\bar{f}$  and  $VV$  modes that arise at tree level, I have included the  $\gamma\gamma$  mode that proceeds through loop diagrams. Though rare, the  $\gamma\gamma$  channel offers an important target for LHC experiments.

Figure 18 shows the partial widths for the decay of a Higgs boson into the dominant  $W^+W^-$  and  $Z^0Z^0$  channels and into  $t\bar{t}$ , for  $m_t = 175 \text{ GeV}/c^2$ . Whether the  $t\bar{t}$  mode will be useful to confirm the observation of a heavy Higgs boson, or merely drains probability from the  $ZZ$  channel favored for a heavy-Higgs search, is a question for detailed detector simulations.

Below the  $W^+W^-$  threshold, the total width of the standard-model Higgs boson is rather small, typically less than 1 GeV. Far above the threshold for decay into gauge-boson pairs, the total width is proportional to  $M_H^3$ . At masses approaching 1 TeV/ $c^2$ , the Higgs boson is an ephemeron, with a perturbative width approaching its mass. The Higgs-boson total width is plotted as a function of  $M_H$  in Figure 19.

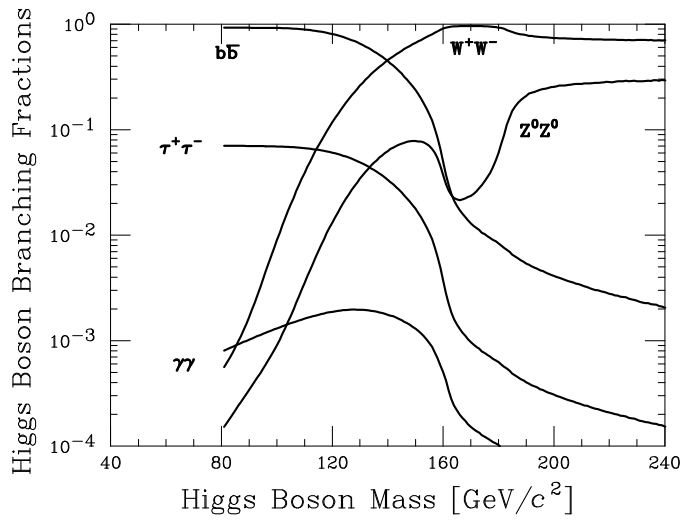


Figure 17. Branching fractions for the prominent decay modes of a light Higgs boson.

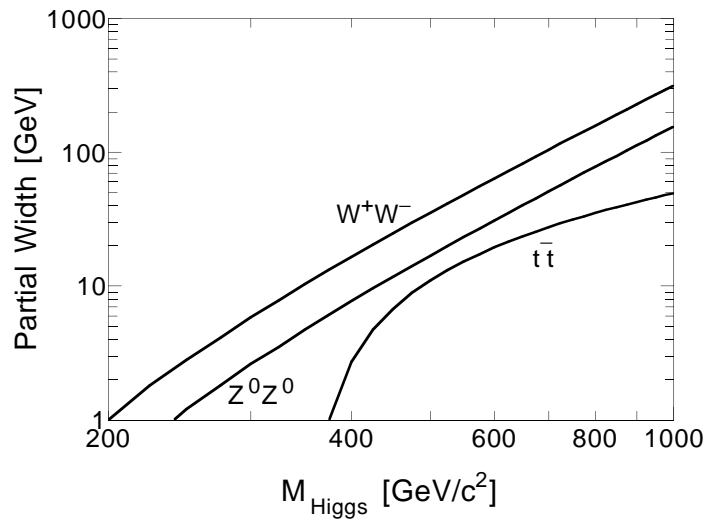


Figure 18. Partial widths for the prominent decay modes of a heavy Higgs boson.



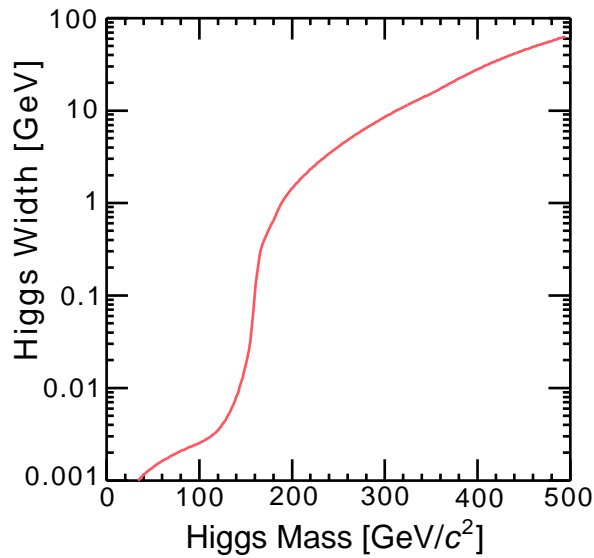
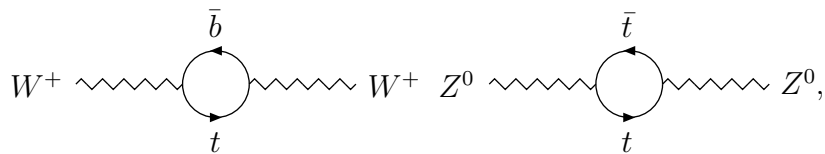


Figure 19. Higgs-boson total width as a function of mass.

#### 5.4 Clues to the Higgs-boson mass

We have seen in §4.1 that the sensitivity of electroweak observables to the (long unknown) mass of the top quark gave early indications for a very massive top. For example, the quantum corrections to the standard-model predictions (5) for  $M_W$  and (6) for  $M_Z$  arise from different quark loops:



$t\bar{t}$  for  $M_W$ , and  $t\bar{t}$  (or  $b\bar{b}$ ) for  $M_Z$ . These quantum corrections alter the link (6) between the  $W$ - and  $Z$ -boson masses, so that

$$M_W^2 = M_Z^2 (1 - \sin^2 \theta_W) (1 + \Delta\rho) , \quad (120)$$

where

$$\Delta\rho \approx \Delta\rho^{(\text{quarks})} = \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}. \quad (121)$$

The strong dependence on  $m_t^2$  is characteristic, and it accounts for the precision of the top-quark mass estimates derived from electroweak observables.

Now that  $m_t$  is known to about 3% from direct observations at the Tevatron, it becomes profitable to look beyond the quark loops to the next most important quantum corrections, which arise from Higgs-boson effects. The Higgs-boson quantum corrections are typically smaller than the top-quark corrections, and exhibit a more subtle dependence on  $M_H$  than the  $m_t^2$  dependence of the top-quark corrections. For the case at hand,

$$\Delta\rho^{(\text{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right), \quad (122)$$

where I have arbitrarily chosen to define the coefficient  $\mathcal{C}$  at the electroweak scale  $v$ . Since  $M_Z$  has been determined at LEP to 23 ppm, it is interesting to examine (see Figure 20) the dependence of  $M_W$  upon  $m_t$  and  $M_H$ . Also indicated in Figure 20 are the direct determinations of  $m_t$  and  $M_W$ , and the values inferred from a universe of electroweak observables, both shown as one-standard-deviation regions. The direct and indirect determinations are in reasonable agreement. Both favor a light Higgs boson, *within the framework of the standard-model analysis*.

The  $M_W$ - $m_t$ - $M_H$  correlation will be telling over the next few years, since we anticipate that measurements at the Tevatron and LHC will determine  $m_t$  within 1 or 2 GeV/ $c^2$  and improve the uncertainty on  $M_W$  to about 15 MeV/ $c^2$ . As the Tevatron's integrated luminosity grows past 10 fb $^{-1}$ , CDF and DØ will begin to explore the region of Higgs-boson masses not excluded by the LEP searches.<sup>72</sup> Soon after that, ATLAS and CMS will carry on the exploration of the Higgs sector at the Large Hadron Collider.

By itself, the  $W$ -boson mass suggests a preference—always within the standard model—for a light Higgs boson. The indication becomes somewhat stronger when all the electroweak observables are examined. Figure 21 shows how the goodness of the LEP Electroweak Working Group's global fit depends upon  $M_H$ . Within the standard model, they deduce a 95% CL upper limit,  $M_H \lesssim 170$  GeV/ $c^2$ . Since the direct searches at LEP have concluded that  $M_H > 113.5$  GeV/ $c^2$ , excluding much of the favored region, either the Higgs boson is just around the corner, or the standard-model analysis is misleading. Things will soon be popping!

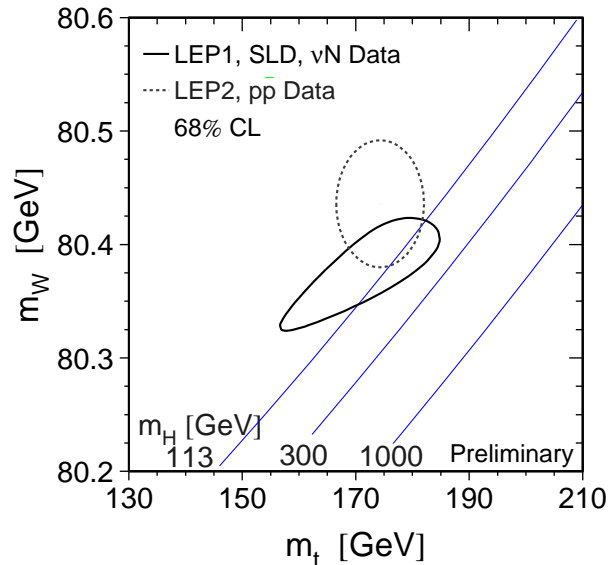


Figure 20. Comparison of the indirect measurements of  $M_W$  and  $m_t$  (LEP I+SLD+ $\nu N$  data, solid contour) and the direct measurements (Tevatron and LEP II data, dashed contour). Also shown is the standard-model relationship for the masses as a function of the Higgs mass. (From the LEP Electroweak Working Group, Ref. <sup>54</sup>.)

## 6 The electroweak scale and beyond

We have seen that the scale of electroweak symmetry breaking,  $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$  GeV, sets the values of the  $W$ - and  $Z$ -boson masses. But the electroweak scale is not the only scale of physical interest. It seems certain that we must also consider the Planck scale, derived from the strength of Newton's constant, and it is also probable that we must take account of the  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  unification scale around  $10^{15-16}$  GeV. There may well be a distinct flavor scale. The existence of other significant energy scales is behind the famous problem of the Higgs scalar mass: how to keep the distant scales from mixing in the face of quantum corrections, or how to stabilize the mass of the Higgs boson on the electroweak scale.

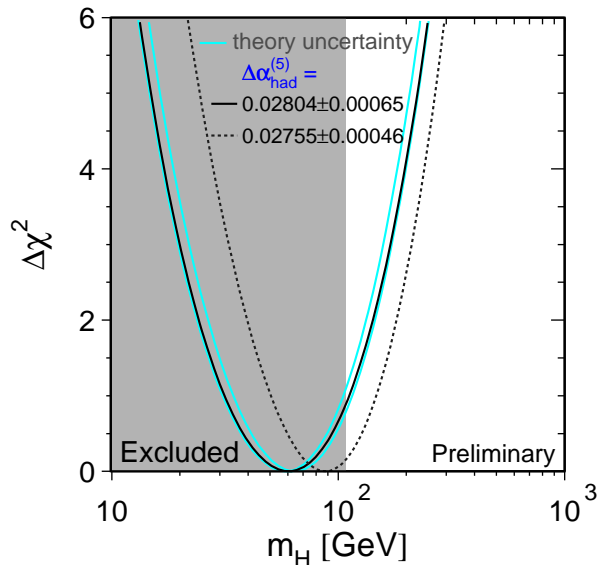


Figure 21.  $\Delta\chi^2 = \chi^2 - \chi_{\min}^2$  from a global fit to precision data *vs.* the Higgs-boson mass,  $M_H$ . The solid line is the result of the fit; the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on  $M_H$  from the direct search at LEP. The dashed curve shows the sensitivity to a change in the evaluation of  $\alpha(M_Z^2)$ . (From the LEP Electroweak Working Group, Ref. <sup>54</sup>.)

### 6.1 Why is the electroweak scale small?

To this point, we have outlined the electroweak theory, emphasized that the need for a Higgs boson (or substitute) is quite general, and reviewed the properties of the standard-model Higgs boson. By considering a thought experiment, gauge-boson scattering at very high energies, we found a first signal for the importance of the 1-TeV scale. Now, let us explore another path to the 1-TeV scale.

The  $SU(2)_L \otimes U(1)_Y$  electroweak theory does not explain how the scale of electroweak symmetry breaking is maintained in the presence of quantum corrections. The problem of the scalar sector can be summarized neatly as

follows.<sup>73</sup> The Higgs potential is

$$V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|(\phi^\dagger\phi)^2 . \quad (123)$$

With  $\mu^2$  chosen to be less than zero, the electroweak symmetry is spontaneously broken down to the  $U(1)$  of electromagnetism, as the scalar field acquires a vacuum expectation value that is fixed by the low-energy phenomenology,

$$\langle\phi\rangle_0 = \sqrt{-\mu^2/2|\lambda|} \equiv (G_F\sqrt{8})^{-1/2} \approx 175 \text{ GeV} . \quad (124)$$

Beyond the classical approximation, scalar mass parameters receive quantum corrections from loops that contain particles of spins  $J = 1, 1/2$ , and 0:

$$m^2(p^2) = m_0^2 + \text{---} \underset{\text{J}=1}{\text{wavy line}} \text{---} + \text{---} \underset{\text{J}=1/2}{\text{loop}} \text{---} + \text{---} \underset{\text{J}=0}{\text{circle}} \text{---} \quad (125)$$

The loop integrals are potentially divergent. Symbolically, we may summarize the content of (125) as

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots , \quad (126)$$

where  $\Lambda$  defines a reference scale at which the value of  $m^2$  is known,  $g$  is the coupling constant of the theory, and the coefficient  $C$  is calculable in any particular theory. Instead of dealing with the relationship between observables and parameters of the Lagrangian, we choose to describe the variation of an observable with the momentum scale. In order for the mass shifts induced by radiative corrections to remain under control (*i.e.*, not to greatly exceed the value measured on the laboratory scale), either  $\Lambda$  must be small, so the range of integration is not enormous, or new physics must intervene to cut off the integral.

If the fundamental interactions are described by an  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  gauge symmetry, *i.e.*, by quantum chromodynamics and the electroweak theory, then the natural reference scale is the Planck mass,<sup>y</sup>

<sup>y</sup>It is because  $M_{\text{Planck}}$  is so large (or because  $G_{\text{Newton}}$  is so small) that we normally consider gravitation irrelevant for particle physics. The graviton-quark-antiquark coupling is generically  $\sim E/M_{\text{Planck}}$ , so it is easy to make a dimensional estimate of the branching fraction for a gravitationally mediated rare kaon decay:  $B(K_L \rightarrow \pi^0 G) \sim (M_K/M_{\text{Planck}})^2 \sim 10^{-38}$ , which is truly negligible!

$$\Lambda \sim M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV} . \quad (127)$$

In a unified theory of the strong, weak, and electromagnetic interactions, the natural scale is the unification scale,

$$\Lambda \sim M_U \approx 10^{15}\text{-}10^{16} \text{ GeV} . \quad (128)$$

Both estimates are very large compared to the scale of electroweak symmetry breaking (124). We are therefore assured that new physics must intervene at an energy of approximately 1 TeV, in order that the shifts in  $m^2$  not be much larger than (124).

Only a few distinct scenarios for controlling the contribution of the integral in (126) can be envisaged. The supersymmetric solution is especially elegant.<sup>74,75,76,77</sup> Exploiting the fact that fermion loops contribute with an overall minus sign (because of Fermi statistics), supersymmetry balances the contributions of fermion and boson loops. In the limit of unbroken supersymmetry, in which the masses of bosons are degenerate with those of their fermion counterparts, the cancellation is exact:

$$\sum_{\substack{\text{fermions} \\ + \text{bosons}}} C_i \int dk^2 = 0 . \quad (129)$$

If the supersymmetry is broken (as it must be in our world), the contribution of the integrals may still be acceptably small if the fermion-boson mass splittings  $\Delta M$  are not too large. The condition that  $g^2 \Delta M^2$  be “small enough” leads to the requirement that superpartner masses be less than about 1 TeV/ $c^2$ .

A second solution to the problem of the enormous range of integration in (126) is offered by theories of dynamical symmetry breaking such as technicolor.<sup>78,79,80</sup> In technicolor models, the Higgs boson is composite, and new physics arises on the scale of its binding,  $\Lambda_{\text{TC}} \simeq O(1 \text{ TeV})$ . Thus the effective range of integration is cut off, and mass shifts are under control.

A third possibility is that the gauge sector becomes strongly interacting. This would give rise to  $WW$  resonances, multiple production of gauge bosons, and other new phenomena at energies of 1 TeV or so. It is likely that a scalar bound state—a quasi-Higgs boson—would emerge with a mass less than about 1 TeV/ $c^2$ .<sup>81</sup>

We cannot avoid the conclusion that some new physics must occur on the 1-TeV scale.<sup>z</sup>

---

<sup>z</sup>Since the superconducting phase transition informs our understanding of the Higgs mech-

## 6.2 Why is the Planck scale so large?

The conventional approach to new physics has been to extend the standard model to understand why the electroweak scale (and the mass of the Higgs boson) is so much smaller than the Planck scale. A novel approach that has been developed over the past two years is instead to *change gravity* to understand why the Planck scale is so much greater than the electroweak scale.<sup>86</sup> Now, experiment tells us that gravitation closely follows the Newtonian force law down to distances on the order of 1 mm. Let us parameterize deviations from a  $1/r$  gravitational potential in terms of a relative strength  $\varepsilon_G$  and a range  $\lambda_G$ , so that

$$V(r) = - \int dr_1 \int dr_2 \frac{G_{\text{Newton}} \rho(r_1) \rho(r_2)}{r_{12}} [1 + \varepsilon_G \exp(-r_{12}/\lambda_G)] , \quad (130)$$

where  $\rho(r_i)$  is the mass density of object  $i$  and  $r_{12}$  is the separation between bodies 1 and 2. Elegant experiments that study details of Casimir and Van der Waals forces imply bounds on anomalous gravitational interactions, as shown in Figure 22. Below about a millimeter, the constraints on deviations from Newton's inverse-square force law deteriorate rapidly, so nothing prevents us from considering changes to gravity even on a small but macroscopic scale.

For its internal consistency, string theory requires an additional six or seven space dimensions, beyond the 3+1 dimensions of everyday experience.<sup>93</sup> Until recently it has been presumed that the extra dimensions must be compactified on the Planck scale, with a compactification radius  $R_{\text{unobserved}} \approx 1/M_{\text{Planck}} \approx 1.6 \times 10^{-35}$  m. The new wrinkle is to consider that the  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  standard-model gauge fields, plus needed extensions, reside on 3 + 1-dimensional branes, not in the extra dimensions, but that gravity can propagate into the extra dimensions.

How does this hypothesis change the picture? The dimensional analysis (Gauss's law, if you like) that relates Newton's constant to the Planck scale changes. If gravity propagates in  $n$  extra dimensions with radius  $R$ , then

$$G_{\text{Newton}} \sim M_{\text{Planck}}^{-2} \sim M^*{}^{-n-2} R^{-n} , \quad (131)$$

where  $M^*$  is gravity's true scale. Notice that if we boldly take  $M^*$  to be as small as  $1 \text{ TeV}/c^2$ , then the radius of the extra dimensions is required to be

---

anism for electroweak symmetry breaking, it may be useful to look to other collective phenomena for inspiration. Although the implied gauge-boson masses are unrealistically small, chiral symmetry breaking in QCD can induce a dynamical breaking of electroweak symmetry.<sup>82</sup> (This is the prototype for technicolor models.) Is it possible that other interesting phases of QCD—color superconductivity,<sup>83,84,85</sup> for example—might hold lessons for electroweak symmetry breaking under normal or unusual conditions?

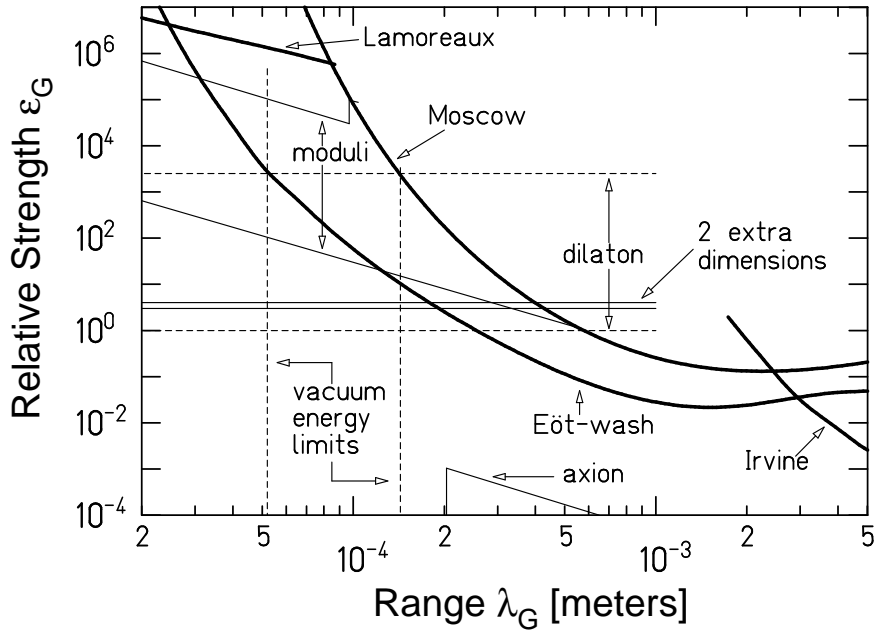


Figure 22. 95% confidence level upper limits (heavy line labeled Eöt-wash) on the strength  $\varepsilon_G$  (relative to gravity) versus the range  $\lambda_G$  of a new long-range force characterized by (130), from Ref. <sup>87</sup>. The region excluded by earlier work<sup>88,89,90</sup> lies above the heavy lines labeled Irvine, Moscow and Lamoreaux, respectively. The theoretical predictions are adapted from Ref. <sup>91</sup>, except for the dilaton prediction, from Ref. <sup>92</sup>.

smaller than about 1 mm, for  $n \geq 2$ . If we use the four-dimensional force law to extrapolate the strength of gravity from low energies to high, we find that gravity becomes as strong as the other forces on the Planck scale, as shown by the dashed line in Figure 23. If the force law changes at an energy  $1/R$ , as the large-extra-dimensions scenario suggests, then the forces are unified at an energy  $M^*$ , as shown by the solid line in Figure 23. What we know as the Planck scale is then a mirage that results from a false extrapolation: treating gravity as four-dimensional down to arbitrarily small distances, when in fact—or at least in this particular fiction—gravity propagates in  $3+n$  spatial dimensions. The Planck mass is an artifact, given by  $M_{\text{Planck}} = M^*(M^*R)^{n/2}$ .

Although the idea that extra dimensions are just around the corner—



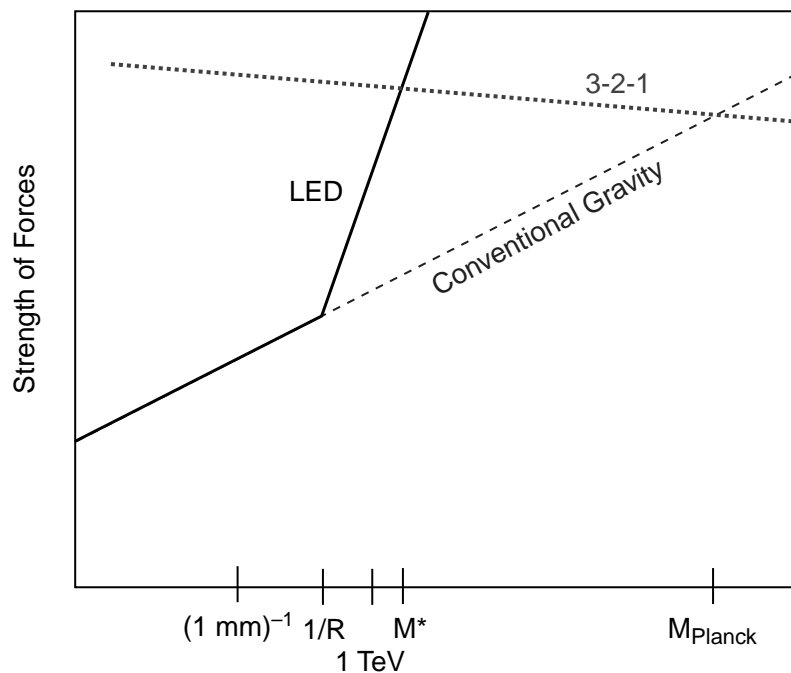


Figure 23. One of these extrapolations (at least!) is false.

either on the submillimeter scale or on the TeV scale—is preposterous, it is not ruled out by observations. For that reason alone, we should entertain ourselves by entertaining the consequences. Many authors have considered the gravitational excitation of a tower of Kaluza–Klein modes in the extra dimensions, which would give rise to a missing (transverse) energy signature in collider experiments.<sup>94</sup> We call these excitations *provatons*, after the Greek word for a sheep in a flock.

The electroweak scale is nearby; indeed, it is coming within experimental reach at LEP2, the Tevatron Collider, and the Large Hadron Collider. Where are the other scales of significance? In particular, what is the energy scale on which the properties of quark and lepton masses and mixings are set? The similarity between the top-quark mass,  $m_t \approx 175 \text{ GeV}/c^2$ , and the Higgs-field vacuum expectation value,  $v/\sqrt{2} \approx 176 \text{ GeV}$ , encourages the hope that in addition to decoding the puzzle of electroweak symmetry breaking in our

explorations of the 1-TeV scale, we might gain insight into the problem of fermion mass. This is an area to be defined over the next decade.

## 7 Outlook

The creation of the electroweak theory is one of the great achievements of twentieth-century science. Its history shows the importance of the interplay between theory and experiment, and the significance of both search-and-discovery experiments and precision, programmatic measurements. We owe a special salute to the heroic achievements of the LEP experiments, of the LEP accelerator team, and of the theorists who devoted themselves to extracting the most (reliable!) information from precision measurements. Their work has been inspiring.

The electroweak story is not done. We haven't fully understood the agent of electroweak symmetry breaking, but we look forward to a decade of discovery on the 1-TeV scale. We probably have much to learn about the non-perturbative aspects of the electroweak theory, about which I've said nothing in these lectures.<sup>95</sup> And we are just learning to define—then to solve—the essential mystery of flavor, the problem of identity. These are the good old days!

For many topics not covered in these lectures, including an introduction to Higgs searches, see my 1998 Granada lectures.<sup>96</sup>

## Acknowledgments

Fermilab is operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy. It is a pleasure to thank Steve Ellis for gracious hospitality at the University of Washington during the writing of these notes. I would also like to take this opportunity to compliment Jonathan Rosner, Hitoshi Murayama, K. T. Mahanthappa, and the TASI staff for their efficient organization of “Flavor Physics for the Millennium,” and for the pleasant and stimulating atmosphere in Boulder.

## References

1. F. Wilczek, “What QCD Tells Us about Nature — and Why We Should Listen,” *Nucl. Phys.* **A663**, 3 (2000) hep-ph/9907340.
2. R. K. Ellis, W. J. Stirling, and B. R. Webber, *QCD and Collider Physics* (Cambridge University Press, Cambridge, 1996).

3. J. M. Conrad, M. H. Shaevitz, and T. Bolton, “Precision Measurements with High Energy Neutrino Beams,” *Rev. Mod. Phys.* **70**, 1341 (1998), hep-ex/9707015.
4. B. Hirosky and F. Bedeschi, preliminary data presented at the XIII Hadron Collider Physics Workshop, January 14–20, 1999, Mumbai, India.
5. G. C. Blazey and B. L. Flaugher, *Ann. Rev. Nucl. Part. Sci.* **49**, 633 (1999), hep-ex/9903058.
6. C. Mesropian and A. A. Bhatti, “ $\alpha_s$  Measurement from Inclusive Jet Cross Section,” Internal CDF Notes 4892, available at [http://www-cdf.fnal.gov/physics/new/qcd/qcd99\\_blessed\\_plots.html#alpha](http://www-cdf.fnal.gov/physics/new/qcd/qcd99_blessed_plots.html#alpha).
7. S. Martí i Garca, “Review of  $\alpha_s$  Measurements at LEP 2,” hep-ex/9704016.
8. I. Hinchliffe and A. Manohar, *Ann. Rev. Nucl. Part. Sci.* **50**, 643 (2000), hep-ph/0004186.
9. D. E. Groom, *et al.* (Particle Data Group), *Euro. Phys. J.* **C15**, 1 (2000); available at <http://pdg.lbl.gov/>.
10. F. Wilczek, “Mass without Mass I: Most of Matter,” *Phys. Today* **52** (11) 11 (November, 1999).
11. C. Quigg, “Top–ology,” *Phys. Today* **50**, 20 (May, 1997); extended version circulated as FERMILAB–PUB–97/091–T, hep-ph/9704332. For more on top physics, see E. H. Simmons, “Top physics,” Lectures at TASI 2000, hep-ph/0011244.
12. R. N. Cahn, “The eighteen arbitrary parameters of the standard model in your everyday life,” *Rev. Mod. Phys.* **68**, 951 (1996).
13. F. Butler, *et al.* (GF11 Collaboration), *Nucl. Phys.* **B430**, 179 (1994).
14. S. Aoki, *et al.* (CP–PACS Collaboration), *Phys. Rev. Lett.* **84**, 238 (2000). For progress toward an unquenched spectrum see A. Ali Khan, *et al.* (CP–PACS Collaboration), “Full QCD light hadron spectrum and quark masses: Final results from CP–PACS,” hep-lat/0010078.
15. F. Wilczek, “Mass without mass II: The medium is the mass-age,” *Phys. Today* **53** (1) 13 (January, 2000).
16. Graham Ross, “Theories of Fermion Masses,” Lectures at TASI 2000.
17. Boris Kayser, “Neutrino Masses and Mixing,” Lectures at TASI 2000.
18. S. Weinberg, *Phys. Rev. D* **13**, 974 (1976), **19**, 1277 (1979); L. Susskind, **20**, 2619 (1979).
19. S. Treiman, *The Odd Quantum*, (Princeton University Press, Princeton, 1999).
20. For an introduction to what was known early on, see A. Pais, “Introducing Atoms and Their Nuclei,” in *Twentieth Century Physics*, edited by L. M. Brown, A. Pais, and B. Pippard (Institute of Physics Publishing, Bristol

- & Philadelphia; American Institute of Physics, New York, 1995), vol. II, p. 43. For a comprehensive treatment, see A. Pais, *Inward Bound: of matter and forces in the physical world* (Clarendon Press, Oxford, 1986).
21. V. L. Fitch and J. L. Rosner, “Elementary particle physics in the second half of the 20th century,” in *Twentieth Century Physics*, edited by L. M. Brown, A. Pais, and B. Pippard (Institute of Physics Publishing, Bristol & Philadelphia; American Institute of Physics, New York, 1995), vol. II, pp. 635-794.
  22. Eugene D. Commins and Philip H. Bucksbaum, *Weak Interactions of Leptons and Quarks* (Cambridge University Press, Cambridge, 1983).
  23. Robert N. Cahn and Gerson Goldhaber, *The experimental foundations of particle physics* (Cambridge University Press, Cambridge and New York, 1989).
  24. J. Chadwick, *Verh. Deutsch. Phys. Ges.* **16**, 383 (1914).
  25. W. Pauli, “Zur älteren und neueren Geschichte des Neutrinos,” in *Collected Scientific Papers*, edited by R. Kronig and V. F. Weisskopf (Interscience, New York, 1964), volume 2, p. 1313.
  26. J. Steinberger, *Ann. Rev. Nucl. Part. Sci.* **47**, *xiii* (1997).
  27. F. Reines, and C. L. Cowan, Jr., *Phys. Rev.* **92**, 830 (1953).
  28. C. L. Cowan, Jr., F. Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire, *Science* **124**, 103 (1956).
  29. F. Ajzenberg-Selove, *Nucl. Phys.* **A490**, 1-225 (1988).
  30. F. Ajzenberg-Selove, *Nucl. Phys.* **A506**, 1-158 (1990).
  31. F. Ajzenberg-Selove, *Nucl. Phys.* **A523**, 1-196 (1991).
  32. C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson, *Phys. Rev.* **105**, 1413 (1957).
  33. M. Bardon, *et al.*, *Phys. Rev. Lett.* **7**, 23 (1961).
  34. A. Possoz, *et al.*, *Phys. Lett.* **70B**, 265 (1977), Erratum **B73**, 504 (1978).
  35. M. Goldhaber, L. Grodzins and A. W. Sunyar, *Phys. Rev.* **109**, 1015 (1958).
  36. L. P. Roesch, V. L. Telegdi, P. Truttmann, A. Zehnder, L. Grenacs and L. Palfy, *Am. J. Phys.* **50**, 931 (1981).
  37. C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Perseus, New York, 1997).
  38. G. Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962); for perspective, see the Nobel Lecture of M. L. Schwartz, *Rev. Mod. Phys.* **61**, 527 (1989).
  39. Updates may be found on the DONUT collaboration’s web server, <http://fn872.fnal.gov/>. For extensive background information, see <http://www.fnal.gov/pub/donut.html>.
  40. For other textbook treatments of the electroweak theory, see T.-P. Cheng

- and L.-F. Li, *Gauge Theory of Elementary Particle Physics* (Oxford University Press, Oxford, 1984); I. J. R. Aitchison and A. J. G. Hey, *Gauge Theories in Particle Physics: A Practical Introduction*, second edition (Adam Hilger, Bristol, 1989).
41. M. K. Gaillard, P. D. Grannis and F. J. Sciulli, “The standard model of particle physics,” *Rev. Mod. Phys.* **71**, S96 (1999) hep-ph/9812285, included in *More Things in Heaven and Earth: A Celebration of Physics at the Millennium*, edited by B. Bederson (Springer-Verlag, New York, 1999), p. 161.
  42. V. L. Ginzburg and L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950); English translation: *Men of Physics: Landau*, Vol. I, edited by D. ter Haar (Pergamon, New York, 1965), p. 138.
  43. P. W. Higgs, *Phys. Rev. Lett.* **12**, 132 (1964). The parallel between electroweak symmetry breaking and the Ginzburg-Landau theory is drawn carefully in R. E. Marshak, *Conceptual Foundations of Modern Particle Physics* (World Scientific, Singapore, 1993), §4.4. For a rich discussion of superconductivity as a consequence of the spontaneous breaking of electromagnetic gauge symmetry, see Steven Weinberg, *The Quantum Theory of Fields*, vol. 2 (Cambridge University Press, Cambridge, 1996), §21.6.
  44. F. J. Hasert, *et al.* (Gargamelle Neutrino Collaboration), *Phys. Lett.* **B46**, 138 (1973).
  45. A. R. Barker, “Kaon Physics: Experiments,” Lectures at TASI 2000.
  46. G. Buchalla, “Kaon and Charm Physics: Theory,” Lectures at TASI 2000.
  47. S. Adler, *et al.* (E787 Collaboration), *Phys. Rev. Lett.* **79**, 2204 (1997), hep-ex/9708031, and *Phys. Rev. Lett.* **84**, 3768 (00), hep-ex/0002015; for an update, see T. K. Komatsubara [for the E787 Collaboration], “Recent results from the BNL E787 experiment,” hep-ex/0009047.
  48. D. Ambrose, *et al.*, (E871 Collaboration), *Phys. Rev. Lett.* **84**, 1389 (2000).
  49. S. L. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
  50. M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
  51. For a description of methods and results for many important measurements, see Young-Kee Kim, “Electroweak Experiments,” Lectures at TASI 2000.
  52. A. Sirlin, “Ten Years of Precision Electroweak Physics,” in *Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy LP99*, ed. J. A. Jaros and M. E. Peskin, *eConf C990809*, 398 (2000), hep-ph/9912227.
  53. M. Swartz, “Precision Electroweak Physics at the Z,” in *Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy*

- LP99*, ed. J. A. Jaros and M. E. Peskin, *eConf C990809*, 307 (2000), hep-ex/9912026.
54. LEP Electroweak Working Group, <http://www.cern.ch/LEPEWWG/>.
  55. W. J. Marciano, “Precision electroweak parameters and the Higgs mass,” in *Physics Potential and Development of Muon Colliders and Neutrino Factories*, edited by D. B. Cline, AIP Conference Proceedings 542 (American Institute of Physics, Woodbury, NY, 2000), p. 48, hep-ph/0003181.
  56. Carl Wieman, “Tests of the Standard Model in Atomic Physics,” Lectures at TASI 2000.
  57. S. C. Bennett and C. E. Wieman, *Phys. Rev. Lett.* **82**, 2484 (1999).
  58. J. Erler and P. Langacker, *Phys. Rev. Lett.* **84**, 212 (2000), hep-ph/9910315; R. Casalbuoni S. De Curtis, D. Dominici, and R. Gatto, *Phys. Lett.* **B460**, 135 (1999) hep-ph/9905568; J. L. Rosner, *Phys. Rev. D* **61**, 016006 (2000) hep-ph/9907524.
  59. J. L. Hewett and T. G. Rizzo, “Low-Energy Phenomenology Of Superstring Inspired  $E_6$  Models,” *Phys. Rep.* **183**, 193 (1989).
  60. M. Veltman, *Phys. Rev. Lett.* **34**, 777 (1975). A. D. Linde, *JETP Lett.* **19**, 183 (1974). J. Dreitlein, *Phys. Rev. Lett.* **33**, 1243 (1977). S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
  61. M. S. Turner, “Cosmological Parameters,” in *Proceedings of Conference on Particle Physics and the Early Universe (COSMO 98)*, edited by D. Caldwell (American Institute of Physics, Woodbury, NY, 1999) AIP conference proceedings 478, pp. 113-128, astro-ph/9904051; M. Fukugita and C. J. Hogan, “Global Cosmological Parameters:  $H_0$ ,  $\Omega_M$ , and  $\Lambda$ ,” §17 of Ref. <sup>9</sup>.
  62. For a preliminary “end-of-LEP” summary, see Peter Igo-Kemenes, “Status of Higgs Boson Searches,” transparencies available at [http://lephiggs.web.cern.ch/LEPHIGGS/talks/pik\\_lepc\\_nov3\\_2000.pdf](http://lephiggs.web.cern.ch/LEPHIGGS/talks/pik_lepc_nov3_2000.pdf). See also the very useful LEP Higgs Working Group summary of the standard-model Higgs search for the November 3, 2000, meeting of the LEPC, available at [http://lephiggs.web.cern.ch/LEPHIGGS/papers/doc\\_nov3\\_2000.ps](http://lephiggs.web.cern.ch/LEPHIGGS/papers/doc_nov3_2000.ps)
  63. B. W. Lee, C. Quigg, and H. B. Thacker, *Phys. Rev. D* **16**, 1519 (1977).
  64. In the high-energy limit, an amplitude for longitudinal gauge-boson interactions may be replaced by a corresponding amplitude for the scattering of massless Goldstone bosons:  $\mathcal{M}(W_L, Z_L) = \mathcal{M}(w, z) + \mathcal{O}(M_W/\sqrt{s})$ . The equivalence theorem can be traced to the work of John M. Cornwall, David N. Levin, and George Tiktopoulos, *Phys. Rev. D* **10**, 1145 (1974), **11**, 972E (1975). It was applied to this problem by Lee, Quigg, and Thacker, Ref. <sup>63</sup>, and developed extensively by Chanowitz and Gailard, Ref. <sup>65</sup>, and others.

65. See, for example, M. Chanowitz and M. K. Gaillard, *Nucl. Phys.* **B261**, 279 (1985); M. Chanowitz, M. Golden, and H. Georgi, *Phys. Rev.* **D36**, 1490 (1987); *Phys. Rev. Lett.* **57**, 2344 (1986).
66. K. G. Wilson, *Phys. Rev.* **D3**, 1818 (1971); *Phys. Rev.* **B4**, 3184 (1971); K. G. Wilson and J. Kogut, *Phys. Rep.* **12**, 76 (1974).
67. For a review, see Urs M. Heller, *Nucl. Phys. B (Proc. Supp.)* **34** (1994) 101 hep-lat/9311058. The numerical analysis that leads to these conclusions is reported in Urs M. Heller, *et al.*, *Nucl. Phys.* **B405**, 555 (1993) hep-ph/9303215.
68. Lower bounds on the Higgs mass date from the work of A. D. Linde, *Zh. Eksp. Teor. Fiz. Pis'ma Red.* **23**, 73 (1976) [English translation: *JETP Lett.* **23**, 64 (1976)]; S. Weinberg, *Phys. Rev. Lett.* **36**, 294 (1976). An early investigation of the effects of heavy fermions appears in P. Q. Hung, *Phys. Rev. Lett.* **42**, 873 (1979). For a review of lower bounds deduced from the requirement of vacuum stability of electroweak potentials, see M. Sher, *Phys. Rep.* **179**, 273 (1989). For useful updates in light of the large mass of the top quark ( $m_t \approx 175 \text{ GeV}/c^2$ ), see G. Altarelli and G. Isidori, *Phys. Lett.* **B337**, 141 (1994); J. Espinosa and M. Quirós, *Phys. Lett.* **B353**, 257 (1995).
69. J. A. Casas, J. R. Espinosa, M. Quirós, and A. Riotto, *Nucl. Phys.* **B436**, 3 (1995); Y. F. Pirogov and O. V. Zenin, *Eur. Phys. J.* **C10**, 629 (1999), hep-ph/9808396.
70. L. Maiani, G. Parisi, and R. Petronzio, *Nucl. Phys.* **B136**, 115 (1978).
71. Two early studies provide valuable introductions to a light Higgs boson: J. Ellis, M. K. Gaillard and D. V. Nanopoulos, *Nucl. Phys.* **B106**, 292 (1976), and A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, *Usp. Fiz. Nauk* **131**, 537 (1980) [English translation: *Sov. Phys.-Uspekhi* **23**, 429 (1980)]. For a first look at the properties of a heavy Higgs boson, see Lee, Quigg, and Thacker, Ref. <sup>63</sup>. A useful general reference is J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Redwood City, California, 1990).
72. M. Carena, *et al.*, "Report of the Tevatron Higgs working group," hep-ph/0010338.
73. M. Veltman, *Acta Phys. Polon.* **B12**, 437 (1981); C. H. Llewellyn Smith, *Phys. Rep.* **105**, 53 (1984).
74. For an approachable introduction, see Joseph D. Lykken, "Introduction to Supersymmetry," in *Fields, Strings, and Duality: TASI 96*, edited by C. Efthimiou and B. Greene (World Scientific, Singapore, 1997). p. 85 hep-th/9612114.
75. Stephen P. Martin, "A Supersymmetry Primer" hep-ph/9709356, latest

- version available at <http://zippy.physics.niu.edu/primer.shtml>.
76. S. Dawson, “The MSSM and Why It Works,” in *Supersymmetry, Supergravity and Supercolliders: TASI 97*, edited by J. Bagger (World Scientific, Singapore, 1999), p. 261 hep-ph/9712464.
  77. H. Murayama, “Supersymmetry,” Lectures at TASI 2000.
  78. R. S. Chivukula, “An Introduction to dynamical electroweak symmetry breaking,” in *Advanced School on Electroweak Theory*, edited by D. Espriu and A. Pich (World Scientific, Singapore, 1998) p. 77, hep-ph/9701322.
  79. R. S. Chivukula, “Lectures on technicolor and compositeness,” Lectures at TASI 2000, hep-ph/0011264.
  80. E. Eichten, and K. Lane, *Phys. Lett.* **90B**, 125 (1980); S. Dimopoulos, and L. Susskind, *Nucl. Phys.* **B155**, 237 (1979).
  81. I will not discuss this alternative further. See M. S. Chanowitz, “Strong  $WW$  scattering at the end of the 90’s: Theory and experimental prospects” hep-ph/9812215.
  82. M. Weinstein, *Phys. Rev. D* **8**, 2511 (1973).
  83. T. Schäfer, “Color Superconductivity,” nucl-th/9911017, to be published in the proceedings of 10th International Conference on Recent Progress in Many-Body Theories (MBX), Seattle, Washington, 10-15 Sep 1999.
  84. F. Wilczek, “The Recent Excitement in High-Density QCD,” *Nucl. Phys.* **A663**, 257 (2000), hep-ph/9908480.
  85. K. Rajagopal and F. Wilczek, “The condensed matter physics of QCD,” hep-ph/0011333, to appear as Chapter 35 in the Festschrift in honor of B. L. Ioffe, *At the Frontier of Particle Physics / Handbook of QCD*, ed. M. Shifman (World Scientific).
  86. I. Antoniadis, *Phys. Lett.* **B246**, 377 (1990); J. D. Lykken, *Phys. Rev. D* **54**, 3693 (1996), hep-th/9603133; N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *Phys. Lett.* **B429**, 263 (1998), hep-ph/9803315; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *Phys. Lett.* **B436**, 257 (1998), hep-ph/9804398.
  87. C. D. Hoyle, *et al.*, “Submillimeter tests of the gravitational inverse-square law: a search for ‘large’ extra dimensions,” hep-ex/0011014.
  88. J. K. Hoskins, R. D. Newman, R. Spero and J. Schultz, *Phys. Rev. D* **32**, 3084 (1985).
  89. V. P. Mitrofanov and O. I. Ponomareva, *Sov. Phys.—JETP* **67**, 1963 (1988).
  90. S. D. Lamoreaux, *Phys. Rev. Lett.* **78**, 5 (1998).
  91. G. C. Long, H. Chang and J. Price, *Nucl. Phys.* **B539**, 23 (1999).
  92. D. B. Kaplan and M. B. Wise, hep-ph/0008116.



93. For an introduction, see J. Lykken, “Physics of Extra Dimensions,” Lectures at TASI 2000.
94. G. Giudice, R. Rattazzi, and J. Wells, *Nucl. Phys.* **B544**, 3 (1999), hep-ph/9811291. E. A. Mirabelli, M. Perelstein, and M. E. Peskin, *Phys. Rev. Lett.* **82**, 2236 (1999), hep-ph/9811337. J. L. Hewett, *Phys. Rev. Lett.* **82**, 4765 (1999), hep-ph/9811356.
95. For the important application to electroweak baryogenesis, see, for example, A. Riotto and M. Trodden, “Recent progress in baryogenesis,” *Ann. Rev. Nucl. Part. Sci.* **49**, 35 (1999), hep-ph/9901362.
96. C. Quigg, “Electroweak symmetry breaking and the Higgs sector,” lectures presented at 27th International Meeting on Fundamental Physics, Sierra Nevada, Granada, Spain, 1-5 Feb 1999. *Acta Phys. Polon.* **B30**, 2145 (1999), hep-ph/9905369.