Fermion Masses in the WS model

Weak interaction phenomenology:

$e = g \sin\vartheta_W \qquad e = \frac{g g'}{\sqrt{g^2+g'^2}}$

$e = g' \cos\vartheta_W$

$\tan\vartheta_W = g'/g$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W} = \frac{1}{2v^2}$$

$$M_W = \frac{38}{\sin\vartheta_W} \text{ Gev}$$

$$M_{Z^0} = \frac{38}{\frac{1}{2}\sin 2\theta_W} \text{ Gev}$$

Muon decay

$$\mathcal{M} = -\frac{i g^2}{16\pi^2} \overline{u}(\nu_\mu)\gamma^\mu(1-\gamma_5)u(\mu)\,\overline{u}(e)\gamma^\nu(1-\gamma_5)v(\bar\nu_e) \times$$

$$\times \frac{g_{\mu\nu} - k_\mu k_\nu/M_W^2}{k^2 - M_W^2}$$

For a review of the weak interactions please read chapter 12 of Halzen + Martin, which is what I covered in class (some of it)

Fermion Masses

Try $\mathcal{L}_m = -m_e \bar\psi \psi$.

$$-m_e \bar e e = -m_e \bar e \left(\frac{1-\gamma_5}{2} + \frac{1+\gamma_5}{2} \right) e$$

$$= -m_e (\underset{\substack{\uparrow \\ \text{doublet}}}{\bar e_L} \underset{\substack{\uparrow \\ \\ \text{singlet}}}{e_R} + \bar e_R e_L)$$

$$\boxed{\text{Violates gauge invariance}}$$

Lepton Masses

Clearly we need another way: (114)

$$\mathcal{L} = -G_e \Big[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix}\phi^+\\ \phi_0\end{pmatrix} e_R + \bar{e}_R (\phi^-, \bar{\phi}^0) \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \Big]$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h+v\end{pmatrix}$$

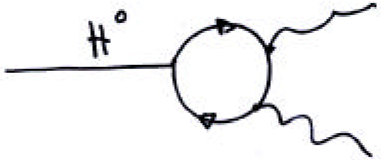
$$\mathcal{L} = -G_e \Big[(\bar{\nu}_e, \bar{e})_L \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h+v\end{pmatrix} e_R + \bar{e}_R \Big(0, \frac{h+v}{\sqrt{2}}\Big) \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \Big]$$

$$\boxed{\mathcal{L}_Y = -\frac{G_e}{\sqrt{2}}(h+v)\left[\bar{e}_L e_R + \bar{e}_R e_L\right]}$$

(115)

Therefore $m_e = -\frac{G_e v}{\sqrt{2}}$ and

$$\mathcal{L}_Y = -m_e(\bar{e}_L e_R + \bar{e}_R e_L) - \frac{m_e}{v}\bar{e} e h$$



the second term predicts that the higgs couples to leptons with a strength proportional to the lepton mass m_e.

Quarks Masses (Very Brief)

Introducing Quarks in the Standard Model

Introducing quarks in the standard model is different than introducing leptons in two ways.

① If you try $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$ doublets you need to find a way to accomodate Cabibbo mixings

② the upper part of the multiplet needs to have mass. (By now we know that neutrinos have mass so this second difference is not valid anymore ... this way can be used to generate masses to neutrinos then)

- We do this by using in addition to the higgs doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ also the SU(2) doublet $i\sigma_2 \Phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix} = \tilde{\Phi}$ $\boxed{Y=-1}$
- The quark doublets can be written as

$$N_L = \begin{pmatrix} p_L \\ n_L \cos\theta + \lambda_L \sin\theta \end{pmatrix} \equiv \begin{pmatrix} p_L \\ n_{cL} \end{pmatrix}$$

$$\lambda_c = \lambda_L \cos\theta - n_L \sin\theta$$

(assuming two quark mixing ....)

Then

$$\mathcal{L}_Y = [N_L \tilde{\Phi} p_R + h.c.] + G_2[\bar{N}_L \Phi n_R + hc] +$$

$$G_3[\bar{N}_L \Phi \lambda_R + h.c.] + G_4[n_R \lambda_c] + G_5[\lambda_R$$

This again will give masses to all quarks and will predict that the Higgs couples to the quarks with couplings proportional to the quark mass:

Running out of time.....

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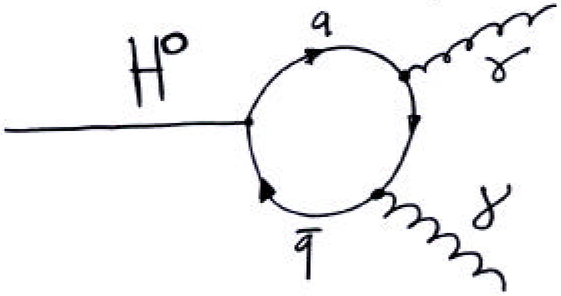
By now it should be clear that the $H^0 \rightarrow \gamma\gamma$ channel at LHC goes through a quark loop with a coupling proportional to the quark mass



Same is true when you produce the Higgs via gg