WS-Model Predictions: Higgs Sector

(101)

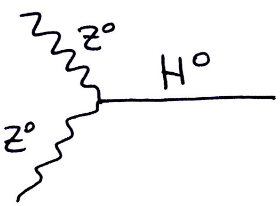
But there is more we can get from this Lagrangian:

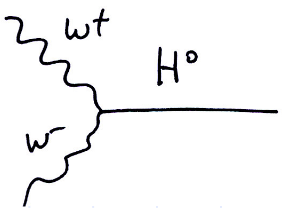
$$\mathcal{L}_{Higgs} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{4}g^2(h+v)^2 W^+_\mu W^{-\mu} + \frac{1}{8}(h+v)^2(g^2+g'^2) Z^0_\mu Z^{0\mu} + V(h)$$

1._ the linear terms in h: $\frac{1}{4}g^2 2hv\, W^+_\mu W^{-\mu}$

$\frac{1}{8}(g^2+g'^2) 2hv\, Z_\mu Z^\mu$

predict





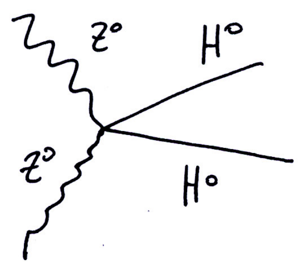
(102)

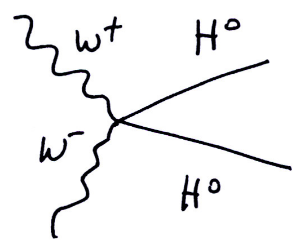
2._ The theory predicts no "pointlike" Couplings of the Higgs to photons!! (later we will see that it only goes through quark loops)

3._ The quadratic terms

$$\frac{1}{4}g^2 h^2 W^+_\mu W^{-\mu} + \frac{1}{8}(g^2+g'^2)h^2 Z^0_\mu Z^{0\mu}$$

predict:



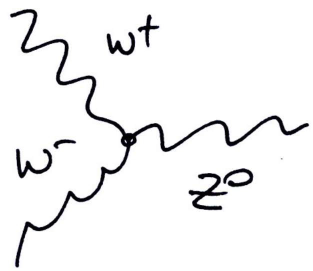


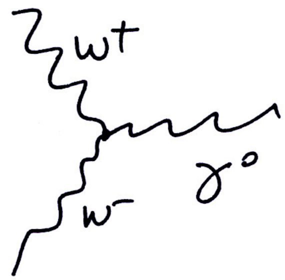
4._ $m_\gamma = 0$

Introducing Leptons in the WS Model

(103)

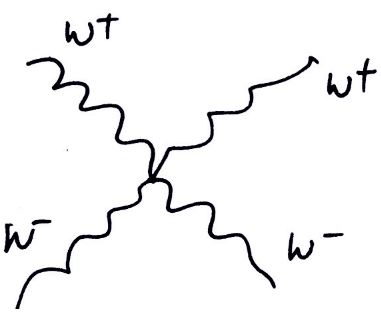
5.- Due to the non-abelian nature of our theory we get terms of the type





They come from the $g\epsilon^{ijk}A_\mu^j A_\nu^k$ term of $F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g\epsilon^{ijk}A_\mu^j A_\nu^k$ when these terms are multiplied by $\partial_\mu A_\nu^i$ in the gauge Lagrangian.

Clearly you could also have



type of terms

INTRODUCING LEPTONS IN THE THEORY (104)

$L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$ is an $SU(2)_L$ doublet with weak hypercharge $\boxed{Y_W = -1}$

$R = e_R$ is an $SU(2)_L$ Singlet with $\boxed{Y_W = -2}$

$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ with $\boxed{Y_W = +1}$ also an $SU(2)_L$ doublet

$SU(2)_L \otimes U_Y(1)$ implies $D_\mu = \partial_\mu - i\frac{g'}{2}Y_W B_\mu - i\frac{g}{2}\vec{\tau}\vec{A}_\mu$

$$\chi_L \to \chi_L' = e^{i\beta(x)Y + i\vec{\alpha}\cdot\vec{\tau}}\chi_L \quad (1)$$

$$\psi_R \to \psi_R' = e^{i\beta(x)Y}\psi_R \quad (2)$$

The vacuum $\langle\phi_0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ n+v \end{pmatrix}$ brakes all the $SU(2)$ generator and also the $U_Y(1)$

Charge Conservation

(105)

Try $\Phi_0 \to \Phi_0' = e^{i g/2\, \vec{\alpha}\cdot\vec{\tau}} \Phi_0$ and you will find that all 3 are broken (the vacuum is not invariant). Same is true for $U_Y(1)$.

But since $1, \sigma^i$ form a basis I am going to redefine the generators in an effort to see if any combination is unbroken (need $m_\gamma = 0$ no matter what)

Try $\hat{Q} = \hat{T}_3 + \frac{1}{2}\hat{Y}$

$$\hat{Q} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{Y}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$$

$$\hat{Q}\Phi_0 \overset{Y=+1}{=} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{u+v}{\sqrt{2}} \end{pmatrix} = 0!$$

$\therefore$ $e^{iY\beta(x)}$ can be redefined to be $e^{i\hat{Q}\beta(x)}$ $\therefore \delta\Phi_0 = i\beta(x)\hat{Q}\Phi_0 = 0$

$\therefore \delta\Phi_0 = 0 \Rightarrow$ one unbroken generator has been found

(106)

Consider the effect of $\hat{Q}$ on a lepton doublet:

(a) $\hat{Q}\chi_L = \left\{ \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} -1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \right\} \chi_L$

$\boxed{Y=-1}$

$$= \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} = \begin{pmatrix} 0\,\nu_L \\ -1\, e_L^- \end{pmatrix}$$

$$\hat{Q}\cdot R = \hat{Q}\, R = \left(0 + \tfrac{1}{2}(-2)\right) e_R = -e_R$$

(b) $\boxed{Y=-2}$

So it is a charge operator and $\hat{Q} = T_3 + \frac{Y}{2}$ is the generator of $U_{em}(1)$ (electromagnetic)

Fortunately or unfortunately this also means that along with a massless photon the theory predicts also a massive scalar left to be found by the exp.

The Lepton couplings to W$^\pm$

(104)

So far we have

Field	T_3	Y	Q	$SU(2)_L$
ν_τ, ν_e, ν_μ	1/2	−1	0	upper part of a doublet
e^-, μ^-, τ^-	−1/2	−1	−1	Lower part of a doublet
H^0	−1/2	+1	0	Lower part of a doublet
e_R, μ_R, τ_R	0	−2	−1	Singlets
Quarks				

Lets write the lepton Lagrangian:

$$\mathcal{L} = \bar{R}\, i\gamma^\mu \underbrace{\left(\partial_\mu + i g' B_\mu\right)}_{Y_w = -2} + \bar{L}\, i\gamma^\mu \underbrace{\left(\partial_\mu + \frac{i}{2} g' B_\mu - i g \frac{\vec{\tau}\vec{A}}{2}\right)}_{Y = -1} L$$

Look at the A^1_μ, A^2_μ couplings

$$\frac{g}{2} \bar{L} \gamma^\mu \left(T^1 A^1_\mu + T^2 A^2_\mu\right) L = \frac{g}{2} \times$$

$$\left(\bar{\nu}\ \ \bar{e}_L\right) \gamma^\mu \begin{pmatrix} 0 & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= \frac{g}{2} \left(\bar{\nu}\ \ \bar{e}_L\right) \gamma^\mu \begin{pmatrix} \sqrt{2}\, W^+_\mu e_L \\ \sqrt{2}\, W^-_\mu \nu_L \end{pmatrix}$$

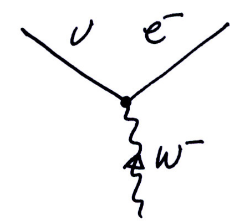
(108)

So $\mathcal{L}_I(W) =$

$$\frac{g}{2} \bar{L} \gamma^\mu \left(T^1 A^1_\mu + T^2 A^2_\mu\right) L =$$

$$\frac{g}{\sqrt{2}} \left\{ \left(\bar{\nu} \gamma^\mu e_L\right) W^+_\mu + \left(\bar{e}_L \gamma^\mu \nu_L\right) W^-_\mu \right\}$$

Which predicts that the $W^\pm$ couples to leptons via the weak interaction



If you compare with the old weak interaction theory based on the G_F coupling you get

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{8 v^2}}$$

The γ and $Z^{\pm}$ couplings to Leptons

(109)

Do the same for γ, Z^0

$$\mathcal{L}_I(A,Z) = -g'\bar{R}\gamma^\mu R B_\mu + \frac{g}{2}\bar{L}\gamma^\mu \tau^3 L A^3_\mu - \frac{g'}{2}\bar{L}\gamma^\mu L B_\mu \rightarrow$$

$$\mathcal{L}_I(A,Z) = -g'\bar{e}_R\gamma^\mu e_R B_\mu + \frac{g}{2}(\bar\nu\ \bar e)\gamma^\mu\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\nu\\e\end{pmatrix}A^3_\mu - \frac{g'}{2}(\bar\nu\gamma^\mu\nu + \bar e\gamma^\mu e)B_\mu$$

$$\mathcal{L}_I(A,Z) = -\frac{g'}{2}\{\underbrace{2\bar e_R\gamma^\mu e_R + \bar\nu_L\gamma^\mu\nu_L + \bar e_L\gamma^\mu e_L}_{X}\}B_\mu + \frac{g}{2}[\underbrace{\bar\nu\gamma^\mu\nu - \bar e_L\gamma^\mu e_L}_{Y}]A^3_\mu$$

$$\mathcal{L}_I(A,Z) = -\frac{g'}{2}\cdot X\cdot(\cos\theta_W A_\mu + \sin\theta_W Z_\mu) + \frac{g}{2}Y(\sin\theta_W A_\mu - \cos\theta_W Z_\mu)$$

(110)

$$\mathcal{L}_I = \Big\{\underbrace{\cos\theta_W}_{\frac{g}{\sqrt{g^2+g'^2}}}\left(\frac{-g'}{2}\right)[2\bar e_R\gamma^\mu e_R + \bar\nu_L\gamma^\mu\nu_L + \bar e_L\gamma^\mu e_L] + \frac{g}{2}[\bar\nu\gamma^\mu\nu - \bar e_L\gamma^\mu e_L]\underbrace{\sin\theta_W}_{\frac{g'}{\sqrt{g^2+g'^2}}}\Big\}A_\mu +$$

nice that they go away otherwise we would have $\nu\gamma$ coupling...

$$+Z_\mu\Big\{-\frac{g'}{2}\sin\theta_W(X) - \frac{g}{2}\cos\theta_W(Y)\Big\}\rightarrow$$

$$\mathcal{L}_I(A,Z) = -\frac{gg'}{\sqrt{g^2+g'^2}}[\bar e_R\gamma^\mu e_R + \bar e_L\gamma^\mu e_L]A_\mu$$

$$+Z_\mu\Big\{-\frac{g'^2}{2\sqrt{g^2+g'^2}}[X] - \frac{g^2}{2\sqrt{g^2+g'^2}}[Y]\Big\}$$

So
$$\boxed{e = \frac{gg'}{\sqrt{g^2+g'^2}}}$$

Predictions:

$$\mathcal{L}_I = \underbrace{-e\left[\bar{e}_R \gamma^\mu e_R + \bar{e}_L \gamma^\mu e_L\right]}_{QED} + \qquad (111)$$

$$+ Z^0_\mu \left\{ \frac{-g'^2}{2\sqrt{g'^2+g^2}} \left(2\bar{e}_R \gamma^\mu e_R + \bar{\nu}_L \gamma^\mu \nu_L + \bar{e}_L \gamma^\mu e_L\right) - \frac{g^2}{2\sqrt{g'^2+g^2}} \left(\bar{\nu} \gamma^\mu \nu - \bar{e}_L \gamma^\mu e_L\right) \right\}$$

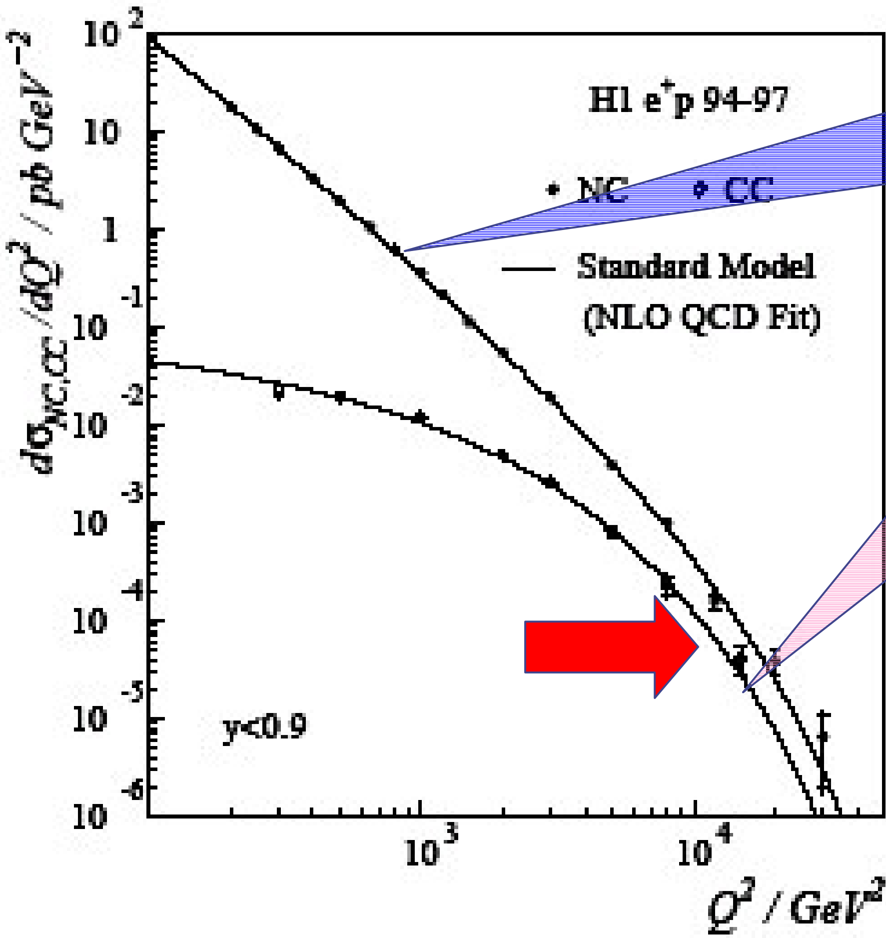
Predictions so far:

1.- We get a photon that couples to charge and therefore we are consistent with the QED.

2.- Z^0 couples to both ν and e^-

3.- the photon does not couple to neutrinos

Standrad Model versus Data I



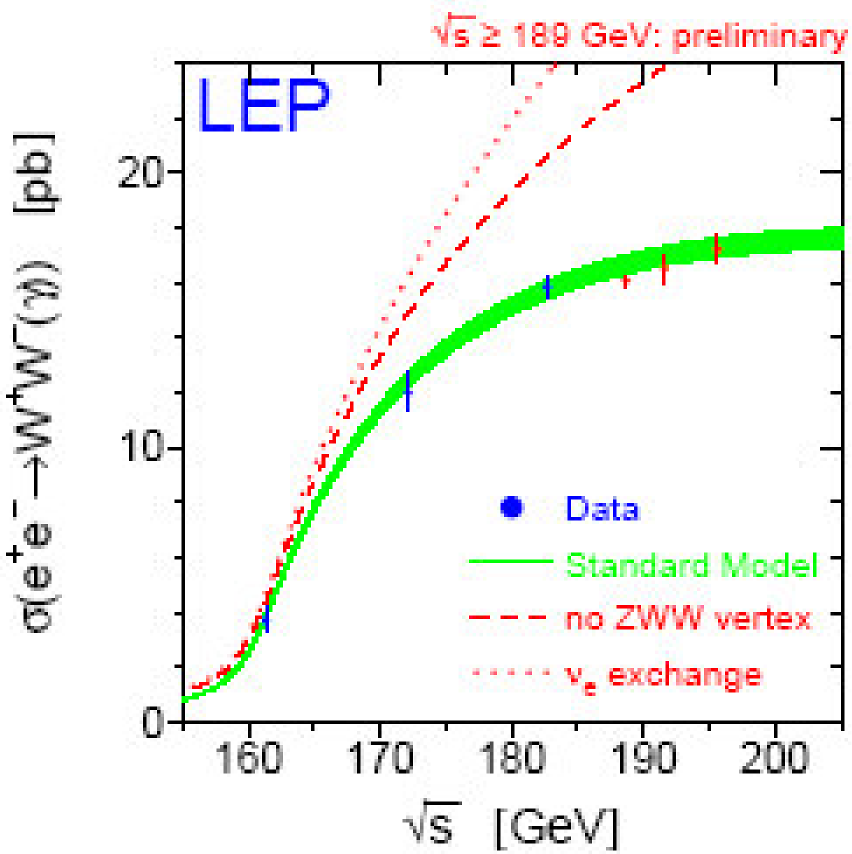
ep-Neutral Current Cross Section (γ/Z^0)

ep-Charged Current Cross Section (γ/Z^0)

Charged and Neutral Currents have the same Strength at high Q^2

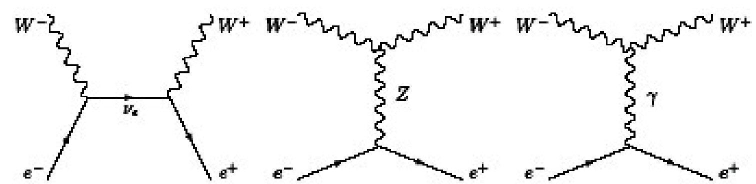
$\Rightarrow$ **Unification !!**

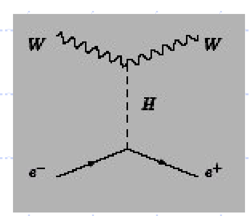
Data vs Standard Model Predictions II



- The LEP data agree with the theory only if we include ZWW Vertex diagram => Non-Abelian theory.

Main diagrams:





Also needed

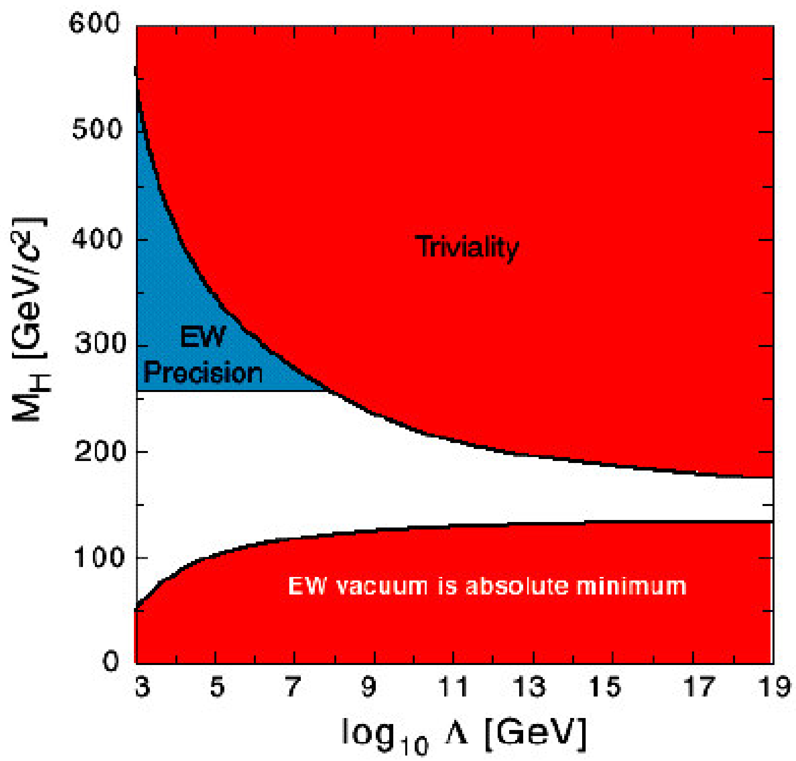
Higgs: what do we expect ? (I)

- General theoretical considerations imply that the Higgs cannot be too heavy:

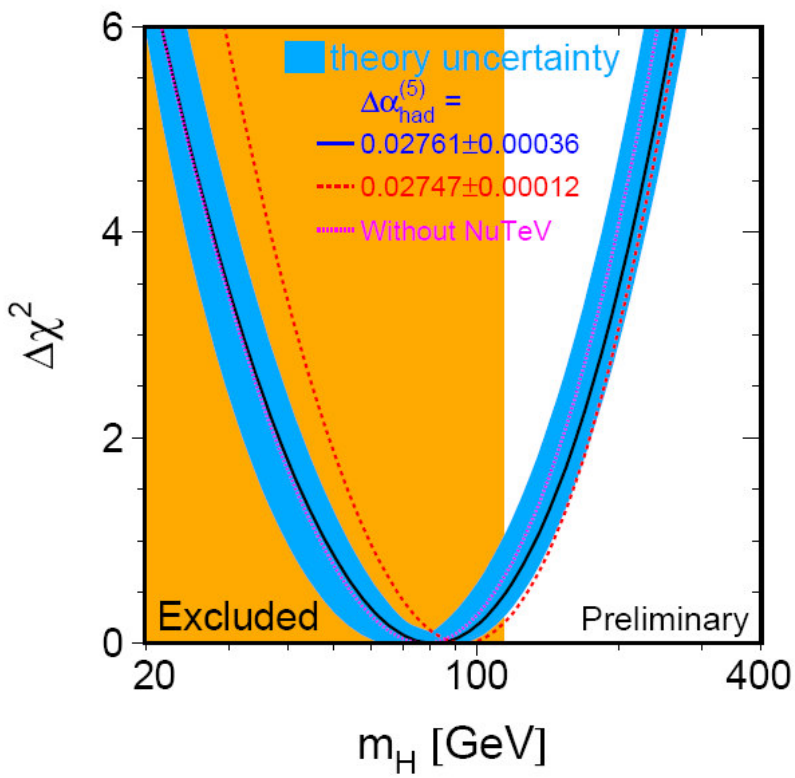
$$\Lambda \leq M_H \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

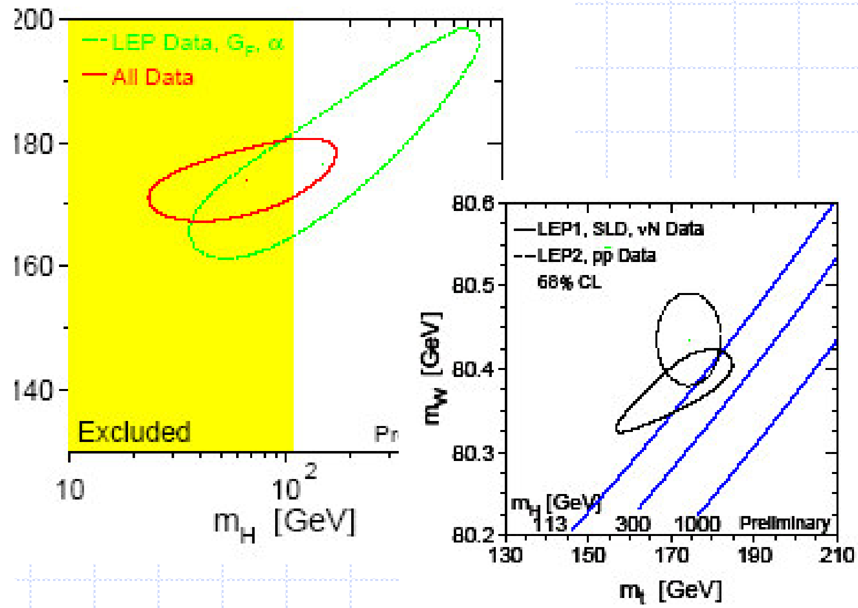
- And cannot be too light either:

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2} F \log\left(\Lambda^2 / v^2\right)$$



LEP Data vs Standard Model Predictions

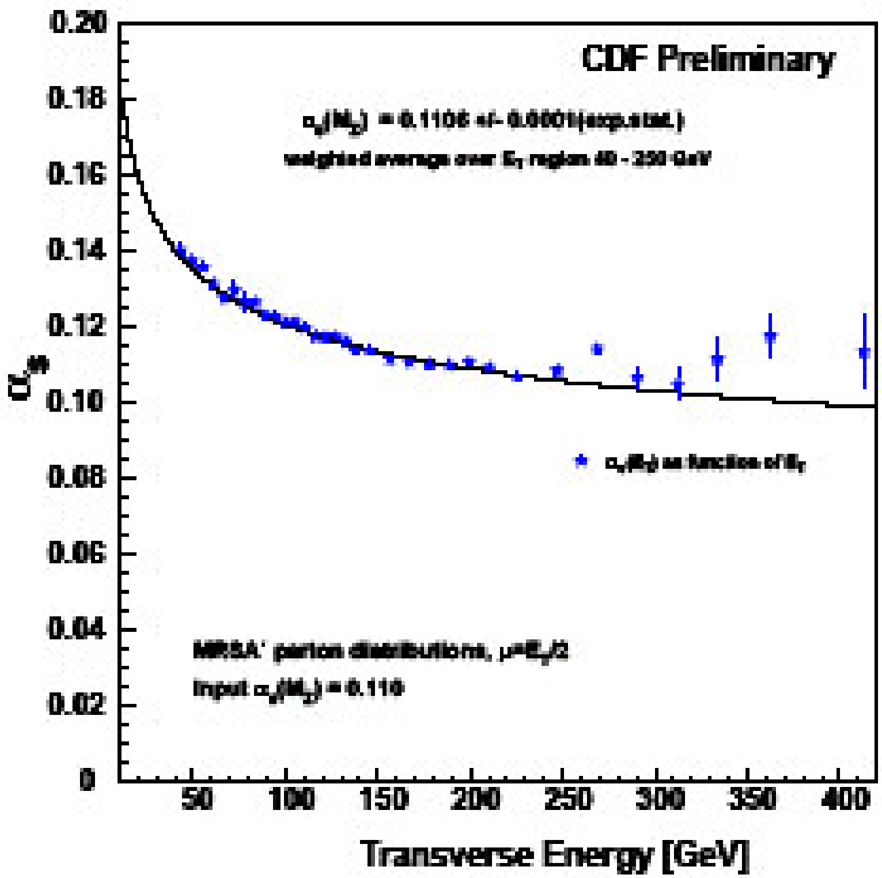




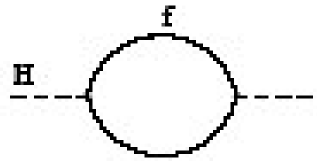
Main diagrams:

- The data are not in agreement with the standard model if we do not include diagrams with a massive S=0 particle. Large correlation with the top-quark mass

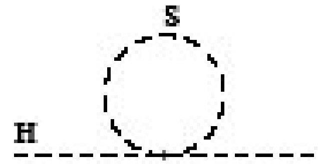
Running of the QCD coupling constant



Is Everything OK if the Higgs is found ?



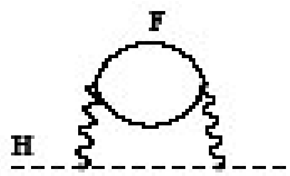
(a)



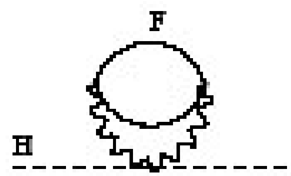
(b)

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_f^2 \ln(\Lambda_{UV}/m_f) + \dots\right].$$

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \ln(\Lambda_{UV}/m_S) + \dots\right].$$



(a)



(b)

$$\Delta m_H^2 = x\left(\frac{g^2}{16\pi^2}\right)^2\left[a\Lambda_{UV}^2 + 48m_F^2 \ln(\Lambda_{UV}/m_F) + \dots\right],$$

Even then…

For the usual Higgs potential, $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|(\varphi^\dagger\varphi)^2$, the value of the potential at the minimum is

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0. \tag{10}$$

Identifying $M_H^2 = -2\mu^2$, we see that the Higgs potential contributes a field-independent constant term,

$$\varrho_H \equiv \frac{M_H^2 v^2}{8}. \tag{11}$$

I have chosen the notation ϱ_H because the constant term in the Lagrangian plays the role of a vacuum energy density. When we consider gravitation, adding a vacuum energy density ϱ_{vac} is equivalent to adding a cosmological constant term to Einstein's equation. Although recent observations [15] raise the intriguing possibility that the cosmological constant may be different from zero, the essential observational fact is that the vacuum energy density must be very tiny indeed, [16]

$$\varrho_{\text{vac}} \lesssim 10^{-46}\ \text{GeV}^4\ . \tag{12}$$

Therein lies the puzzle: if we take $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV and insert the current experimental lower bound [28] $M_H \gtrsim 105\ \text{GeV}/c^2$ into (11), we find that the contribution of the Higgs field to the vacuum energy density is

$$\varrho_H \gtrsim 8 \times 10^7\ \text{GeV}^4, \tag{13}$$

some 54 orders of magnitude larger than the upper bound inferred from the cosmological constant.