Higgs Mechanism

Higgs mechanism

88

Consider the same theory as before but now require that the theory is gauged. That is, it is invariant under local U(1) transform.

$$\boxed{\begin{array}{ll} D_\mu = \partial_\mu - ieA_\mu & A_\mu' = A_\mu - \partial_\mu \alpha(x) \\ U = e^{-ie\alpha(x)} & \Phi' = U\Phi \end{array}}$$

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^* D^\mu\phi - \frac{m^2}{2}\phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu + ieA_\mu)\rho e^{-i\theta}(\partial^\mu - ieA^\mu)\rho e^{i\theta} +$$

$$\boxed{\Phi = \rho e^{i\theta}} \qquad - \frac{m^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4$$

$$\mathcal{L} = \frac{1}{2}\left\{\partial_\mu\rho e^{-i\theta} + \rho(-i\partial_\mu\theta)e^{-i\theta} + ieA_\mu\rho e^{-i\theta}\right\} \cdot$$

$$\left\{\partial_\mu\rho e^{i\theta} + i\partial_\mu\theta\rho e^{i\theta} - ieA^\mu\rho e^{i\theta}\right\} +$$

$$- \frac{m^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4$$

89

$$\mathcal{L} = \frac{1}{2}\left\{\partial_\mu\rho + \rho \cdot i \cdot (A_\mu \cdot e - \partial_\mu\theta)\right\} \cdot$$

$$\left\{\partial^\mu\rho - i\rho(A^\mu \cdot e - \partial_\mu\theta)\right\} - \frac{m^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4$$

Call: $\Theta = \frac{1}{e}\theta$

$$\mathcal{L} = \frac{1}{2}\left\{\partial_\mu\rho + ie\rho(A_\mu - \partial_\mu\Theta)\right\} \cdot$$

$$\left\{\partial^\mu\rho - ie\rho(A_\mu - \partial_\mu\Theta)\right\} - \frac{m^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4$$

Call: $A_\mu' = A_\mu - \partial_\mu\Theta$

$$\mathcal{L} = \frac{1}{2}\left\{\partial_\mu\rho + ie\rho[A_\mu']\right\}\left\{\partial^\mu\rho - ie\rho[A_\mu']\right\} +$$

$$- \frac{m^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4$$

So what just happened is that the vector boson A_μ which had no mass and two degrees of freedom (2-transv. polarizations)

The Goldstone Bosons Disappear…

ate the Goldstone Boson θ that (90)
had no mass and one degree of freedom and now we have a massive vector Boson with 3 degrees of freedom (two transverse and one longitudinal polarizations)

So $\mathcal{L} = \frac{1}{2}\partial^\mu\rho\,\partial_\mu\rho + e^2\rho^2 A'_\mu A'^\mu - \frac{m^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4 +$

$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (added so that A_μ has a kinetic term also)

$\rho = n + v$ (same trick)

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho}_{1/2\,\partial_\mu n\,\partial^\mu n} + e^2(n+v)^2 A''_\mu A''^\mu - \frac{m^2}{2}(n^2+v^2+2nv) + $$

$$- \frac{\lambda}{4}(n^2+v^2+2nv)(n^2+v^2+2nv)$$

$$e^2(n^2+v^2+2nv)A'_\mu A'^\mu = v^2 e^2 A'_\mu A'^\mu + HOT$$

$$\therefore\ M_A^2/2 = e^2v^2$$

$$\boxed{M_A^2 = 2e^2v^2}$$

(91)

As for n, it gets its mass just as before from the n^2 coefficient of the $-\frac{m^2}{2}(n+v)^2 +$ $-\frac{\lambda}{4}(n+v)^4$ terms and

$$\boxed{M_n^2 = -2m^2 > 0}$$

So: The Goldstone Boson gets "eaten" by the Vector boson and the vector boson gets mass and 3 degrees of freedom.
The second scalar becomes a massive one.

BEFORE HIGGS		AFTER HIGGS	
ρ	1 DoF	ρ	1 DoF
θ	1 DoF	A_μ	3 DoF
A_μ	2 DoF		4 DoF
	4 DoF		

The Weinberg Salam Model

If you do not know why the coefficient of $A_\mu A^\mu$ is equal to the $\frac{m_A^2}{2}$, here is the reason: Consider the Lagrangian for $m \neq 0$ $S=1$ field (92)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$$

$$\frac{\partial \mathcal{L}}{\partial \partial_\mu A_\nu} = - F^{\mu\nu} \qquad \frac{\partial \mathcal{L}}{\partial A_\nu} = m^2 A^\nu$$

$$\therefore \; -\partial_\mu F^{\mu\nu} - m^2 A^\nu = 0 \Rightarrow$$

$$-\Box A^\nu + \underbrace{\partial^\nu (\partial_\mu A^\mu)} - m^2 A^\nu = 0$$

I can always choose a gauge such that $\partial_\mu A^\mu = 0$

$\partial_\mu A'^\mu = \partial_\mu A^\mu - \Box \Theta$

$$\therefore (\Box + m^2) A^\nu = 0$$

is the equation for a massive vector field

The Weinberg Salam Model (93)

It is based on the $SU_L(2) \otimes U_Y(1)$ group:

Recall that $SU(2)$ has 3 generators and $U(1)$ has one. So there are 4 gauge fields $A_\mu^1, A_\mu^2, A_\mu^3, B_\mu$. And the covariant derivative can be written as:

$$D_\mu = \partial_\mu - \frac{i}{2} g' Y B_\mu - \frac{i}{2} g \; \vec{\tau} \vec{A}_\mu$$

For the moment take $Y=-1$ for the B_μ term

Note:: Y=-1 for lepton doublets only !!

$$D_\mu = \partial_\mu + \frac{i}{2} g' B_\mu - \frac{i}{2} g \vec{\tau} \cdot \vec{A}_\mu$$

This derivative acts on higgs doublets and Lepton doublets:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Recall that $e_L = \frac{1}{2}(1-\gamma_5)e$; $e_R = \frac{1}{2}(1+\gamma_5)e$

The Weinberg Salam Lagrangian

94

- Therefore one can write the gauge field Lagrangian

$$\mathcal{L}_G = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Tr(\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu})$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\mu A^i_\nu + g\epsilon^{ijk} A^j_\mu A^k_\nu$$

$$\mathbb{F}_{\mu\nu} = T^i F^i_{\mu\nu}$$

- The Lepton field Lagrangian can be written in terms of the doublet and singlet terms

$$\mathcal{L} = \underbrace{\bar{R} i\gamma^\mu (\partial_\mu + i g' B_\mu) R}_{\text{singlets}} +$$

$$\bar{L} i\gamma^\mu (\partial_\mu + \frac{i}{2} g' B_\mu - i g \frac{\vec{\tau}\cdot\vec{A}_\mu}{2}) L$$

Whatever happens this theory has to have a photon with $m_\gamma = 0$ at the end. So one of the generators will leave the vacuum invariant and there will be a massive scalar coming also from the unbroken generator.

95

Three generators have to be broken to give us 3 Goldstone Bosons which will give mass to the remaining 3 gauge fields.

Define the complex Higgs field as

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (4 \text{ degrees of freedom})$$

as an SU(2) doublet and

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi)$$

Note:: Υ=-+1 for the Higgs doublet !!

$$\mathcal{L}_{Higgs} = (\partial_\mu \phi^\dagger + i g'/_2 B_\mu \mathbb{1} \phi^\dagger + i \frac{g}{2} \vec{\tau}\cdot\vec{A}_\mu \phi^\dagger) \cdot$$

$$(\partial_\mu \phi - i g'/_2 B_\mu \phi^\dagger \mathbb{1} - i \frac{g}{2} \vec{\tau}\cdot\vec{A}_\mu \phi) +$$

$$- V(\phi^\dagger \phi)$$

Aims: From 1+3 massless gauge fields (B_μ, A^i_μ) we want $(W^\pm, Z^0)$, $A_\mu \Rightarrow \mathcal{L}_{HIGGS}$ has to give us 3 Goldstone bosons + H^0 (into)

WS continued...

96

Take $V = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$

and if you like you could also have

Couplings $\mathcal{L}_I = -G_e[\bar{R}\phi^\dagger L + \bar{L}\phi R]$

which are gauge invariant and will deal with them later.

As before you can show that if $\mu^2 < 0$

$$V(\phi^\dagger\phi) = \mu^2(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2) + \lambda(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2)^2$$

if $\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \begin{pmatrix}\phi_1+i\phi_2\\ \phi_3+i\phi_4\end{pmatrix}$ and $\frac{\partial V}{\partial\phi_i} = 0$

and $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$ $\quad v = \sqrt{\frac{-\mu^2}{\lambda}}$

This vacuum breaks both the $SU(2)$ and the $U_Y(1)$ symmetries (try it)

$Q = T_L^3 + \frac{1}{2}Y$ is not broken

(Charge is conserved) $\Rightarrow$ massless photon + massive scalar Higgs

97

Back to the covariant derivative:

$$D_\mu^\dagger = \partial_\mu + \frac{i}{2} g T^\ell A^\ell_\mu + i g'/_2 B_\mu \rightarrow$$

$$D_\mu^\dagger = \partial_\mu + i\frac{g}{2}\begin{pmatrix} A_\mu^3 & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3\end{pmatrix} + i\, g'/_2 \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu\end{pmatrix} \rightarrow$$

$$D_\mu^\dagger = \partial_\mu + \frac{i}{2}\begin{pmatrix} g'B_\mu + gA_\mu^3 & (A_\mu^1 - iA_\mu^2)g \\ (A_\mu^1+iA_\mu^2)g & g'B_\mu - gA_\mu^3\end{pmatrix}$$

$$(D_\mu\phi)^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \partial_\mu \eta\end{pmatrix} + \frac{i}{2\sqrt{2}}\begin{pmatrix} g(A_\mu^1 - iA_\mu^2)(\eta+v) \\ (g'B_\mu - gA_\mu^3)(\eta+v)\end{pmatrix}$$

We have taken the field to be $\phi = \underbrace{e^{\frac{i\vec{\xi}\cdot\vec{\tau}}{2v}}}_{\text{3 broken generators, 3 Goldstone bosons}}\begin{pmatrix}0\\ \frac{\eta+v}{\sqrt{2}}\end{pmatrix}$

At the end

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \frac{1}{8}(\eta+v)^2 g^2(A_\mu^1+iA_\mu^2)(A_\mu^1-iA_\mu^2) + \frac{1}{8}(\eta+v)^2(B_\mu g' - gA_\mu^3)^2$$

Define $W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$

$$Z_\mu = \frac{-gA_\mu^3 + g'B_\mu}{\sqrt{g^2+g'^2}}\ ;\ A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$$

The $W^\pm$, Z^0 and H^0 Masses

Therefore (98)

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \frac{1}{2}\partial_\mu h\,\partial^\mu h + \frac{1}{8}g^2(h+v)^2\,2\,W^+_\mu W^{\mu -} + \frac{1}{8}(h+v)^2\left(\sqrt{g^2+g'^2}\right)^2 Z^0_\mu Z^{0\mu}$$

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \frac{1}{2}\partial_\mu h\,\partial^\mu h + \frac{1}{4}g^2(h+v)^2 W^+_\mu W^{-\mu} + \frac{1}{8}(h+v)^2(g^2+g'^2)\,Z^0_\mu Z^{0\mu}$$

$$\frac{1}{2}m_W^2 = +\frac{g^2}{8}v^2 \Rightarrow m_W^2 = \frac{g^2v^2}{4} \Rightarrow \boxed{M_W = \frac{gv}{2}}$$

$$\frac{1}{2}m_Z^2 = \frac{1}{8}(g^2+g'^2)v^2 \Rightarrow m_Z^2 = (g^2+g'^2)\frac{v^2}{4} \Rightarrow$$

$$\boxed{m_{Z^0} = \frac{1}{2}v\sqrt{g^2+g'^2}}$$

$$\therefore \boxed{\frac{m_W}{m_{Z^0}} = \frac{g}{\sqrt{g'^2+g^2}}}$$

Next we evaluate the Higgs mass: (99)

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - \mu^2\frac{1}{2}(h+v)^2 - \frac{\lambda}{4}(h+v)^4 \rightarrow$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{\mu^2}{2}h^2 - \frac{\lambda}{4}(h^2+2hv+v^2)(h^2+2hv+v^2)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{\mu^2}{2}h^2 - \frac{\lambda}{4}(h^2v^2+4h^2v^2+h^2v^2)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - \underbrace{\underbrace{\underbrace{\frac{h^2}{2}\left(\mu^2+\frac{\lambda}{2}\{v^2+4v^2+v^2\}\right)}_{-\frac{h^2}{2}(-\lambda v^2+3\lambda v^2)}}_{-\lambda\frac{h^2}{2}v^2 2}}_{-\frac{h^2}{2}(2\lambda v^2)}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - (2\lambda v^2)\frac{h^2}{2} + \dots$$

$$\boxed{m_{H^0} = 2\lambda v^2 = -2\mu^2}$$

The Photon and Z^0 Fields

Define $g'/g = \tan\theta_W$

Then

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu$$

$$Z^0_\mu = \sin\theta_W B_\mu - \cos\theta_W A^3_\mu$$

and

$$B_\mu = \cos\theta_W A_\mu + \sin\theta_W Z_\mu$$

$$A^3_\mu = \sin\theta_W A_\mu - \cos\theta_W Z_\mu$$