Total: 15 points

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Homework Assignment II

Problem 1: We have already shown in class that the gauge transformation for a Yang-Mills field is:

$$\mathbb{A}'_\mu = U \mathbb{A}_\mu U^{-1} - \frac{i}{g} \partial_\mu U \cdot U^{-1}$$

This transformation leaves the scalar lagrangian invariant under:

$$\Phi \to \Phi' = U(\theta)\Phi \; ; \; U(\theta) = e^{-i\vec{T}\cdot\vec{\theta}}$$

Show that the gauge field Lagrangian given by:

$$\mathcal{L} = -\frac{1}{4} Tr(\mathbb{F}^{\mu\nu}\mathbb{F}_{\mu\nu})$$

is also invariant under this transformation, where:

$$\mathbb{F}_{\mu\nu} = T^i F^i_{\mu\nu} \; ; \; F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk} A^j_\mu A^k_\nu$$

Hints:

(a) Show that the Y-M field change under U is given by

$$\delta A^i_\mu = \epsilon^{ijk}\theta^j A^k_\mu - \frac{1}{g}\partial_\mu \theta^i$$

(b) Calculate:

$$\delta(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu) = \epsilon^{ijk}\theta^j(\partial_\mu A^k_\nu - \partial_\nu A^k_\mu) + \epsilon^{ijk}[(\partial_\mu\theta^j)A^k_\nu - \partial_\nu\theta^j \cdot A^k_\mu]$$

(c) Calculate: $\epsilon^{ijk}\delta(A^j_\mu A^k_\nu) =$

$$-\frac{\epsilon^{ijk}}{g}\left((\partial_\mu\theta^j)A^k_\nu - (\partial_\nu\theta^j)A^k_\mu\right) +$$

$$+\epsilon_{ijk}\epsilon_{jem}\theta^e A^k_\nu A^m_\mu + \epsilon_{ijk}\epsilon_{kem} A^m_\nu A^j_\mu \theta^e$$