(5)

(9) The expansion for the quantized photon field is:

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2k^0}} \sum_\lambda \epsilon_\mu(\lambda \vec{k}) \left[a(\lambda \vec{k}) e^{-ikx} + a^\dagger(\lambda \vec{k}) e^{ikx} \right]$$

Where: $\left[a(\lambda \vec{k}), a^\dagger(\lambda' \vec{k}') \right] = \delta_{\lambda\lambda'} \delta^{(3)}(\vec{k} - \vec{k}')$

Define: $a_\mu(\vec{k}) = \sum_\lambda \epsilon_\mu(\lambda \vec{k})\, a(\lambda \vec{k})$ (λ summs over polarisations)

$$\left[a_\mu(\vec{k}), a_\nu^\dagger(\vec{k}') \right] - g_{\mu\nu} \delta^3(\vec{k} - \vec{k}')$$

The polarization vectors are defined as:

$$\epsilon_\mu^{L} = (1,0,0,1) \quad \epsilon_\mu^{(-)} = \frac{1}{\sqrt{2}}(0,1,i,0), \; \epsilon_\mu^{(+)} = \frac{1}{\sqrt{2}}(0,1,-i,0)$$

And finally you get:

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2k^0}} \left[a_\mu(\vec{k}) e^{-ikx} + a_\mu^\dagger(\vec{k}) e^{ikx} \right]$$

Wick's theorem for photons gives you:

$$A^\circ_\mu(x) A^\circ_\nu(y) = - g_{\mu\nu} D_F(x-y)$$

Show that:

$$a_{IN}(k\lambda) = -i \int \frac{d^3x}{\sqrt{(2\pi)^3 2k^0}} e^{ikx} \epsilon_\mu(k\lambda) \overleftrightarrow{\partial_0} A^\mu(x)$$

(look in the scalar field case. It is similar)