5) The quantized Dirac field satisfying the equations in (1) can be written as:

$$\psi_\alpha(x) = \frac{1}{(2\pi)^{3/2}} \int d^3\vec{p} \sqrt{\frac{M}{E_{\vec{p}}}} \sum_{r=1}^{2} \left\{ u_{r\alpha} b_r e^{-ipx} + v_{r\alpha} d_r^\dagger e^{ipx} \right\}$$

where $u_r(\vec{p}) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M} \end{pmatrix} \chi_r$; $v_r(\vec{p}) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+M} \\ 1 \end{pmatrix} \chi_r$

(5) $\chi_r = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $r=1,2$ and $b_1(\vec{p})$ destroys an $e^-\uparrow$

$b_2(\vec{p})$ destroys an $e^-\downarrow$, $d_1(p)$ destroys an $e^+\uparrow$ and $d_2(p)$ an $e^+\downarrow$

The Dirac fields obey anti-commutation relations:

$$\left\{\psi_\alpha(\vec{x},t), \psi^\dagger_\beta(\vec{x}',t)\right\} = \delta_{\alpha\beta}\,\delta^3(\vec{x}-\vec{x}'), \quad \left\{\psi_\alpha(\vec{x},t), \tilde{\psi}_\beta(\vec{x}',t)\right\} = \gamma^0_{\alpha\beta}\,\delta^3(\vec{x}-\vec{x}')$$

Hence, the operators obey also anti-commutation relations such as:

$$\left\{b_r(\vec{p}), b^\dagger_{r'}(\vec{p}')\right\} = \delta_{rr'}\,\delta^3(\vec{p}-\vec{p}') \quad \text{(Same for } d_r\text{)}$$

Recall that : $\tilde{u}_r(\vec{p})\, u_{r'}(\vec{p}) = \delta_{rr'}$, $\tilde{v}_r(\vec{p})\, v_{r'}(\vec{p}) = \delta_{rr'}$

$$\sum_{r=1}^{2} u_{r\alpha} \bar{u}_{r\beta} = \left(\frac{\gamma\cdot p + m}{2m}\right)_{\alpha\beta}; \quad \sum_{r=1}^{2} v_{r\alpha} \tilde{v}_{r\beta} = \left(\frac{\gamma\cdot p - m}{2m}\right)_{\alpha\beta}$$

Use Wick's theorem for Fermions: $T(\psi_\alpha \bar{\psi}_\beta) - N(\psi_\alpha \tilde{\psi}_\beta) = \overset{\bullet}{\psi}_\alpha \overset{\bullet}{\tilde{\psi}}_\beta$
and show that:

$$\overset{\bullet}{\psi}_\alpha(x)\, \overset{\bullet}{\tilde{\psi}}_\beta(y) = (i\not{\partial} + m)\, D_F(x-y) = (i\not{\partial} + m) \int \frac{d^3p}{2E_p(2\pi)^3}\, e^{-ip(x-y)}$$

Be careful with the normal product operator, **N,** acting on fermionic operators:

$$N\left(b_2(\vec{p})\, b_2^\dagger(\vec{p}')\right) = -b_2^\dagger(\vec{p}')\, b_2(\vec{p})$$