## Problem set II

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Problem 1: In the $\lambda \phi^{4}$ theory, the interaction term is given by:

$$
H_{I}=\frac{1}{4!} \lambda \phi^{4}
$$

Show that, to the lowest order in $\lambda$, the differential cross-section for two particle elastic scattering in the centre-of-mass frame is given by

$$
\frac{d \sigma}{d \Omega}=\frac{\lambda^{2}}{128 \pi^{2} s}
$$

where $\boldsymbol{s}=\left(\boldsymbol{p}_{\mathbf{1}}+\boldsymbol{p}_{2}\right)^{\mathbf{2}}$ and $\boldsymbol{p}_{\mathbf{1}}, \boldsymbol{p}_{\mathbf{2}}$ are the 4-momenta of the incoming particles.

Problem 2: Calculate the Compton scattering cross section which includes the two Feynman diagrams:


Show that the result is the Klein Nishina (1929) formula :

$$
\frac{d \sigma}{d \Omega}=\frac{\alpha^{2}}{4 m^{2}}\left(\frac{k_{f}}{k_{i}}\right)^{2}\left[\frac{k_{f}}{k_{i}}+\frac{k_{i}}{k_{f}}+4\left(\varepsilon_{f} \cdot \varepsilon_{i}\right)^{2}-2\right]
$$

Which at the low energy limit reduces to the Thomson cross section:

$$
\frac{d \sigma}{d \Omega}=\frac{\alpha^{2}}{m^{2}}\left(\varepsilon_{f} \cdot \varepsilon_{i}\right)^{2} \quad \frac{\alpha}{m}=r_{0}=2.8 \times 10^{-13} \mathrm{~cm}
$$

