Problem set I

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<u>Problem 1:</u> The K^- meson and the $\overline{K}^0$ are an isospin ½ doublet, the Λ^0 is an isospin singlet and the three sigma particles Σ^+, Σ^0, Σ^- form an isospin triplet. Calculate he ratios of cross sections in the five reactions:

I. $K^- + p \rightarrow \pi^0 \Lambda^0$
II. $K^- + p \rightarrow \pi^0 \Sigma^0$
III. $K^- + p \rightarrow \pi^+ \Sigma^-$
IV. $K^- + p \rightarrow \pi^- \Sigma^+$

assuming that the first reaction is described by the same amplitudes in states of total isospin, **I**, as the latter three.

<u>Problem 2:</u> The deuteron is an isospin singlet. Calculate the ratios of the scattering cross sections for the reactions:

I. $p + p \rightarrow d + \pi^+$
II. $p + n \rightarrow d + \pi^0$

assuming charge independence of the interactions.

<u>Problem 3:</u> An infinitely long straight wire of negligible cross-sectional area is at rest and has a uniform linear charge density λ_0 in the inertial frame K'. The frame K' (and the wire) move with velocity **v** parallel to the direction of the wire with respect to the laboratory frame K.

a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the Laboratory frame.
b) Calculate the charge and current densities associated with the wire in its rest frame and the laboratory frame.
c) From the charge and current densities in the laboratory frame calculate directly the electric and magnetic fields in the laboratory. Compare the results with those from (a) and comment on them.

Problem 4: Using the Lorentz and Parity transformation properties of the Dirac spinors show that:

I. $\bar{\Psi}\Psi$ is a scalar.
II. $\bar{\Psi}\gamma^5\Psi$ is a pseudoscalar.
III. $\bar{\Psi}\gamma^\mu\Psi$ is a vector.
IV. $\bar{\Psi}\gamma^\mu\gamma^5\Psi$ is an axial vector.
V. $\bar{\Psi}[\gamma^\mu,\gamma^\nu]\Psi$ is a tensor.

Use the above to define an interaction Lagrangian which is Lorentz invariant and
a) always conserves parity
b) always violates parity.

Problem 5:

a) Show that the helicity operator is conserved regardless of the fermion mass.
b) Show that the Chirality (Handedness) is only conserved for massless fermions.
c) A positive energy Dirac spinor in the momentum space can be written as,

$$\Psi(p,\uparrow\downarrow)=\sqrt{(E+M)}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\vec{p}}{(E+m)} \end{pmatrix}\chi_{\uparrow\downarrow}$$

where $\vec{\sigma}\vec{p}\chi_{\uparrow\downarrow}=(\pm 1)\chi_{\uparrow\downarrow}$. (1) Show that the following two expressions are eigenvectors of the helicity operator and are non-zero at the relativistic limit.
$\frac{1}{2}(1-\gamma_5)\Psi(p,\downarrow)$; $\frac{1}{2}(1+\gamma_5)\Psi(p,\uparrow)$. (2) Show also that the expressions:
$\frac{1}{2}(1-\gamma_5)\Psi(p,\uparrow)$; $\frac{1}{2}(1+\gamma_5)\Psi(p,\downarrow)$ are of the order of $\frac{m}{E}$. Hence, they vanish at the relativistic limit.

d) Show that for $m=0$, $\gamma_5 u(p,s)=+\vec{\Sigma}\,\hat{p}\,u(p,s)$ for particles and $\gamma_5 \upsilon(p,s)=-\vec{\Sigma}\,\hat{p}\,\upsilon(p,s)$ for antiparticles which leads of course to different projection operators for particles and antiparticles:

$$u_R(p)=\frac{(1+\gamma_5)}{2}u(p) \qquad u_L(p)=\frac{(1-\gamma_5)}{2}u(p)$$
$$\upsilon_R(p)=\frac{(1-\gamma_5)}{2}\upsilon(p) \qquad \upsilon_L(p)=\frac{(1+\gamma_5)}{2}\upsilon(p)$$