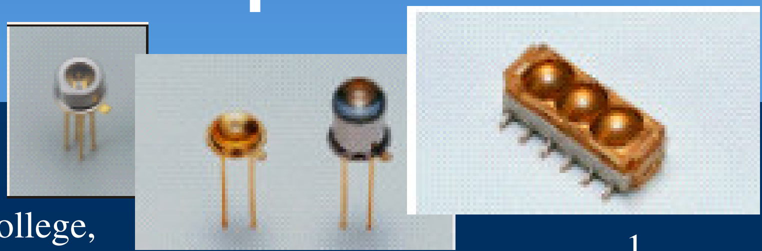
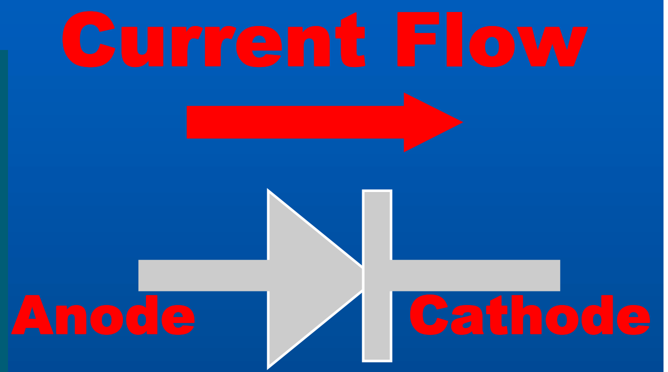
Junction Diodes

- **Most elementary solid state junction electronic devices.**
- **They conduct in one direction (almost correct).**
- **Useful when one converts from AC to DC (rectifier).**
- **But today diodes have a wide range of applications from LEDs, photodiodes, a variety of sensors and in optical communication**



The Diode Symbol

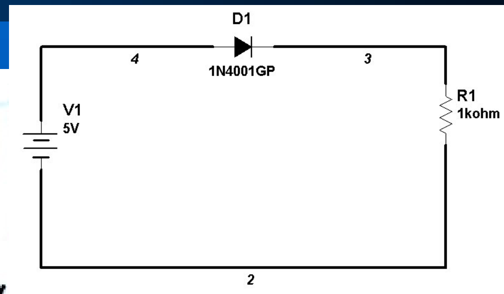
- The Diode Symbol:
- Current flows from Anode to Cathode

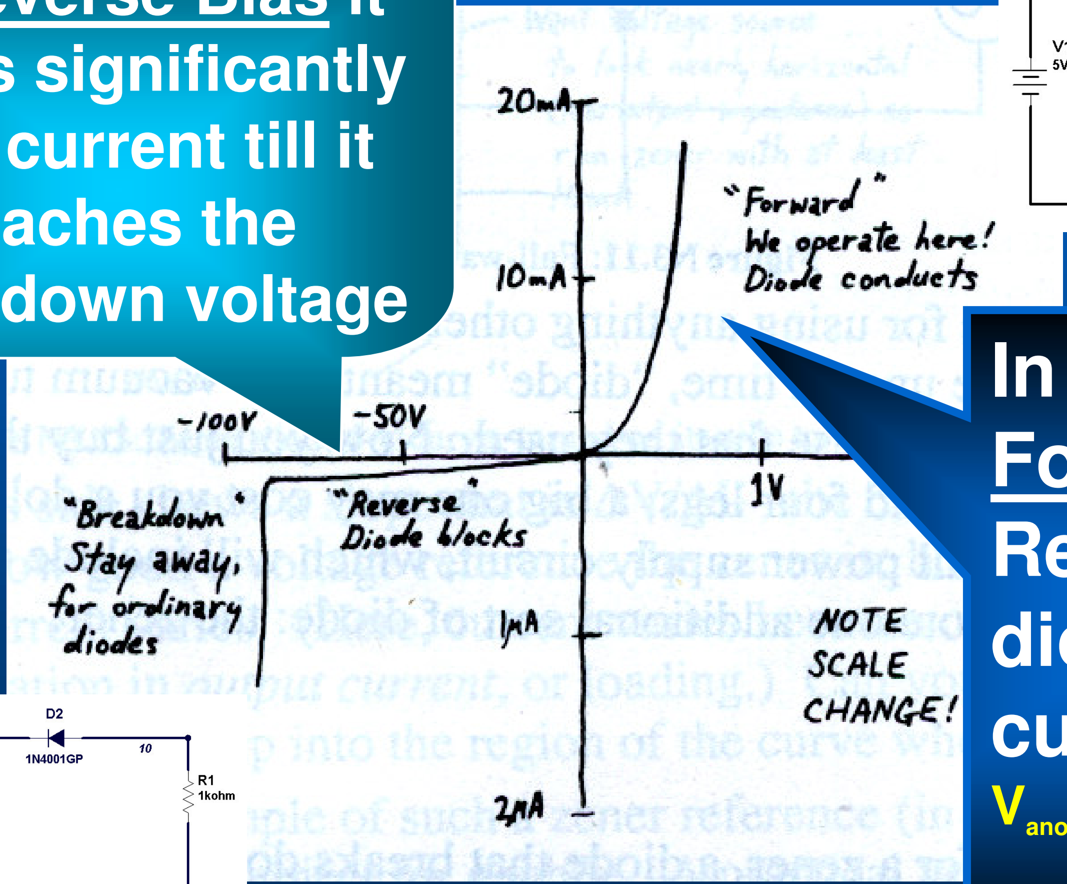


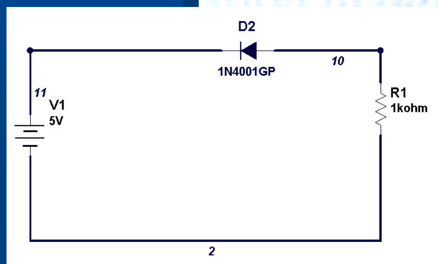
- This is how they look :
- Anode   (Higher V)
- Cathode   (Lower V)

The Diode Characteristics

In **Reverse Bias** it draws significantly less current till it reaches the breakdown voltage



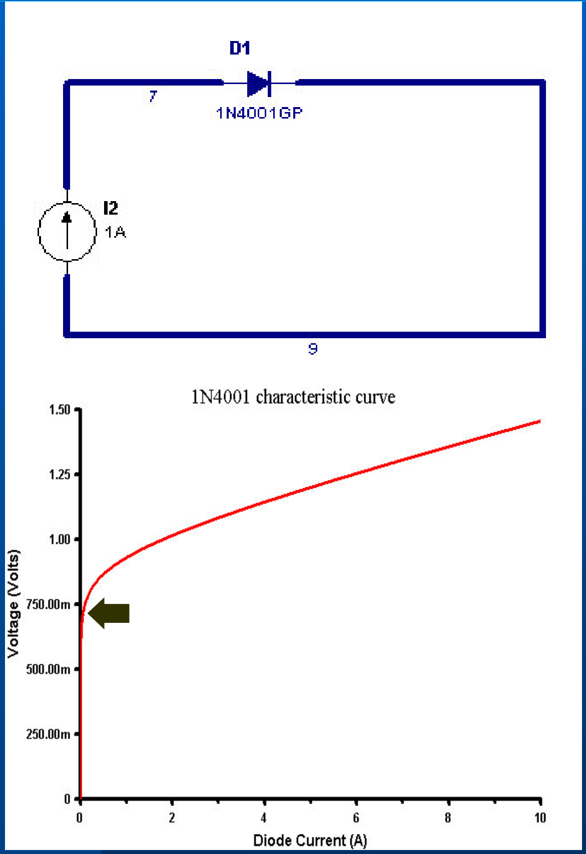




In the **Forward Bias** Region the diode draws current if

$V_{anode} = V_{cathode} + 0.7\ V$

Forward Bias Characteristics



- In ‘forward bias’ the diode draws current according to:

$$I(V)= I_S \{e^{(V/nV_T)}-1\}$$

- I_S is called Saturation Current
- V is the Diode Voltage
- V_T = KT/q = 25.2 mV at T=20 C^0
- K = 1.38 10^{-23} Jules/Kelvin (Boltzmann’s constant)
- q = 1.6 10^{-19} Cb
- n = 2 for discrete components
- n = 1 for diodes within ICs

Voltage Drop vs Current

- One can solve for V(I):

$$I(V) = I_S\{e^{(V/nV_T)} - 1\}$$

- to get $V_2 - V_1 = nV_T \ln(I_2/I_1)$
- or $V_2 - V_1 = 2.3\, nV_T \log(I_2/I_1)$

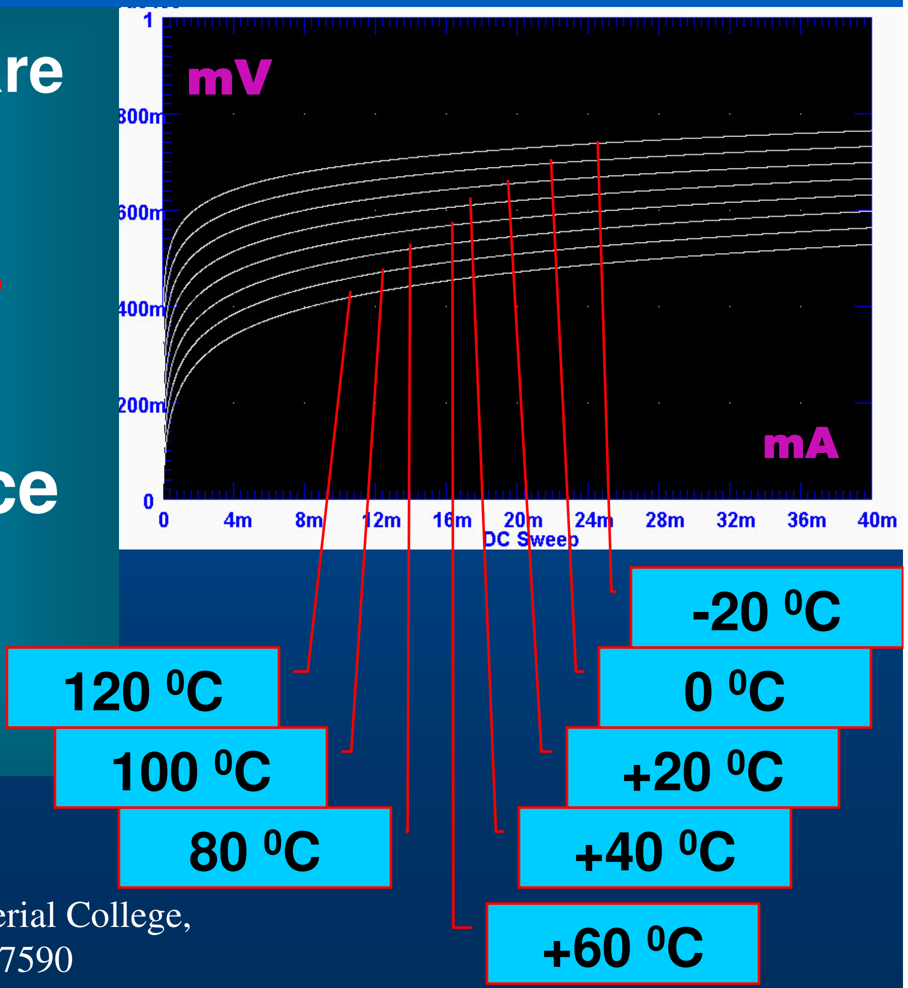
- <u>Conclusion:</u> A factor of 10 in current results to 60 mV in voltage drop (n=1) or 120 mV n=2

Temperature Dependence (1N4001 in forward bias)

- Both I_S and the exponent are temperature dependent in

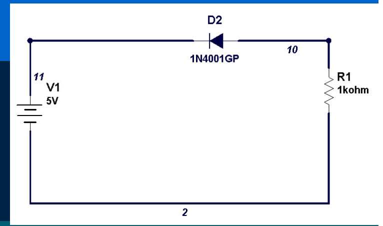
$$I(V)= I_S\{e^{(V/nV_T)}-1\}$$

- In fact most of the temperature dependence comes from I_S
- Rule : -2 mV per ^{0}C



Costas Foudas, Imperial College,
Rm: 508, x47590

Diodes in reverse bias



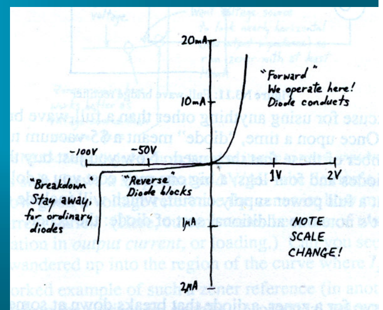
- In reverse bias the V<0 and as it becomes more negative the exponential term vanishes

$$I(V) = I_S\{e^{(V/nV_T)}-1\} \Rightarrow I_R = -I_S$$

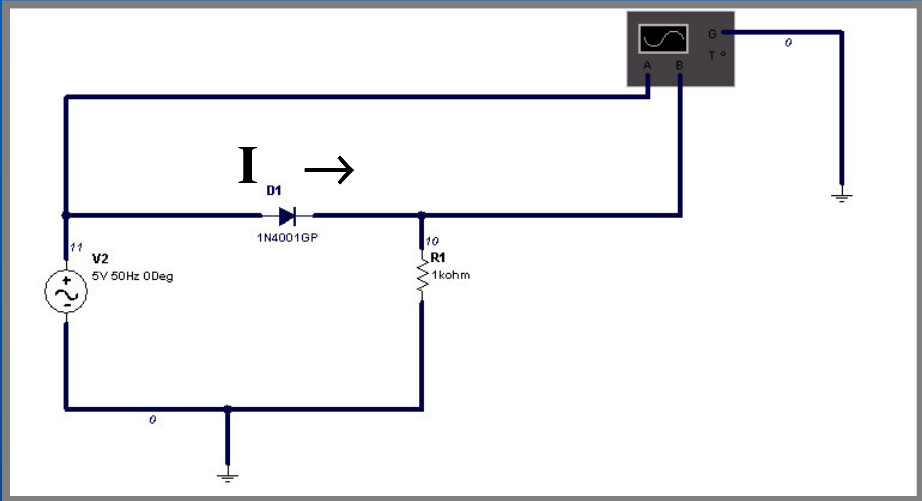
hence the name 'saturation'. $I_S = 10^{-14}\text{-}10^{-15}$ A

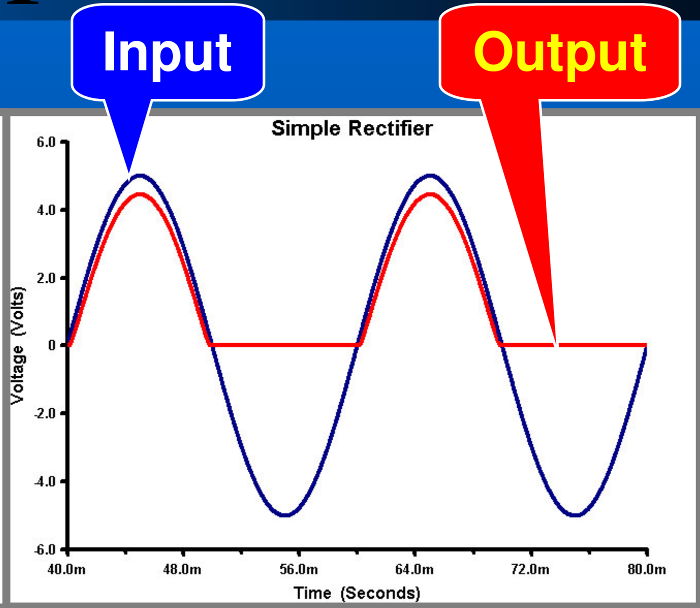
- In practice the reverse current is much larger, of the order of nA, because of leakage effects which dominate and have different temperature dependence.
- Rule of thumb: <u>I_S doubles every 5 ^{0}C</u>

  <u>In reality I_R doubles every 10 ^{0}C (leakage)</u>



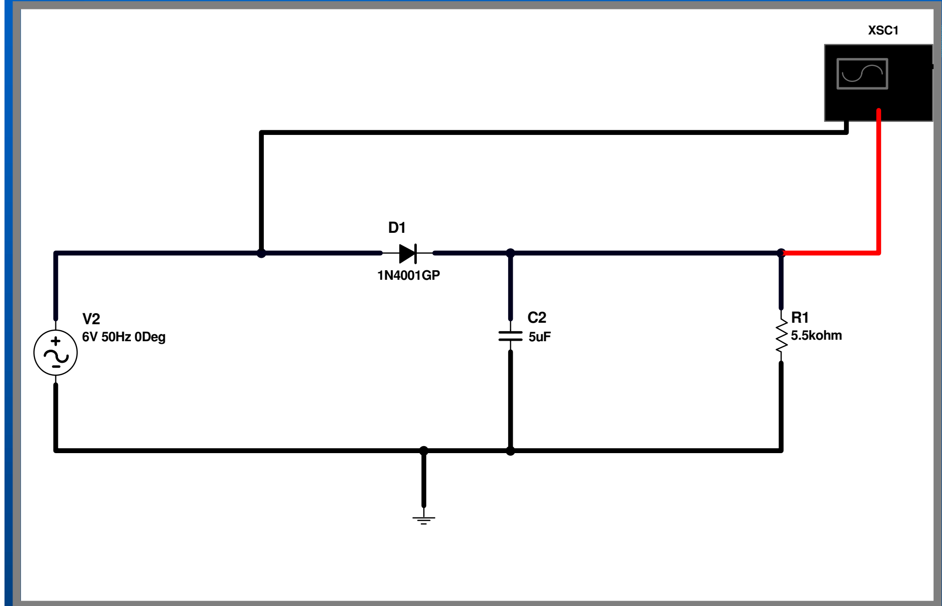
Exercise 1-I: Simple Rectifier

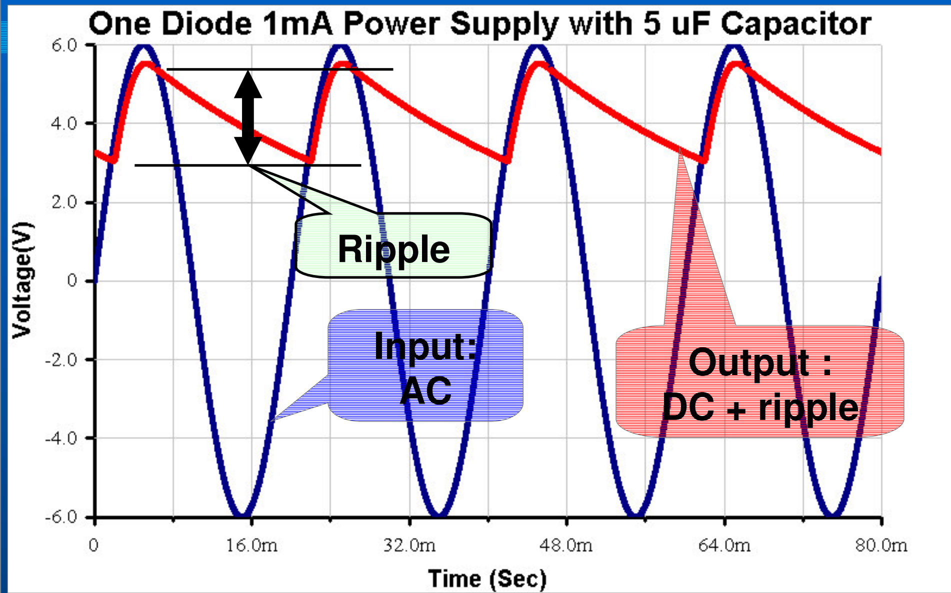




- Only the positive part of the signal (shown in red) goes through
- But the output is hardly DC….

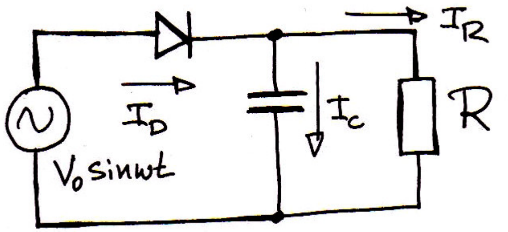
Exercise 1-II: Add 5μF Capacitor





- Assume a output current of I_{out} = 1 mA
- When the supply voltage is below ~ V_{out} – 0.7 the diode stops conducting and the capacitor discharges through the load resistor
- What capacitor do I need for 1% ripple noise (= 55 mV)?

Exercise 1-III: Calculating C



$I_D = I_c + I_R$

$$Q = CV \Rightarrow \frac{dQ}{dt} = C\frac{dV}{dt} \Rightarrow I_c = C\frac{dV}{dt}$$

During the discharge phase $I_c = I_R$ $(I_D = 0)$

$$\therefore \quad \frac{V}{R} = C\frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{V}{RC} \quad \Bigg\} \Rightarrow$$

Assume that $RC >> T \Rightarrow V = V_0$
(ideal diode)

$$\frac{dV}{dt} = \frac{V_0}{RC} \Rightarrow \Delta V = V_{ripple} = \frac{V_0}{RC}\Delta t \xrightarrow[\text{if } RC>>T]{\Delta t \simeq T}$$

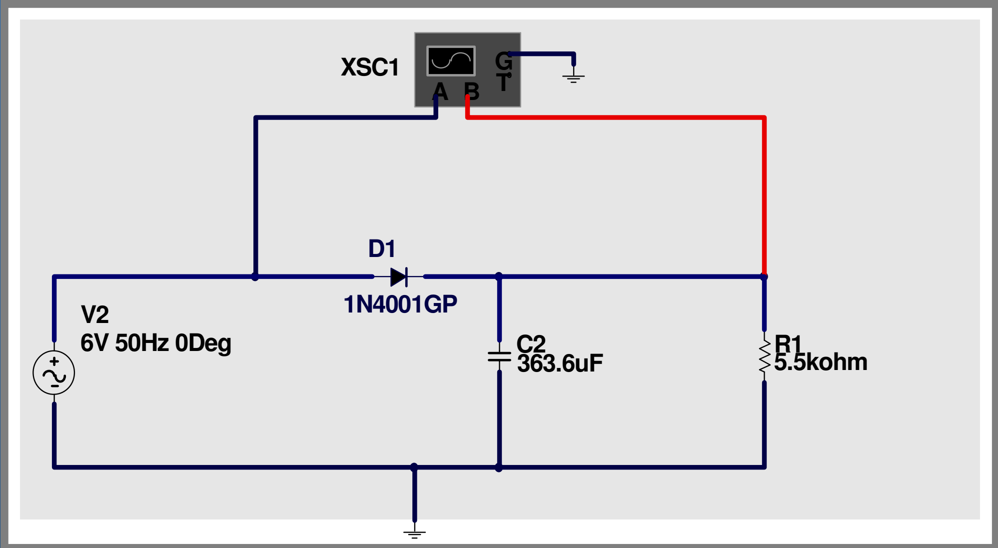
$$V_{ripple} = \frac{V_0 T}{RC} = \frac{V_0}{fRC}$$

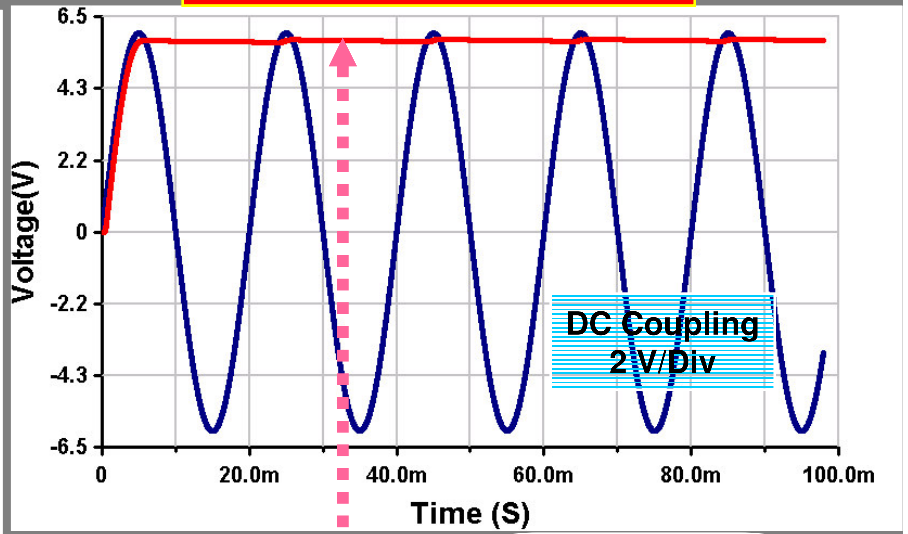
$$C = \frac{V_0}{fRV_{ripple}} = \frac{100}{50 \times 5.5\,10^3}\,F = 363.6\,\mu F$$

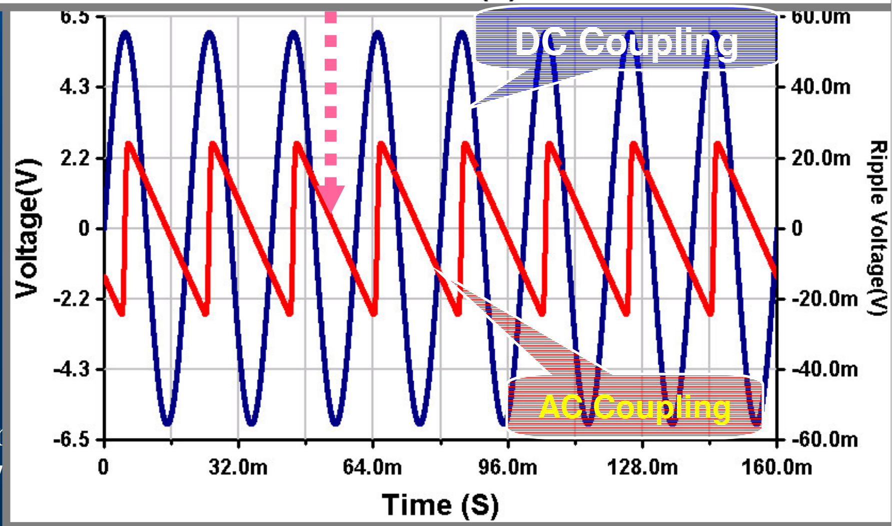
- This calculation is only valid when RC>>T
- Notice that when $C = 5\mu F$ this approx. is not valid.

Exercise 1- IV: C =363.6 μF

The output is almost DC



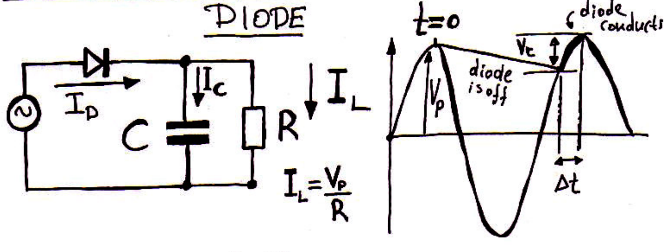




- The resulting ripple is 45.3 mV ~ 1%
- The capacitor required is large !!!!

Exercise 1-VI: Average Diode Current

AVERAGE CURRENT THROUGH THE DIODE ①



$V(t) = V_p \cos(\omega t)$

To find the average current we need to find the time that the diode conducts (Δt).

$$V_p - V_r = V_p \cos(\omega \Delta t) \underset{\omega\Delta t \ll}{\simeq} V_p\left(1 - \frac{(\omega \Delta t)^2}{2}\right) \Rightarrow$$

$$\cancel{V_p} - V_r \cong \cancel{V_p} - \frac{V_p}{2}(\omega \Delta t)^2 \rightarrow \boxed{\omega \cdot \Delta t = \sqrt{\frac{2V_r}{V_p}}} \quad (1)$$

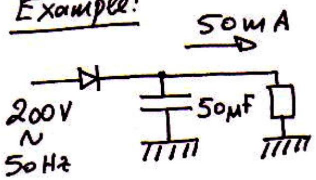
($\omega.\Delta t$ is small indeed)

$$\left.\begin{array}{l} I_D = I_C + I_L \\ Q = CV \rightarrow I_C = C\frac{dV}{dt} \end{array}\right\} \rightarrow \left.\begin{array}{l} I_D = C\frac{\Delta V}{\Delta t} + \frac{V_p}{R} \\ \Delta V = V_r \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} I_D = C\frac{V_r}{\Delta t} + \frac{V_p}{R} \\ V_r = \frac{V_p}{fRC} \end{array}\right\} \Rightarrow I_D = \cancel{C}\frac{V_p}{\Delta t \cdot f \cdot R\cancel{C}} + \frac{V_p}{R} \overset{(1)}{\Longrightarrow}$$

$$\boxed{I_D^{AVERAGE} = \frac{V_p}{R}\left[1 + \pi\sqrt{\frac{2V_p}{V_r}}\right]}$$

Example: ②



$R_L = \frac{200V}{50mA} = 4k\Omega$

$$V_r = \frac{V_p}{fRC} = \frac{200\,V}{50\,sec^{-1}\, 4\, 10^3\Omega\, 50\, 10^{-6}F} = \frac{1}{50}\, 10^3 \text{ Volts}$$

$V_r = 20$ Volts

$$I_D^{AVERAGE} = \frac{200V}{4k\Omega}\left[1 + 3.14\sqrt{\frac{2\,200}{20}}\right]$$

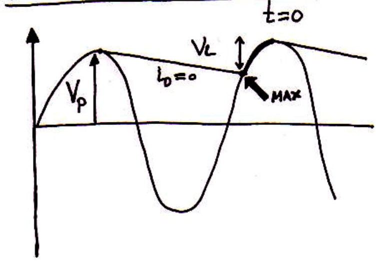
$$\boxed{I_D^{AVERAGE} = 752\ mA}$$

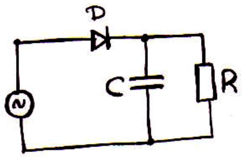
∴ Although we only draw 50 mA our diode needs to be able to survive 752 mA average currents!!

- **Average Current 752 mA**

Exercise 1-VII: Maximum Diode Current

MAXIMUM CURRENT THROUGH THE DIODE ③





$V_r = \frac{V_p}{fCR}$ ①

$\Delta t \cdot \omega = \sqrt{\frac{2V_r}{V_p}}$ ②

$V(t) = V_p \cos(\omega t) \Rightarrow$

$\frac{dV}{dt} = -\omega V_p \sin(\omega t)$

@ $t = -\Delta t$ $\Rightarrow$

$$\frac{dV}{dt} = +\omega V_p \sin(\omega \Delta t) \approx V_p \omega^2 \Delta t$$

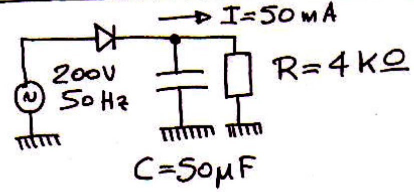
$$i_D^{MAX} = C \frac{dV_c}{dt}\Big|_{MAX} + \frac{V_p}{R}$$

$$i_D^{MAX} = C\omega^2 V_p \Delta t + \frac{V_p}{R} \overset{②}{=} C\omega^2 V_p \frac{1}{\omega}\sqrt{\frac{2V_r}{V_p}} + \frac{V_p}{R}$$

$$i_D^{MAX} = \frac{V_p}{R}\left(1 + \omega RC\sqrt{\frac{2V_r}{V_p}}\right)$$

$$i_D^{MAX} \overset{①}{=} \frac{V_p}{R}\left(1 + \frac{2\pi V_p}{V_r}\sqrt{\frac{2V_r}{V_p}}\right) \Rightarrow \boxed{i_D^{MAX} = \frac{V_p}{R}\left[1 + 2\pi\sqrt{\frac{2V_p}{V_r}}\right]}$$

BACK TO THE PREVIOUS EXAMPLE ④

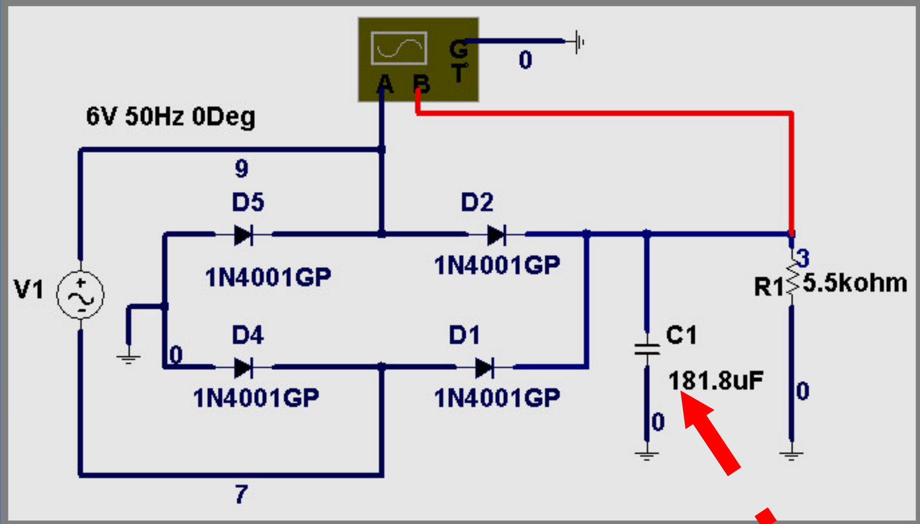


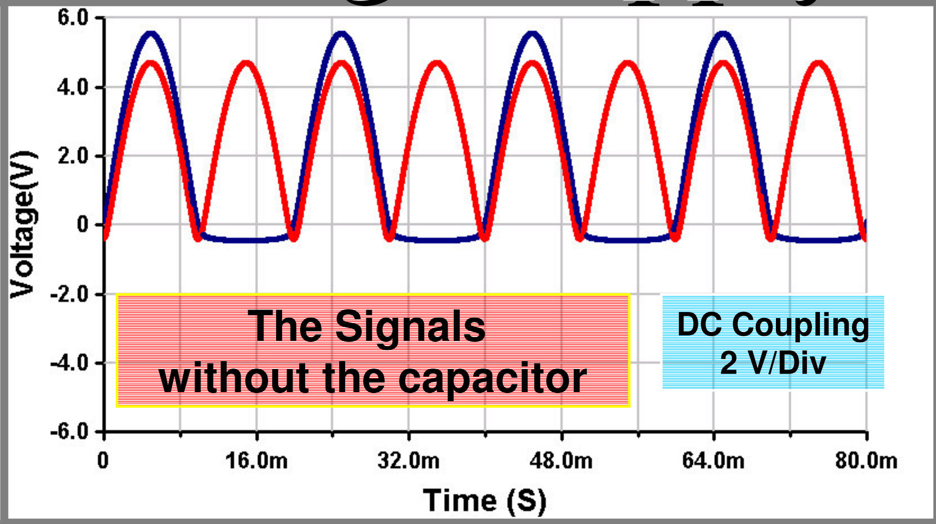
$$i_D^{MAX} = 50\,mA\,(1 + 6.28\sqrt{20})$$
$$= 1.455\,A \Rightarrow$$
$$i_D^{MAX} \cong 1.5\,A$$

In conclusion: Although our load needs only $I_L = 50\,mA$, our diode needs to be able to withstand average currents of ~ 750 mA and pick currents of ~ 1.5A

- **Maximum Current 1.5 A**

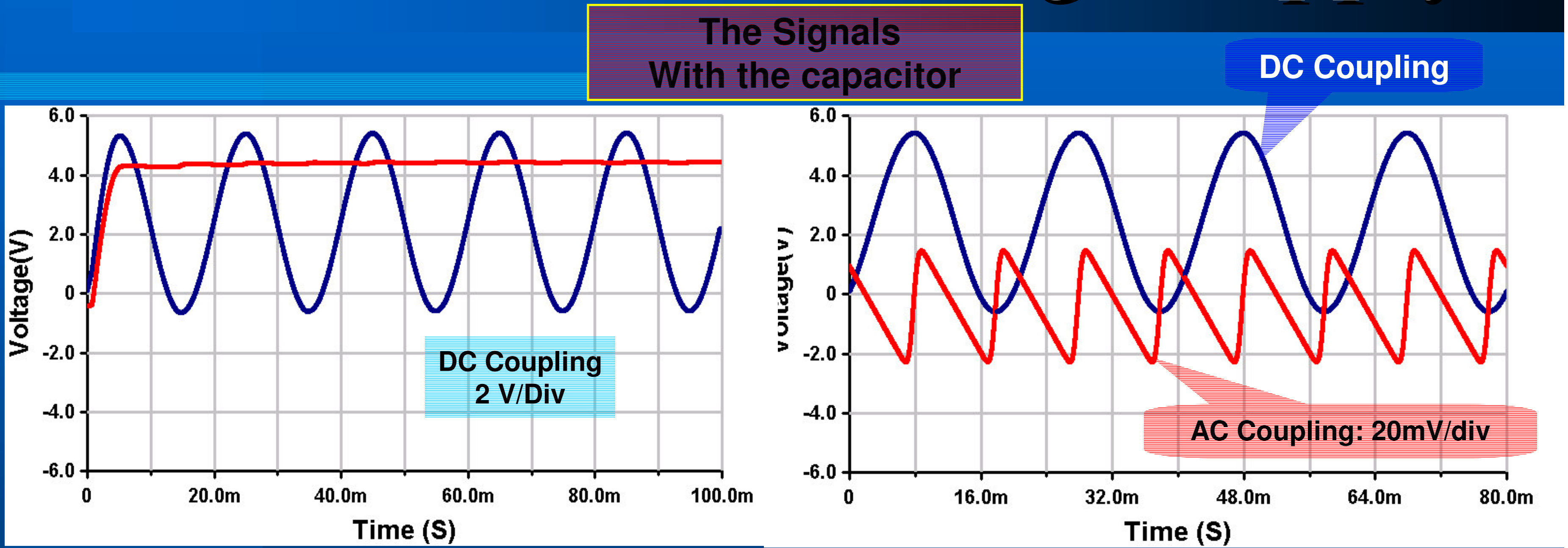
Exercise 1- V: Bridge Supply





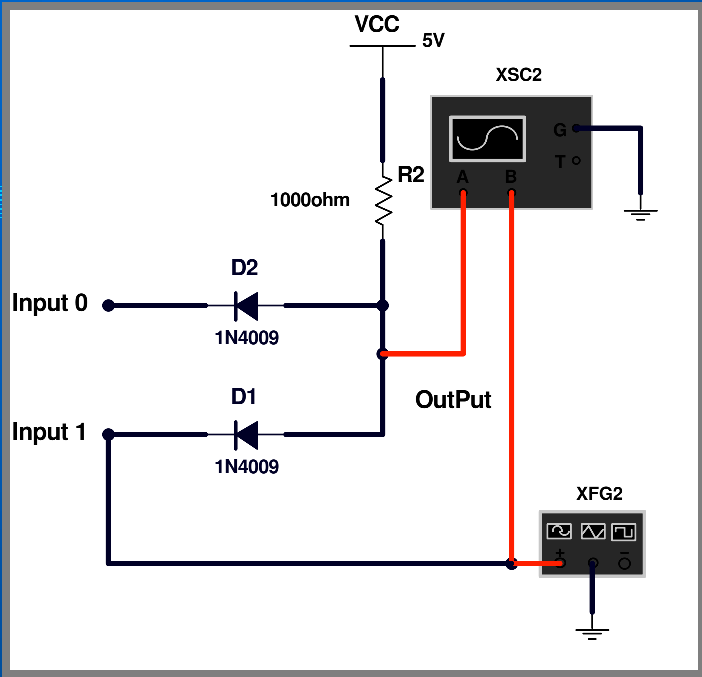
- The Diode Bridge utilises both the positive and the negative part of the AC pulse.
- Hence, rectifying such a signal is easier and a smaller capacitor (half as big) does the same job.

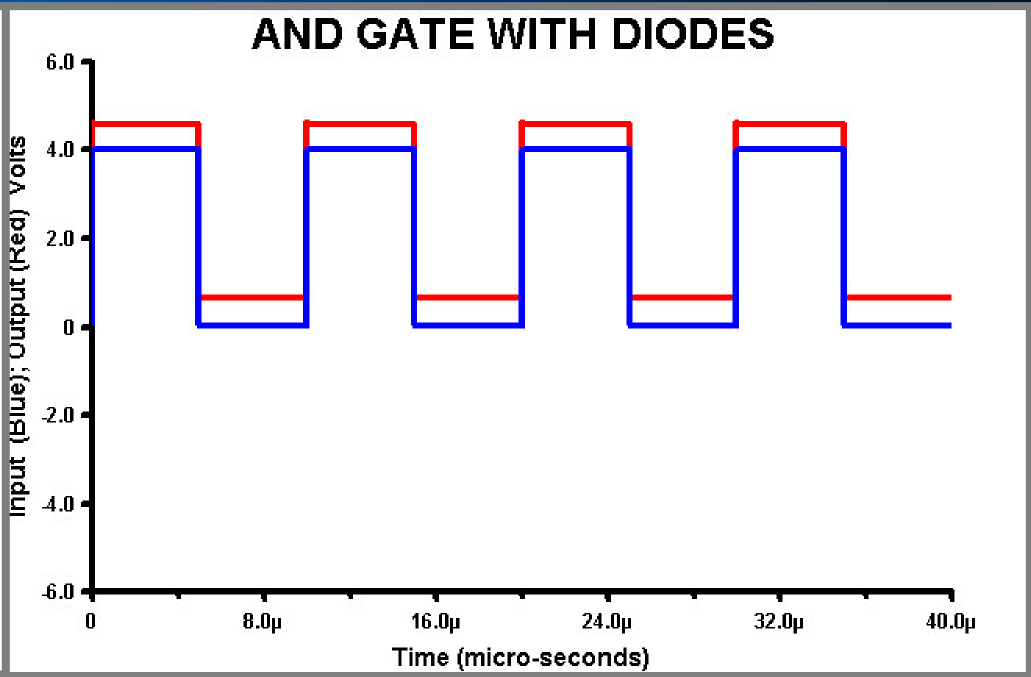
Exercise 1- VI: Bridge Supply



- Since the output of the bridge rectifier has a period which is half the period of the one diode rectifier we need a filter capacitor which is half as large.
- The ripple voltage is shown to be 37 mV (even better than before).

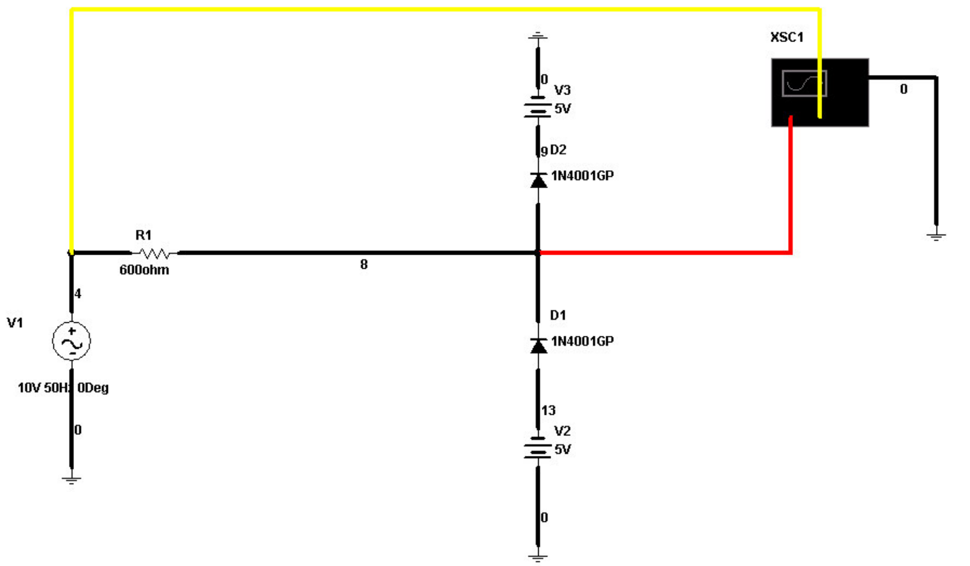
Exercise 2: AND Gate

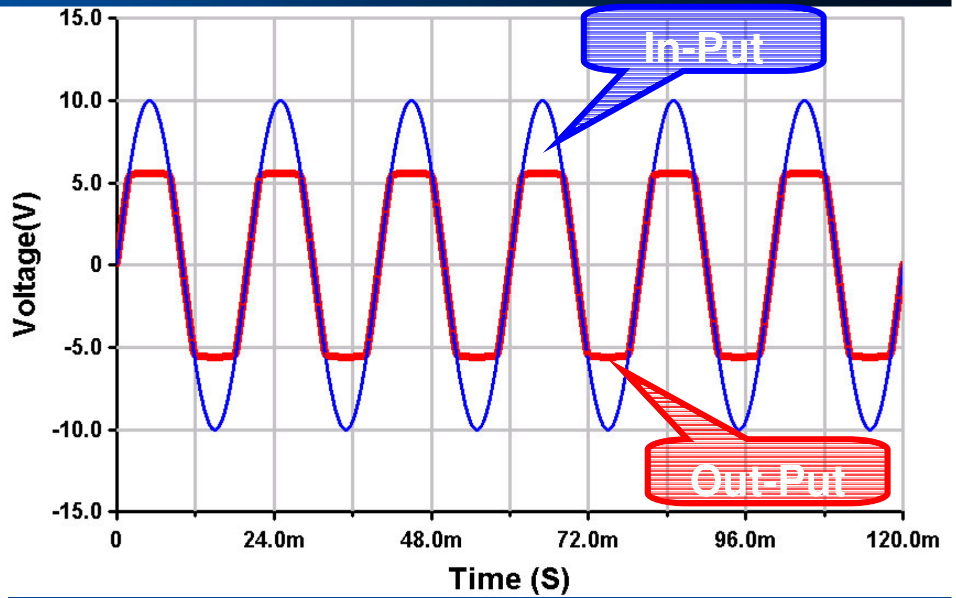




- **Diodes in AND gate configuration.**
- **One input disconnected (floats)**
- **The output (red) is 0.7 Volts shifted from the input (blue)**

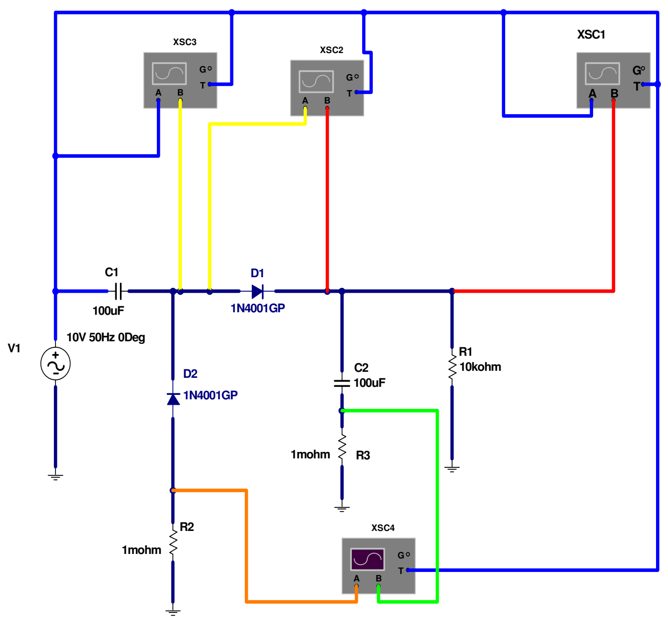
Clipping Circuits with Diodes

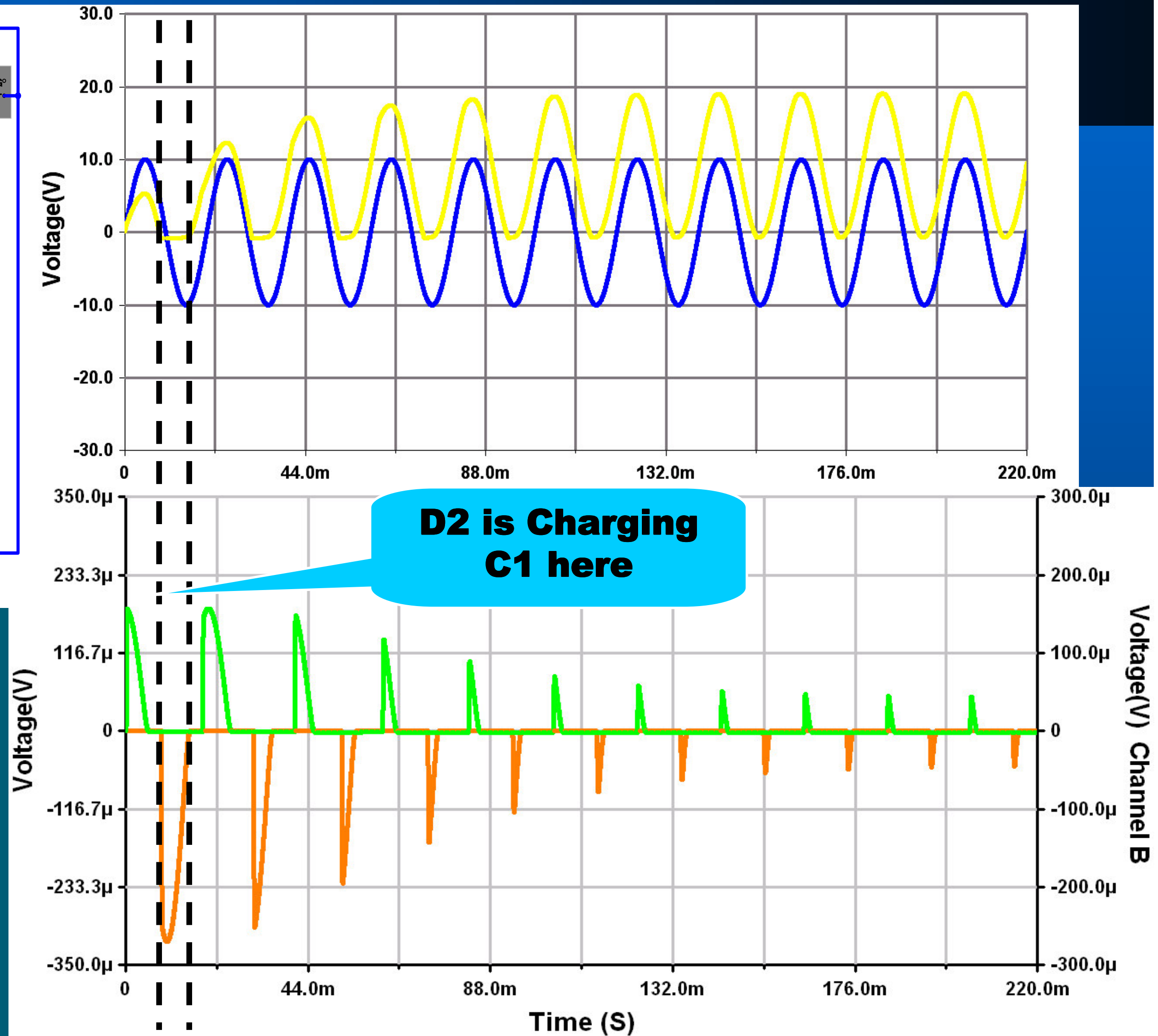




- Often one needs to limit the range of some input analogue pulse or even to clip spikes on a digital signal.
- This can easily be done with a diode and a voltage source

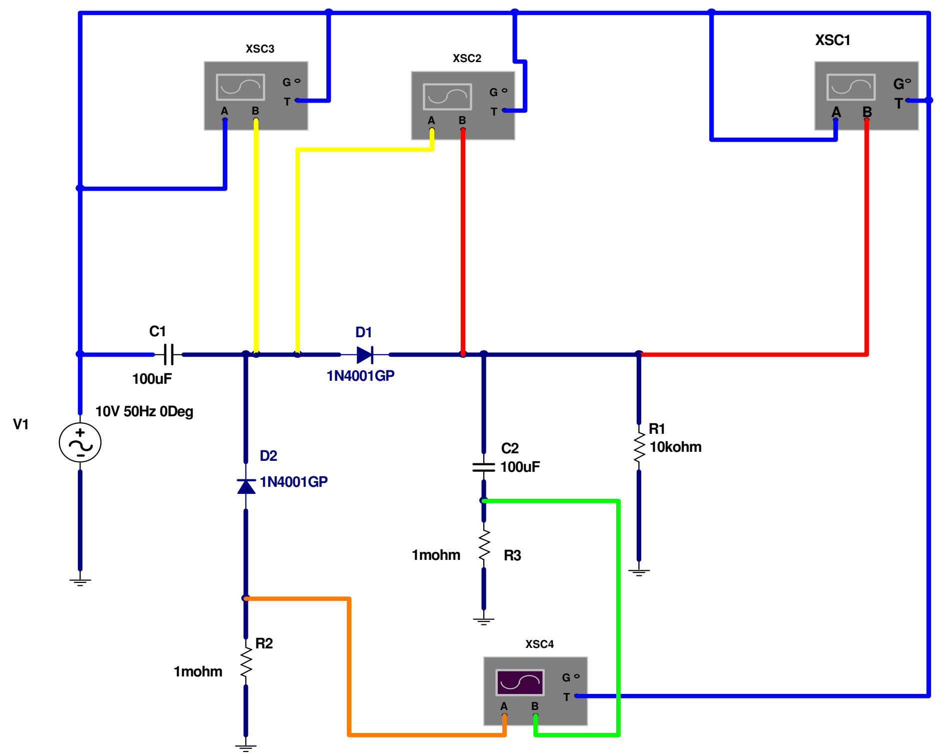
Voltage Doubling Circuits

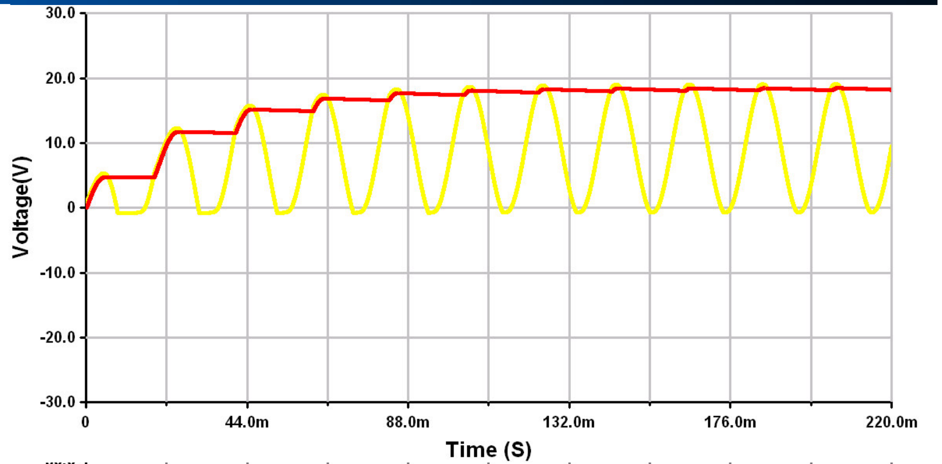


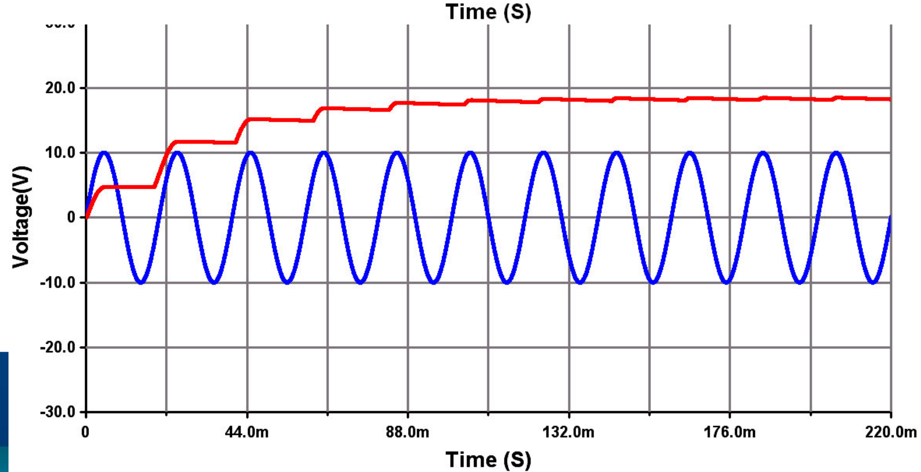


- C1 Charges during the first half of the reverse phase.
- C1 is in series with the source during the forward phase. Hence it doubles the output voltage.
- The output voltage will approach to $2 * V1 - 2 * V_D$

Voltage Doubling Circuits

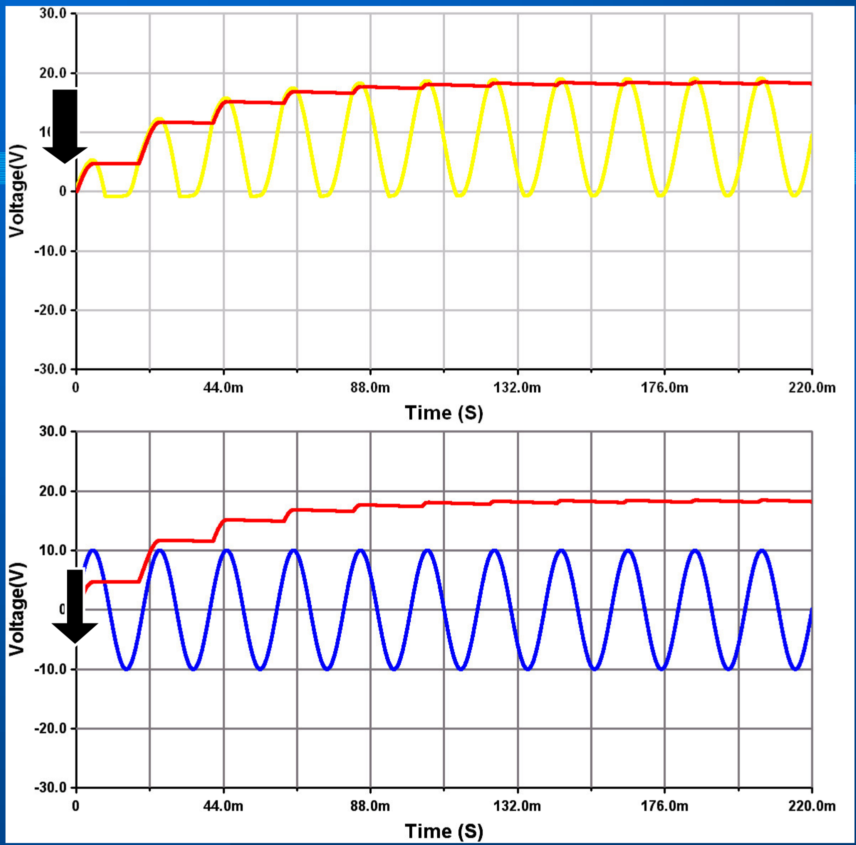




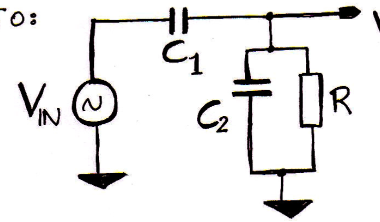


- The First step is about 5 Volts (V/2)
- The charge stored in the cap. each time can be calculated.
- Hence, the time it takes to stabilize can also be calculated

Voltage Doubling Circuits



DURING "TURN ON" THE CIRCUIT IS EQUIVALENT TO:



$Z_1 = \frac{1}{j\omega C_1}$ ①

$Z_2 = \frac{R}{1+j\omega R C_2}$ ②

$Z_{TOT} = Z_1 + Z_2$ ③

$$①②③ \Rightarrow \underset{\sim}{V_{out}} = \frac{Z_2}{Z_1+Z_2}\underset{\sim}{V_{IN}} = \underset{\sim}{A}\cdot\underset{\sim}{V_{IN}}$$

$$A = \frac{Z_2}{Z_1+Z_2} = |A|e^{i\phi}$$ *and after some complex number math. one gets:*

$$\tan\Phi = \frac{1}{\omega R(C_1+C_2)}$$

$$|A| = \frac{1}{\sqrt{(1/\omega^2R^2C_1)+\left(\frac{C_1+C_2}{C_1}\right)^2}}$$

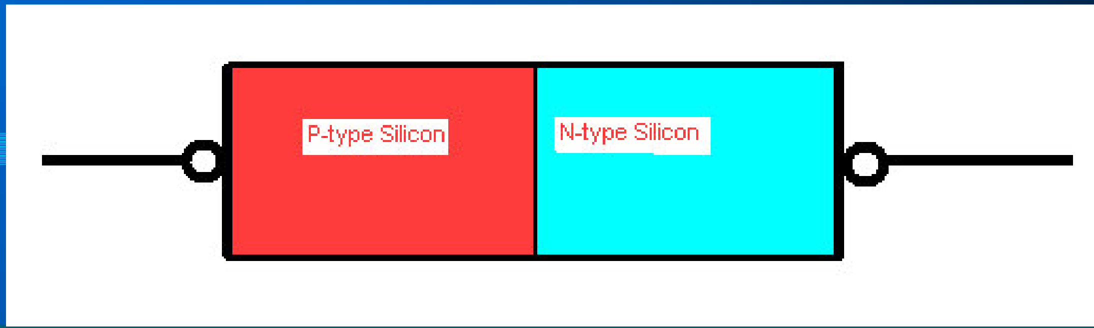
With $R = 10k\Omega$, $C_1 = C_2 = 100\mu F$ $\Rightarrow |A| \simeq \frac{1}{2}$ Hence the first time is 5 Volts.

- **Here is the proof that the first step is V/2**
- **The next step can be calculated: $V_D \approx 0.7$ Volts but you need the diode equation in order to calculate the current and then the charge…**

Appendix I: Semiconductors

- For those of you who are interested to see how the diodes and in general the semiconductors work.
- For the students who are not taking the Instrumentation course.

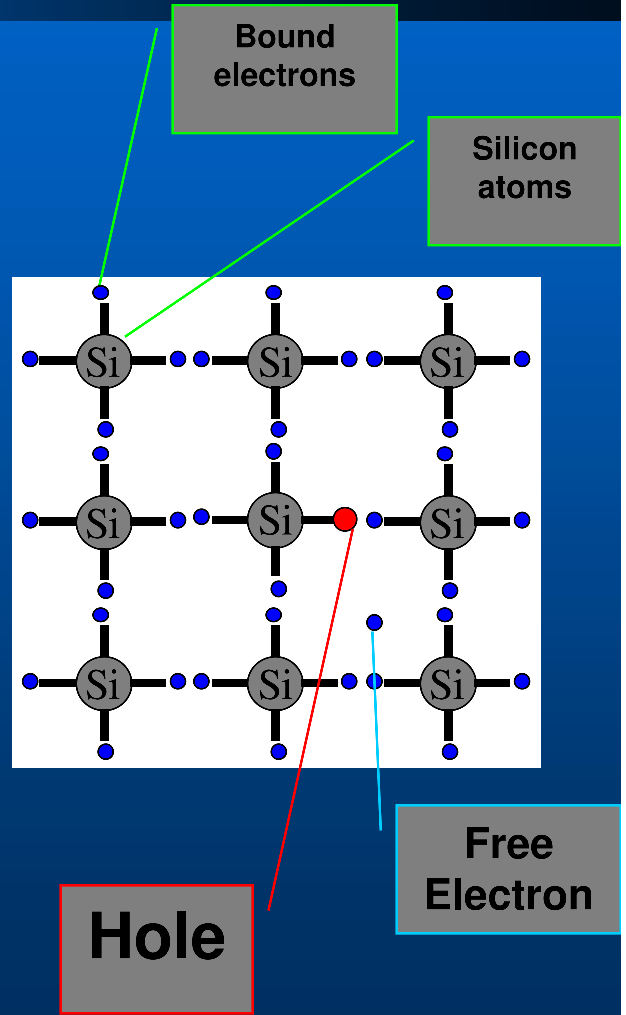
Semiconductor Physics I



- A semiconductor diode is a pn-junction. Next:
- Silicon properties; electron and hole carriers
- What us a p-type and what is an n-type semiconductor
- Electrical properties of semiconductors

Semiconductor Physics II
Silicon Lattice

- Silicon atoms are held in place by 4 covalent bonds.
- At sufficiently low Temperature there are no free electrons or holes (carriers)
- At room temperature some bonds brake (thermal excitation) and *n* free electrons and *p* holes are created.
- $n = p = n_i$

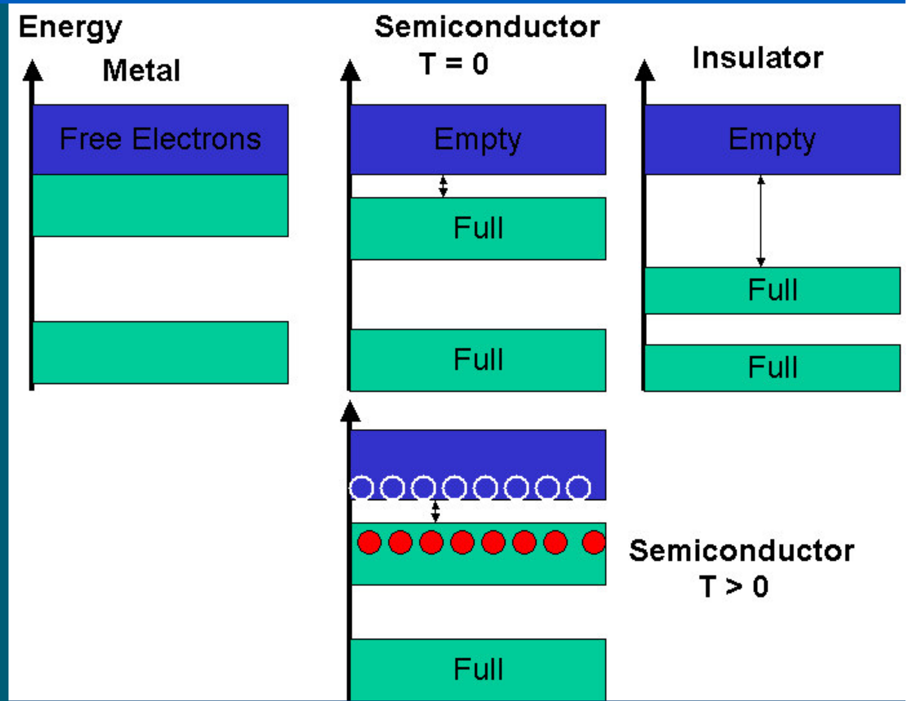


Semiconductor Physics III

- The number of carriers n_i is obviously temperature dependent:
- $n_i^2 = BT^3 e^{(-E_G/KT)}$ *(proof in App. II)*
- $B = 5.4\ 10^{31}\ K^{-3}\ cm^{-6}$
- $E_G = 1.12$ eV (Si)
- $K = 8.62\ 10^{-5}$ eV/K
- At 300 K $n_i = 1.5\ 10^{10}$ to be compared with $5\ 10^{22}$ Si-Atoms/cm^3
- The semiconductor conductivity rises with temperature

Semiconductor Physics IV
Energy Bands

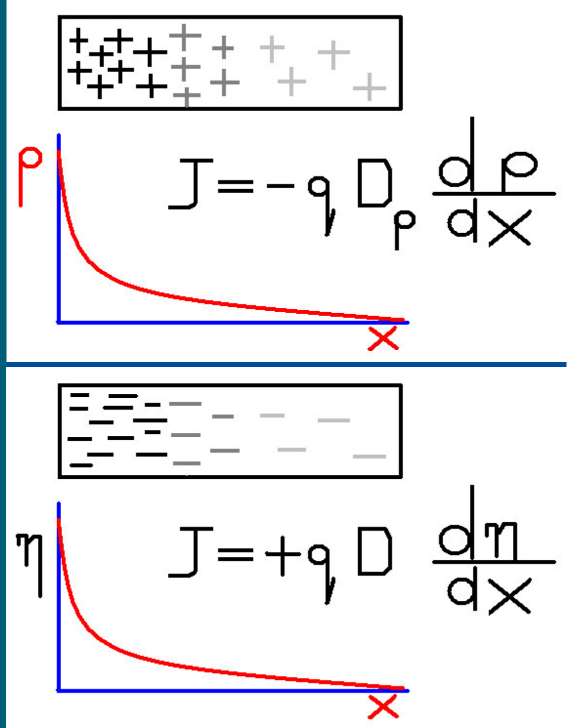
- Metals have no gap between the conduction band and the last bound band.
- Insulators have a large gap
- Semiconductors have a small gap ~ 1eV
- At T>0 thermally excited electrons move to the conduction band leaving holes behind.



This is why the conductivity rises with Temperature

Semiconductor Physics III
Diffusion Current

- **The net current created by thermal excitation is of course zero because the motion is random and the silicon carrier density is uniform.**
- **But suppose that the carrier density is made non-uniform.**
- **In this case the thermal excitation results to the Diffusion Current: I_D.**



Where p, n are the hole and electron densities

Semiconductor Physics IV

Drift Velocity and Current

- The carriers (electrons/holes can also drift under the influence of an electric field E.
- The Drift velocity is proportional to the Electric field but different for electrons and holes:

$$\vec{V} = \mu_p \vec{E} \qquad \vec{V} = \mu_\eta \vec{E}$$

- The constant μ_p is the mobility for electrons and μ_n the mobility for holes

Semiconductor Physics V

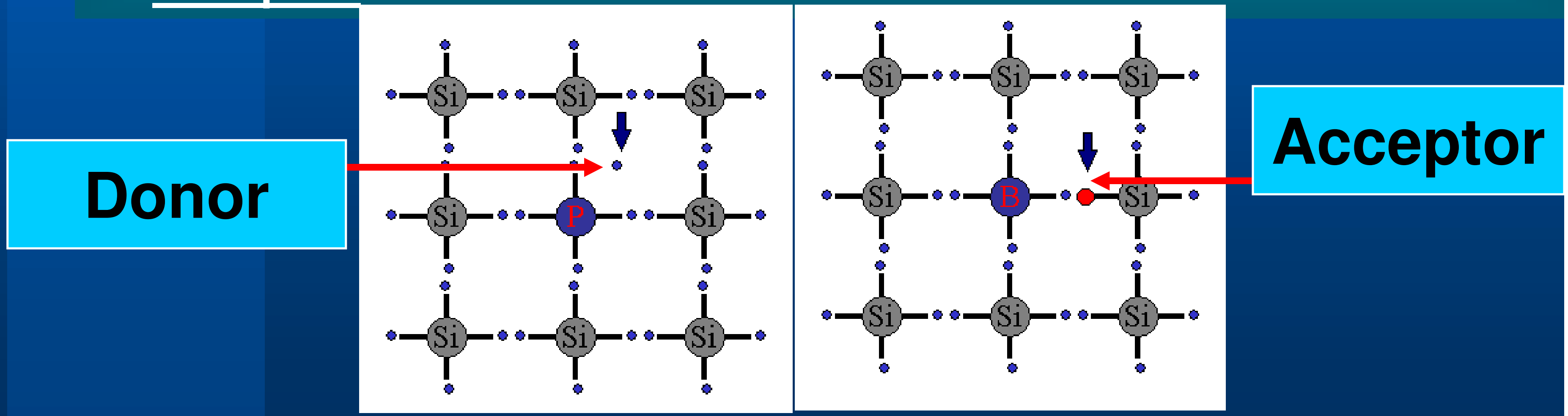
Drift Current Density

- For Si the electron mobility is 1350 cm^2/Vs and the hole mobility is 480 cm^2/Vs.
- From first year physics one can calculate that the current density due to the drift velocity is:

$$\vec{J} = q(p\mu_p + n\mu_n)\vec{E}$$

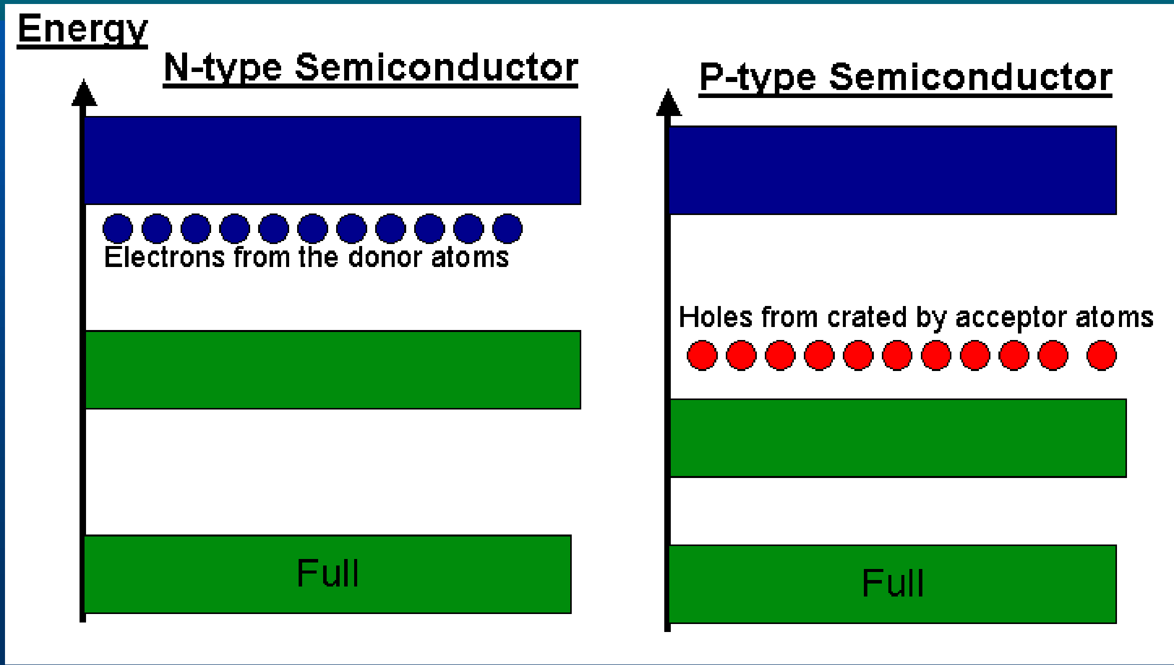
Doped Semiconductors (VI)

- Doped semiconductors come either as n- or p-type (electrons or holes are predominately the carriers)
- Introducing small Phosphorus (which has 5 valence electrons) impurities generates an n-semiconductor since one electron remains free. Hence Phosphorus is a <u>Donor.</u>
- Boron has three valence electrons. Introducing Boron in Si will crate holes because the Boron takes one electron from Si to form covalent bonds in the Si lattice. Hence Boson is an <u>Acceptor.</u>



Doped Semiconductors (VII)

- The donor electrons occupy energy states just below the conductivity band
- The holes created by the acceptors occupy energy states which are just above the zone with the bounded electrons.



Doped Semiconductors (VIII)

- The carriers created via implant of impurities are called majority carriers. In general carriers can always be created via thermal excitation. Therefore there are always also minority carriers.
- In a p-type semiconductor the majority carriers are the holes and the minority carriers are the electrons.
- In an n-type semiconductor the majority carriers are electrons and the minority carriers are holes
- Electron and hole densities obey :

N-type	P-type
$n_{n0} = N_D$	$p_{p0} = N_A$
$n_{n0}\, p_{n0} = n_i^2$	$n_{p0}\, p_{p0} = n_i^2$
$p_{n0} = n_i^2/N_D$	$n_{p0} = n_i^2/N_A$

Notation:

$n_{n0}/\, p_{n0}$ = number of electrons/holes in n-type Semiconductor at thermal equilibrium

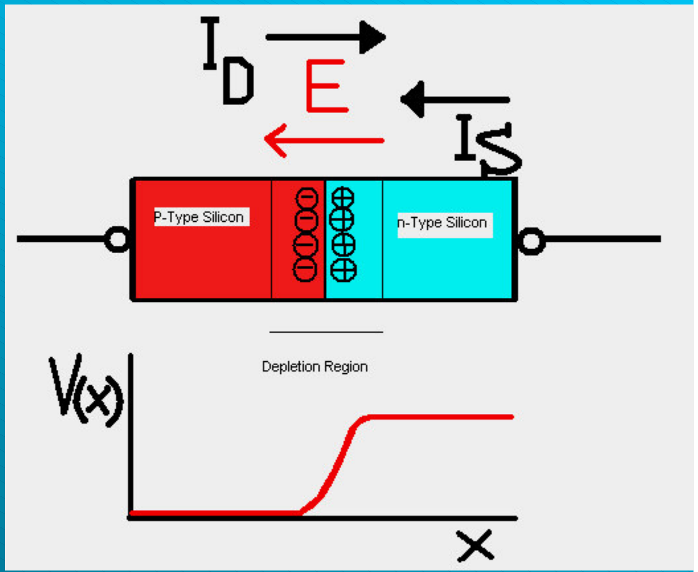
N_D = number of donors

n_{p0}/p_{p0} = number of electrons/holes in p-type Semiconductor at thermal equilibrium

N_A = number of acceptors

pn-junction in open circuit (VII)

- As discussed before n-type of carriers will diffuse towards p-type and p-type of carriers will diffuse towards n-type creating I_D. There they will recombine. This will leave '+' charges in the n-region and '–' charges in the p-region mostly close to the junction.
- Therefore a carrier-depleted region is created close to the junction.
- This charge creates a field that eventually stops the process.
- In addition minority carriers (thermally excited holes from n-type region or electrons from the p-type region) will through the depletion region. This will crate I_S (drift current).
- Since no net current is flowing through the diode we have : $I_D = I_S$
- The Junction potential is given by:



$$V_0=V_T \ln(N_A N_D/n_i^2)$$

$$I_D = I_S$$

Depletion Region Depth (VIII)

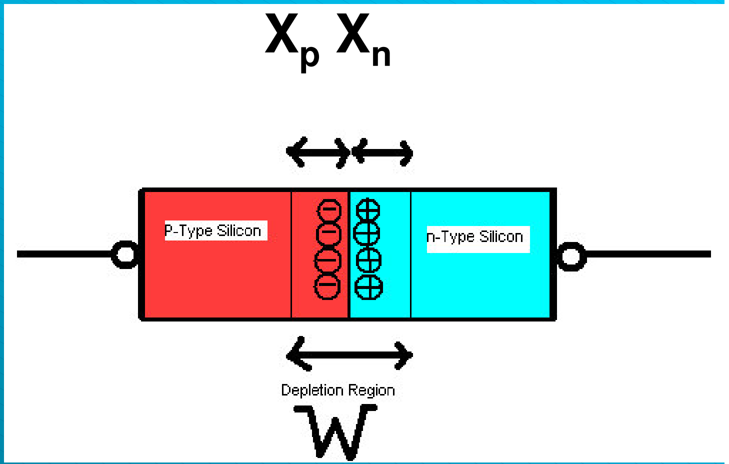
- The charges from both sides of the depletion region are equal. Therefore: $qx_pAN_A = qx_nAN_D \Rightarrow x_n/x_p = N_A/N_D$ where x_p, x_n are the depths of each side of the depletion region and A is the area.
- The depletion region total depth is simply $W_{dep}=x_p+x_n$:

$$W_{dep}= [(2\varepsilon_s/q)(1/N_A + 1/N_D)V_0]^{1/2}$$

<u>For Silicon:</u>

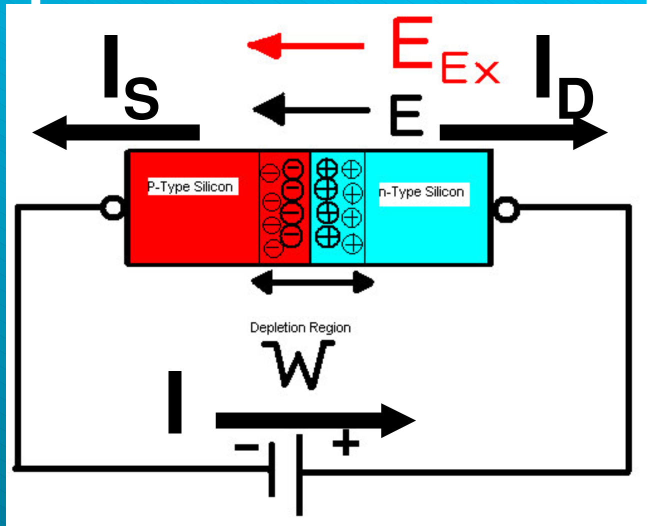
$\varepsilon_s= 1.04\ 10^{-12}$ F/cm

$W = 0.1 - 1.0\ \mu m$



pn-junction in reverse bias (IX)

- In reverse bias the n-side of the junction is connected to the '+' and the p-side to the '–' of a source.
- The external field has the same direction with the depletion region field resulting to a stronger field in the depletion region.
- The depletion region grows as more holes are transported to the n-side and more electrons in the p-side
- No current flows except of a small leakage current….
- $$I = I_S - I_D$$



The Depletion Capacitance (X)

THE DEPLETION CAPACITANCE ①

We see that the diode in reverse bias behaves like a capacitor:

$$q_J = q_N = e\cdot N_D\cdot x_n\cdot A \quad (A) \quad \left\{\begin{array}{l} e = \text{electron charge} \\ A = \text{area} \\ x_n, N_D \text{ as before}\end{array}\right.$$

$$\frac{x_n}{x_p} = \frac{N_A}{N_D} \text{ (from before)} \Rightarrow \frac{x_n}{x_n+x_p} = \frac{N_A}{N_A+N_D} \Rightarrow$$

$$x_n = (x_n+x_p)\frac{N_A}{N_A+N_D} = W_{dep}\frac{N_A}{N_A+N_D} \quad (B)$$

$$(A)+(B) \rightarrow q_J = e N_D W_{dep}\frac{N_A}{N_A+N_D} A \Longrightarrow$$

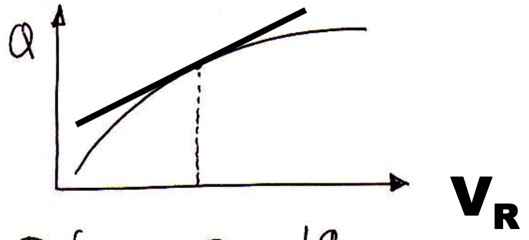
$$\boxed{q_J = e\frac{N_A N_D}{N_A+N_D} W_{dep} A}$$

$$W_{dep} = \sqrt{\frac{2\varepsilon_s}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(\underbrace{V_0}_{\text{depl. Voltage}}+\underbrace{V_R}_{\text{reverse Voltage}})}$$

The difference with the capacitor is that the Q, V do not obey a linear relationship

$$q_J = e\frac{N_A N_D}{N_A+N_D}\cdot A\cdot\sqrt{\frac{2\varepsilon_s}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0+\mathbf{V_R})}$$

Therefore $q_J \sim \sqrt{V_R} \Rightarrow$ ②



Define $C_j = \frac{dq_J}{dV_R} \Rightarrow$

$$\frac{dQ_J}{dV_R} = e\frac{N_A N_D}{N_A+N_D} A \frac{dW_{dep}}{dV_R} \quad ①$$

$$\frac{dW_{dep}}{dV_R} = \frac{1}{2}\frac{1}{W_{dep}}\frac{2\varepsilon_s}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right) \quad ②$$

$$①② \Rightarrow \frac{dQ_J}{dV_R} = e\frac{\cancel{N_A N_D}}{\cancel{N_A+N_D}} A\frac{1}{2}\frac{1}{W_{dep}}\frac{2\varepsilon_s}{e}\cancel{\frac{N_A+N_D}{N_A N_D}}$$

$$\frac{dQ_J}{dV_R} = \frac{\cancel{e}A}{\cancel{2}W_{dep}}\frac{\cancel{2}\varepsilon_s}{\cancel{e}} = \varepsilon_s\frac{A}{W_{dep}}$$

$$\therefore C_J = \frac{dQ_J}{dV_R} = \frac{\varepsilon_s A}{W_{dep}} \quad \text{(just like a capacitor)}$$

The Depletion Capacitance (XI)

③

$$C_j = \frac{A\varepsilon_s}{\sqrt{\frac{2\varepsilon_s}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_0+V_R)}} \rightarrow$$

$$C_j = \frac{A\varepsilon_s}{\sqrt{V_0\left(1+\frac{V_R}{V_0}\right)}\sqrt{\frac{2\varepsilon_s}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}} \rightarrow$$

$$C_j = \frac{1}{\sqrt{1+\frac{V_R}{V_0}}} \times \frac{A\sqrt{\varepsilon_s}}{\sqrt{\frac{2}{e}\left(\frac{N_A+N_D}{N_A N_D}\right)V_0}} \rightarrow$$

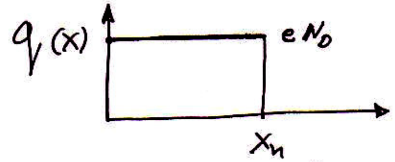
$$C_j = \frac{1}{\sqrt{1+V_R/V_0}} \times \frac{A\sqrt{\varepsilon_s e N_A N_D}}{\sqrt{2V_0(N_A+N_D)}} \quad ①$$

If we call $C_{j0} = \frac{A\sqrt{\varepsilon_s e N_A N_D}}{\sqrt{2V_0(N_A+N_D)}}$ then

$$C_j = \frac{C_{j0}}{\left(1+\frac{V_R}{V_0}\right)^{1/2}}$$

④

We derived the previous formula under the assumption that the junction charge is distributed as



This is obviously a simplification and in general we expect a more complicated and smooth charge distribution.

The capacitance formula is then approximated by:

$$C_j = \frac{C_{j0}}{\left(1+\frac{V_R}{V_0}\right)^{m}}$$

where C_{j0} = junction capacitance

m = grading coefficient

both are used in SPICE to model a diode.

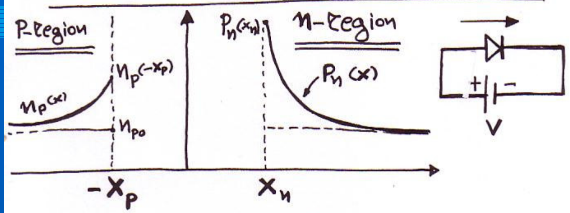
The Diode in Forward Bias XII

- Majority carriers are supplied on both sides.
- They neutralize some of the uncovered bound charge.
- The depletion layer narrows and the depletion voltage reduces
- The diffusion current grows
- $$I = I_D - I_s$$

The Diode in Forward Bias XIII

- **The Saturation current can then be calculated**

THE pn JUNCTION IN FORWARD-BIAS ①



$$p_n(x_n) = p_{no}\, e^{V/V_T} \quad \text{(LAW OF JUNCTION)}$$

$$p_n(x) = p_{no} + [p_n(x_n) - p_{no}]\, e^{-(x-x_n)/L_p}$$

L_p = Diffusion length; $L_p = \sqrt{D_p \tau_p}$

τ_p = excess-minority lifetime

$L_p \sim 1\text{-}100\,\mu m$; $\tau_p \sim 1ns - 10\mu sec$

$$J_p = -q D_p \frac{dp(x)}{dx} = -q D_p p_{no}[e^{V/V_T} - 1]\left(-\frac{1}{L_p}\right) e^{-\frac{(x-x_n)}{L_p}}$$

$$J_p = \underbrace{+q \frac{D_p}{L_p} p_{no}[e^{V/V_T} - 1]}_{j_p}\, e^{-(x-x_n)/L_p}$$

$$J_p = j_p\, e^{-\frac{(x-x_n)}{L_p}}$$

②

So

$$j_p = q \frac{D_p}{L_p} p_{no} (e^{V/V_T} - 1)$$

and similarly

$$j_n = q \frac{D_n}{L_n} n_{po} (e^{V/V_T} - 1)$$

$$I = A(j_p + j_n) \Rightarrow$$

$$I = A\left(\frac{q D_p p_{no}}{L_p} + \frac{q D_n n_{po}}{L_n}\right)(e^{V/V_T} - 1)$$

$$I = A q n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)(e^{V/V_T} - 1)$$

recall that for a diode $I = I_s(e^{V/V_T} - 1)$

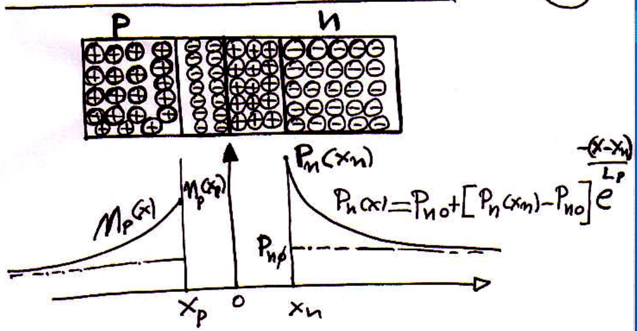
$\therefore$

$$\boxed{I_s = A q n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)}$$

Diffusion Capacitance

- **Diode display a capacitance also in forward bias.**
- **Results to a *transit time* τ_T**

DIFFUSION CAPACITANCE (1)



$$Q_p = A \cdot q \cdot \int_{x_n}^{\infty} (P_n(x) - P_{no})\, dx \rightarrow$$

$$Q_p = A \cdot q \cdot \int_{x_n}^{\infty} [P_n(x_n) - P_{no}]\, e^{-\frac{x-x_n}{L_p}} dx \rightarrow$$

$$Q_p = A \cdot q\, [P_n(x_n) - P_{no}] \int_{x_n}^{\infty} e^{-\frac{x-x_n}{L_p}} dx \rightarrow$$

$$\left.\begin{array}{l} Q_p = A \cdot q \cdot L_p (P_n(x_n) - P_{no}) \\ P_n(x_n) = P_{no}\, e^{V/V_T} \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} Q_p = A \cdot q \cdot L_p P_{no} (e^{V/V_T} - 1) \\ J_p = q \frac{D_p}{L_p} P_{no} (e^{V/V_T} - 1) \end{array}\right\} \Rightarrow$$

(2)

$$\frac{Q_p}{J_p} = \frac{A L_p^2}{D_p} \Rightarrow Q_p = \frac{I L_p^2}{D_p}$$

$$\tau = \frac{L_p^2}{D_p} \Rightarrow Q_p = \tau_p I_p$$

IN GENERAL: $Q = \tau_p I_p + \tau_n I_n$

or one can write $Q = \tau_T \cdot I$

τ_T = TRANSIT TIME

In most devices one of the sides is more heavily dopped e.g $N_A >> N \rightarrow I = I_p$, $\tau_T = \tau_p$

$$C_d = \frac{dQ}{dV} = \frac{\tau_T}{V_T} I$$

in practice $C_j = 2 C_{jo}$

Appendix II:
Number of Carriers in Semiconductors

- **Fermi Statistics is used to calculate the number of electron and hole carriers in a semiconductor.**
- **The explicit calculation is shown**

Appendix II: Number of Carriers in a Semiconductor

NUMBER OF CARRIERS IN A SEMICONDUCTOR (1)

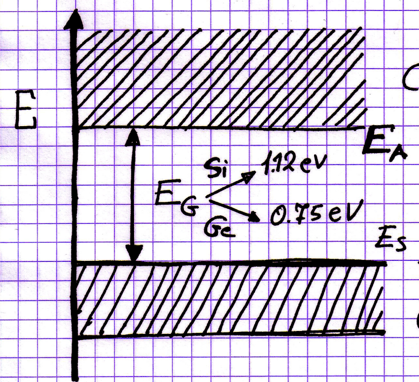
We want to calculate the number of electrons per unit volume which contribute to the conductivity of a semiconductor

- The e^- obey the Fermi-Dirac Statistics (fermions)

$$n(E)dE = N(E)F(E)dE$$

$$N(E) = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2} \quad \text{(Phase space)}$$

$$F(E) = \frac{1}{1+e^{(E-E_F)/KT}}$$



CONDUCTIVITY BAND (e^- in this band contribute to the conductivity of the semiconductor)

BOUND e^- BAND (e^- cannot move and they do not contribute to the conductivity)

(2)

The number of e^- carriers in the conductivity band is given by:

$$n = \int_{E_A}^{+\infty} N(E)F(E)\,dE \quad (1)$$

$$N(E) = \frac{4\pi}{h^3}(2m_n)^{\frac{3}{2}}(E-E_A)^{\frac{1}{2}} \quad (2)$$

(1) (2) → $$n = \frac{4\pi}{h^3}(2m_n)^{3/2}\int_{E_A}^{\infty}\frac{(E-E_A)^{1/2}}{1+e^{(E-E_F)/KT}}$$

@ room temp $KT \sim 0.03$ eV, $E_F = \frac{1}{2}(E_A+E_S)$

$\therefore E-E_F \sim \frac{1.12}{2}$ eV $\gg 0.03$ eV

and the "1" can be ignored in the denominator

Therefore:

$$n = \frac{4\pi}{h^3}(2m_n)^{3/2}\int_{E_A}^{\infty}(E-E_A)^{1/2}e^{\frac{E-E_F}{KT}}\,dE$$

$$n = \frac{4\pi}{h^3}(2m_n)^{3/2}e^{-\frac{E_A-E_F}{KT}}\int_0^{\infty}x^{1/2}e^{-\frac{x}{KT}}\,dx \quad (3)$$

But $I = \int_0^{\infty}\sqrt{\varepsilon}\,e^{-\varepsilon/KT}\,d\varepsilon$, $\varepsilon = y^2$ $\Big\}$ $I = \int_0^{\infty} y\,e^{-y^2/KT}\,2y\,dy = 2\int_0^{\infty}y^2e^{-y^2/KT}\,dy \quad (4)$

Appendix II: II

But $\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$ (3)

$$\frac{\partial}{\partial a}\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\pi}\, a^{-3/2}\left(-\frac{1}{2}\right)$$

$$\int_0^\infty (-x^2)\, e^{-ax^2}dx = -\frac{1}{4}\sqrt{\pi}\, a^{-3/2}$$

$$\int_0^\infty x^2 e^{-ax^2}dx = \frac{1}{4}\frac{\pi^{1/2}}{a^{3/2}} \quad (5)$$

(3)(4)(5)

$$n = \frac{4\pi}{h^3}(2m_n)^{3/2}\, e^{-\frac{E_A-E_F}{kT}}\, 2\int_0^\infty y^2 e^{-y^2/kT}dy$$

$$n = \frac{\cancel{4}\pi}{h^3}(2m_n)^{3/2}\, e^{-\frac{E_A-E_F}{kT}}\, 2\,\cancel{\frac{1}{4}}\,\frac{\pi^{1/2}}{(1/kT)^{3/2}}$$

$$n = \frac{2\pi}{h^3}(2m_n)^{3/2}\, e^{-\frac{E_A-E_F}{kT}}\,(kT)^{3/2}\,\pi^{1/2}$$

$$n = \frac{2}{h^3}(2\pi m_n kT)^{3/2}\, e^{-(E_A-E_F)/kT}$$

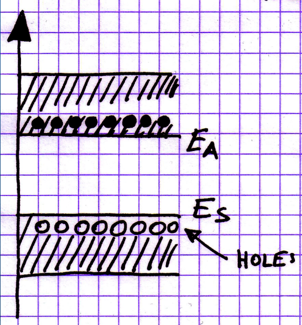
(4)

$$n = 2\left[\frac{2\pi m_n KT}{h^3}\right]^{\frac{3}{2}} e^{-\frac{E_A-E_F}{KT}}$$

Next we have to calculate <u>the number of holes</u>:

a hole ≡ absence of an electron

∴ $F_{HOLE} = 1 - F(E)$



$$P = \int_{-\infty}^{E_S} N(E)\, F_{HOLE}\, dE$$

Appendix II: III

as in the case with electrons we have ⑤

$$N(E) = \frac{4\pi}{h^3}(2m_p)^{3/2}(E_S - E)^{1/2}$$

and

$$F_{HOLE}(E) = 1 - F(E) = \frac{1}{1 + e^{-\frac{E-E_F}{KT}}}$$

Therefore

$$P = \frac{4\pi}{h^3}(2m_p)^{3/2}\int_{-\infty}^{E_S}\frac{(E_S-E)^{1/2}\,dE}{1+e^{-\frac{E-E_F}{KT}}}$$

$$\Rightarrow$$

$$P = 2\left[\frac{2m_p\pi KT}{h^2}\right]^{3/2} e^{-\frac{E_F-E_S}{KT}}$$

Given that the semiconductor had no impurities $n = P \Rightarrow$ ⑥

$$m_n^{3/2}\, e^{-\frac{E_A-E_F}{KT}} = m_p^{3/2}\, e^{-\frac{E_F-E_S}{KT}}$$

$$e^{-\frac{E_A-E_F}{KT}+\frac{E_F-E_S}{KT}} = \left(\frac{m_p}{m_n}\right)^{3/2}$$

$$e^{2E_F/KT}\, e^{-\frac{E_A+E_S}{KT}} = \left(\frac{m_p}{m_n}\right)^{3/2}$$

$$\frac{2E_F}{KT} - \frac{E_A+E_S}{KT} = \frac{3}{2}\ln\left(\frac{m_p}{m_n}\right)$$

$$\frac{2E_F}{KT} = \frac{E_A+E_S}{KT} + \frac{3}{2}\ln\left(\frac{m_p}{m_n}\right)$$

$$E_F = \frac{E_A+E_S}{2} + \frac{3}{4}KT\ln\left(\frac{m_p}{m_n}\right)$$

Appendix II: IV

THE PRODUCT $n \cdot p$

(7)

Using the expressions for n, p we have

$$n \cdot p = 2\left(\frac{2\pi m_n KT}{h^2}\right)^{3/2} \times 2\left(\frac{2\pi m_p KT}{h^2}\right)^{3/2} \times e^{-\frac{E_A - E_F}{KT}} \, e^{-\frac{E_F - E_S}{KT}} \Longrightarrow$$

$$n \cdot p = 4\left(\frac{2\pi KT}{h^2}\right)^3 (m_p)^{3/2} (m_n)^{3/2} \times e^{\frac{-E_A + \cancel{E_F} - \cancel{E_F} + E_S}{KT}}$$

$$\boxed{n \cdot p = 4\left(\frac{2\pi KT}{h^2}\right)^3 (m_p \cdot m_n)^{3/2} \times e^{-E_G/KT}}$$