ΔΥΝΑΜΙΚΟ (Συνέχεια)

$V_f - V_i = -\int \vec{E}\cdot d\vec{\ell}$. Σημειακό φορτίο Q, $\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$

$V_\infty \to 0$

ΑΝ $Q > 0 \to V > 0$

$Q < 0 \to V < 0$

ΔΥΝΑΜΙΚΟ ΑΠΟ ΟΜΑΔΑ ΦΟΡΤΙΩΝ

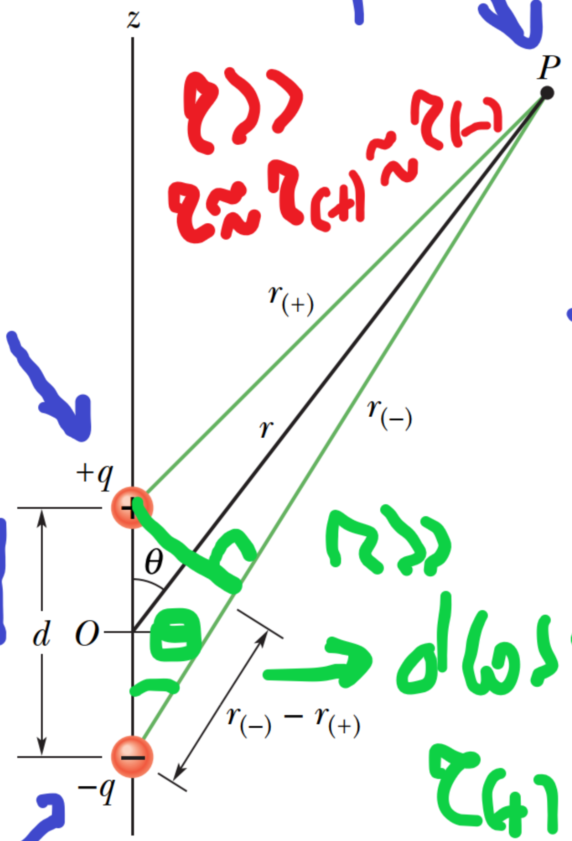
$$V_P = \sum_{i=1}^{N} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}$$

ΔΥΝΑΜΙΚΟ ΗΛΕΚΤΡΙΚΟΥ ΔΙΠΟΛΟΥ

2

3/4/20

$V_P = ?$



$r \gg$

$r \approx r_{(+)} \approx r_{(-)}$

$r \gg$

$\rightarrow d\cos\theta$

$r_{(+)} \cdot r_{(-)} \approx r^2$

$|\vec{p}| = q \cdot d$

$$V_P = \frac{q}{4\pi\varepsilon_0 r_{(+)}} - \frac{q}{4\pi\varepsilon_0 r_{(-)}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_{(+)}} - \frac{1}{r_{(-)}}\right)$$

$$\rightarrow V_P = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_{(-)} - r_{(+)}}{r_{(-)} \cdot r_{(+)}}\right)$$

ΥΠΟΘΕΤΟΥΜΕ ΟΤΙ ΤΟ $r \gg$

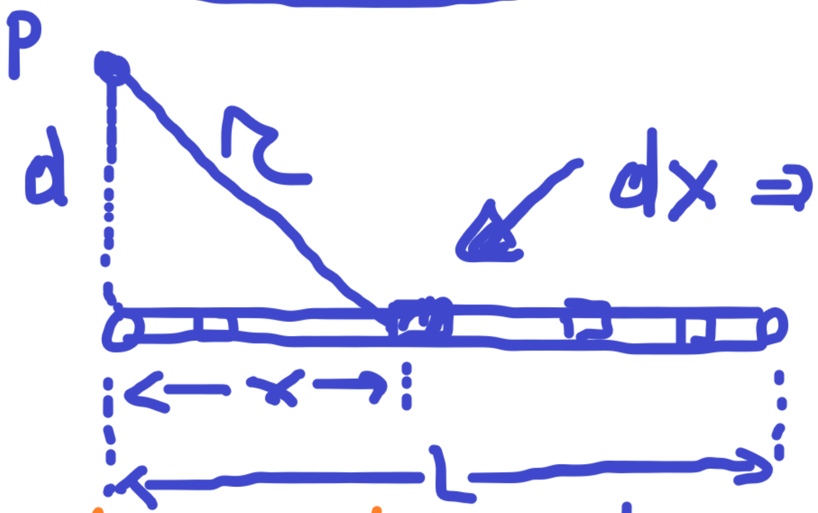
ΑΝ $r \gg \rightarrow$

$$\boxed{V_P = \frac{(q}{4\pi\varepsilon_0}\frac{d)\cos\theta}{r^2}}$$

ΔΥΝΑΜΙΚΟ ΔΙΠΟΛΟΥ

$$\boxed{V_P = \frac{|\vec{p}|\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}}$$

ΠΑΡΑΔΕΙΓΜΑ: ΕΥΘΕΙΑ ΦΟΡΤΙΟΥ ΓΡΑΜΜΙΚΗΣ ΠΥΚΝΟΤΗΤΑΣ λ



$dx \Rightarrow dQ = \lambda\, dx$

ΔΙΔΕΤΑΙ:

$$J = \int \frac{dx}{(x^2+d^2)^{1/2}} = \ln\left[x+\sqrt{x^2+d^2}\right]$$

$$\left(\lambda = \frac{dQ}{dx} \qquad [\lambda] = \frac{C}{m}\right)$$

$$\boxed{\ln\left(\frac{a}{b}\right) = \ln a - \ln b}$$

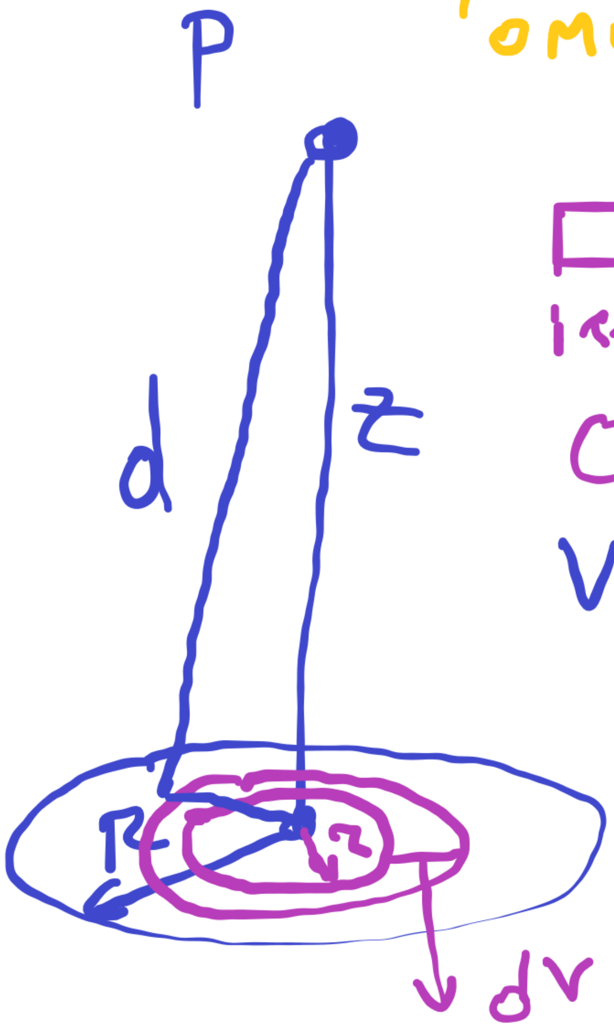
$$\int_0^L dV_P = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{\sqrt{d^2+x^2}} \Rightarrow$$

$$V_P = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{\sqrt{x^2+d^2}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(x+\sqrt{x^2+d^2}\right)\Big|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left\{\ln\left(L+\sqrt{L^2+d^2}\right) - \ln d\right\}$$

$$\rightarrow \boxed{V_P = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L+\sqrt{L^2+d^2}}{d}\right]}$$

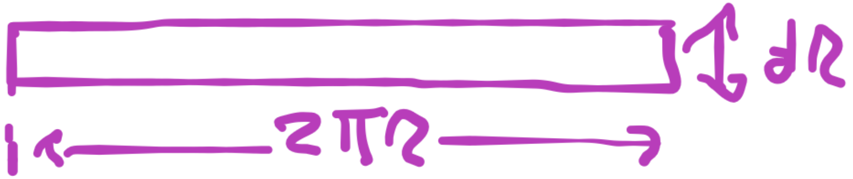
ΔΥΝΑΜΙΚΟ ΦΟΡΤΙΣΜΕΝΟΥ ΔΙΣΚΟΥ

3/4/20 4



ΟΜΟΙΟΜΟΡΦΑ

$\sigma = \frac{dQ}{dA} \rightarrow [\sigma] = \frac{C}{m^2}$ $\qquad df = f'\,dx$



$dQ = 2\pi r \cdot dr \cdot \sigma$

$\frac{1}{2}dr^2$

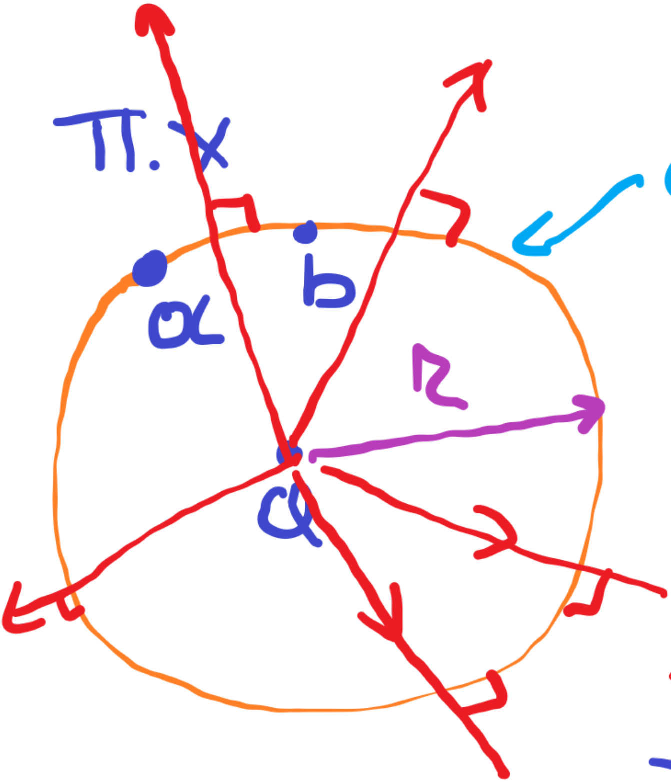
$$V_P = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{dQ}{d} = \frac{1}{\cancel{4}_{2}\cancel{\pi}\varepsilon_0}\int_0^R \frac{\cancel{2\pi} r \cdot dr\, \sigma}{\sqrt{r^2+z^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2+z^2}} \Rightarrow$$

$$V_P = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{\frac{1}{2}\,dr^2}{\sqrt{r^2+z^2}} = \frac{\sigma}{4\varepsilon_0}\int_0^R \frac{d(r^2+z^2)}{\sqrt{r^2+z^2}} = \frac{\sigma}{4\varepsilon_0}\left[\frac{(r^2+z^2)^{1/2}}{1/2}\right]_0^R$$

$$\boxed{V_P = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right]}$$

ΙΣΟΔΥΝΑΜΙΚΕΣ ΕΠΙΦΑΝΕΙΕΣ

Π.χ.

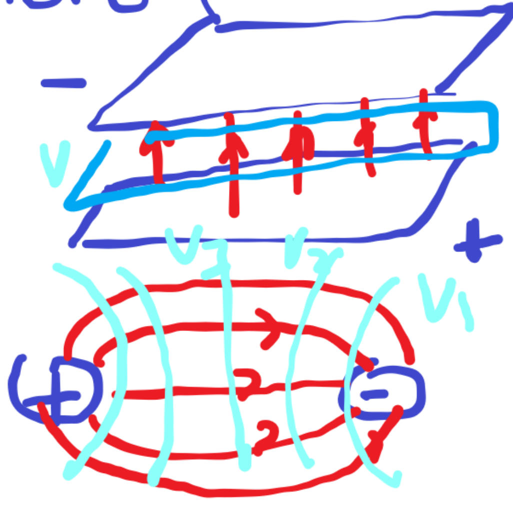


ΟΛΑ ΤΑ ΣΗΜΕΙΑ ΤΗΣ ΣΦΑΙΡΑΣ ΕΧΟΥΝ ΤΟ ΙΔΙΟ ΔΥΝΑΜΙΚΟ ⇒ Η ΣΦΑΙΡΑ ΕΙΝΑΙ ΙΣΟΔΥΝΑΜΙΚΗ ΕΠΙΦΑΝΕΙΑ

ΟΙ ΙΣΟΔΥΝΑΜΙΚΕΣ ΕΠΙΦΑΝΕΙΕΣ ΕΙΝΑΙ ΠΑΝΤΑ ΚΑΘΕΤΕΣ ΣΤΙΣ ΔΥΝΑΜΙΚΕΣ ΓΡΑΜΜΕΣ ΤΟΥ ΠΕΔΙΟΥ

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\left.\begin{aligned} V_b - V_a &= -\int_a^b \vec{E}\cdot d\vec{\ell} \\ V_b = V_a &\Rightarrow \underbrace{\int_a^b \vec{E}\cdot d\vec{\ell}}_{0} = 0 \end{aligned}\right\} \Rightarrow \vec{E} \perp d\vec{\ell}$$



ΥΠΟΛΟΓΙΣΜΟΣ ΠΕΔΙΟΥ ΑΠΟ ΤΟ ΔΥΝΑΜΙΚΟ 3/4/20 (6)

$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right)$$

$$E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right)$$

ΓΙΑΤΙ ΑΥΤΟ ΕΙΝΑΙ ΣΩΣΤΟ

$$V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{\ell} = \int_1^2 \vec{\nabla} V \cdot d\ell = \int_1^2 dV = \underline{V(2) - V(1)}$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

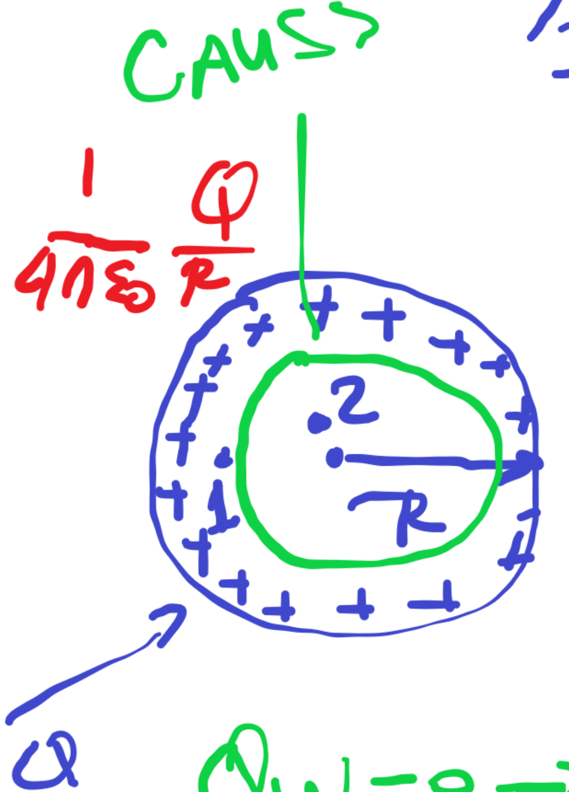
$$df = \vec{\nabla} f \cdot d\vec{x}$$

$$V(x,y,z) \xrightarrow{\vec{E} = -\vec{\nabla} V} \vec{E}$$

ΔΥΝΑΜΙΚΟ ΦΟΡΤΙΣΜΕΝΟΥ ΑΓΩΓΟΥ

3/4/20 7

$\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$



$Q_{IN}=0 \rightarrow$

$\vec{E}_{IN}=0$

$$V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{\ell} \Rightarrow V_2 - V_1 = 0$$

$\hookrightarrow E_{IN}=0$!!

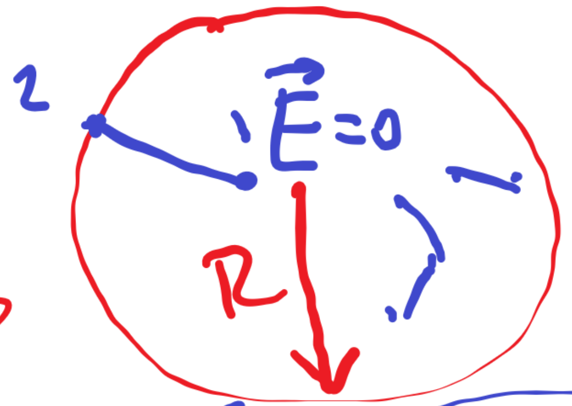
$$V_2 = V_1 = 0$$

$$\boxed{\text{ΤΟ ΔΥΝΑΜΙΚΟ ΣΤΟ ΕΣΩΤΕΡΙΚΟ ΤΟΥ ΑΓΩΓΟΥ ΕΙΝΑΙ ΤΟ ΙΔΙΟ ΠΑΝΤΟΥ}}$$

$\vec{E}=0$

$V=$ ΣΤΑΘΕΡΟ

8



ΑΓΩΓΟΣ

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

GAUSS $\rightarrow \vec{E}=0$

$$\Delta V_{21} = -\int \vec{E}\cdot d\vec{\ell} = 0$$

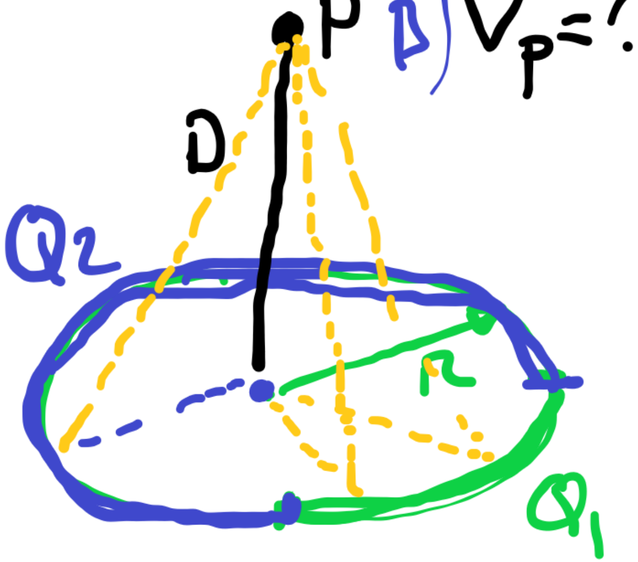
ΑΡΑ ΟΛΑ ΤΑ ΣΗΜΕΙΑ ΜΕΣΑ ΣΤΗ ΣΦΑΙΡΑ ΕΧΟΥΝ ΤΟ ΙΔΙΟ ΔΥΝΑΜΙΚΟ ΜΕ ΤΗΝ ΕΠΙΦΑΝΕΙΑ

ΚΕΦΑΛΑΙΟ 24 ΑΣΚΗΣΗ 24.23

$\frac{1}{4\pi\varepsilon_0} = 8.99\times10^{9}\,\frac{Nm^2}{C^2}$ 3.4.20 (9)

α) $V_{KENTPO} = ?$

β) $V_p = ?$



$R = 8.2\,cm$

$Q_1 = 4.2\,pC$

$Q_2 = -6\,Q_1$

$D = 6.71\,cm$

ΛΥΣΗ:

(α) $V_K = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} - \frac{6Q_1}{R}\right)$

$V_K = -2.3\,V$

(β) $V_p = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{\sqrt{D^2+R^2}} - \frac{6Q_1}{\sqrt{D^2+R^2}}\right) \rightarrow$

$V_p = -1.78\,V$

24 - 24

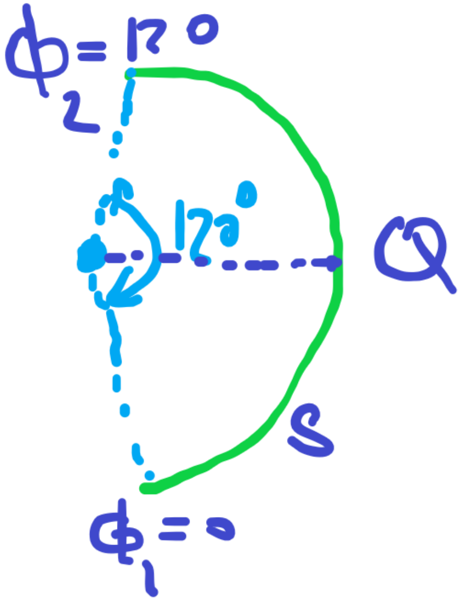
3/4/20 10

$Q = -25.6\ pC$

$R = 3.71\ cm$

$\phi = 120^0$

$V_K = ?$



ΤΟ Q ΕΙΝΑΙ ΟΜΟΙΟΜΟΡΦΑ ΚΑΤΑΝΕΜΗΜΕΝΟ $\Rightarrow$

$$\lambda = \frac{Q}{S}$$

ΛΥΣΗ

$$V_P = \int_{\phi_1}^{\phi_2} \frac{1}{4\pi\varepsilon_0} \frac{dq}{R} = \frac{1}{4\pi\varepsilon_0} \int_{\phi_1}^{\phi_2} \frac{\lambda ds}{R}$$

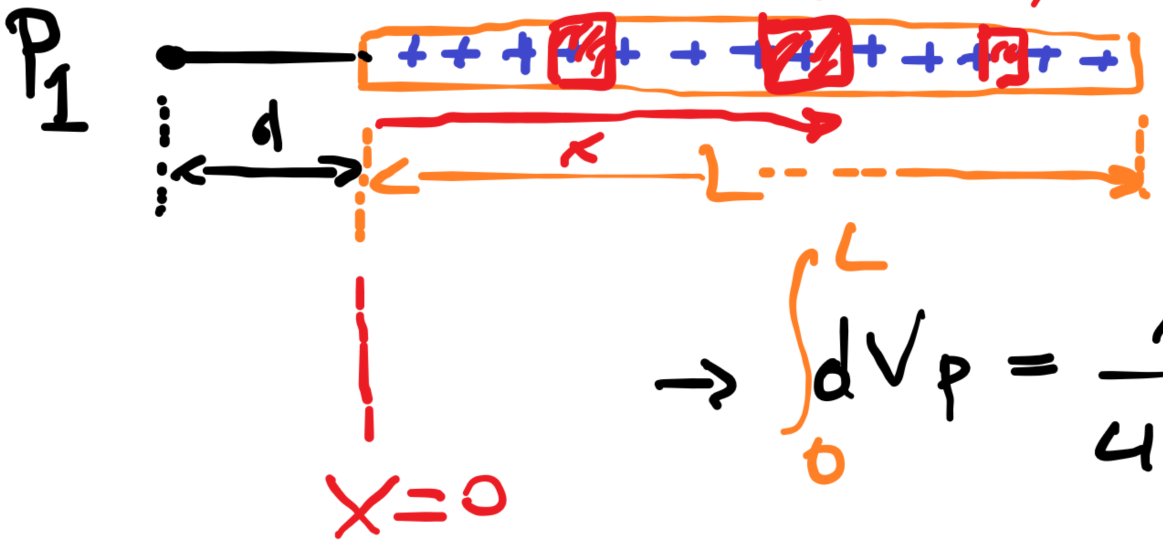
$$= \frac{1}{4\pi\varepsilon_0 R} \lambda \underbrace{\int_{\phi_1}^{\phi_2} R d\phi}_{S} = \frac{Q}{4\pi\varepsilon_0 R}$$

$$V_P = \frac{Q}{4\pi\varepsilon_0 R} = -6.2\ V$$

ΑΣΚΗΣΗ 24·30

$L = 12\ \text{cm}$ $\quad Q = 56.1\ \text{fC}$ $\quad d = 2.5\ \text{cm}$ $\quad V_p = ?$

$dx\,; \quad dQ = \lambda\, dx \qquad \lambda = \dfrac{Q}{L}$



$$dV_p = \frac{1}{4\pi\varepsilon_0}\,\frac{dQ}{d+x} \qquad 0 \le x \le L$$

$$\rightarrow \int_0^L dV_p = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dx}{d+x} \Rightarrow$$

$$V_p = \frac{\lambda}{4\pi\varepsilon_0}\underbrace{\int_0^L \frac{dx}{d+x}}_{\ln(x+d)\big|_0^L} = \frac{\lambda}{4\pi\varepsilon_0}\left\{\ln(L+d) - \ln d\right\}$$

$d(d+x)$ $\qquad \lambda = Q/L$

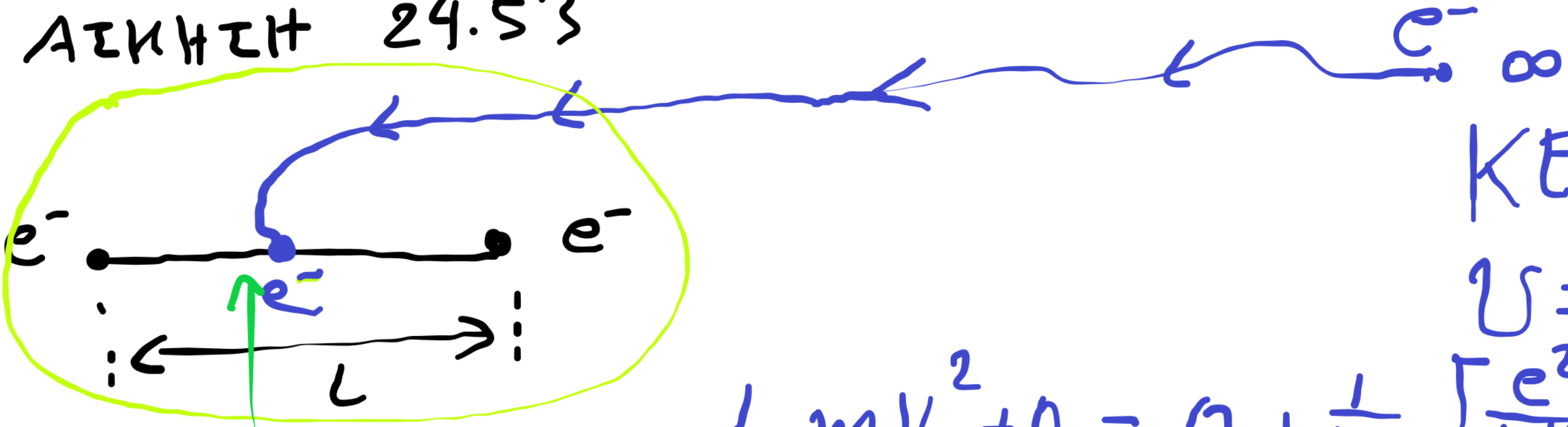
$$\Rightarrow V_p = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{L+d}{d}\right) \Rightarrow \boxed{V_p = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)}$$

$$V_p = 7.39\times10^{-3}\ \text{V}$$

ΑΣΚΗΣΕΙΣ ΓΙΑ ΤΟ ΣΠΙΤΙ

24.16, 24.25, 24.38, 24.39, 24.51

ΑΣΚΗΣΗ 24.53



$L = 2\,cm$

KE = 0

$U \neq 0$

$KE = \frac{1}{2} m V_0^2$

$U = 0$

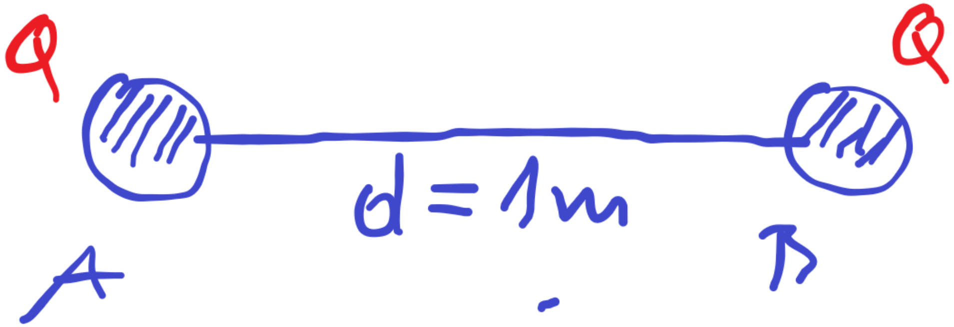
$$\frac{1}{2} m V_0^2 + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\left[\frac{e^2}{L/2} + \frac{e^2}{L/2}\right] \Rightarrow$$

$$\frac{m V_0^2}{2} = 2\,\frac{1}{4\pi\varepsilon_0}\,\frac{2e^2}{L} = \frac{\cancel{4}e^2}{\cancel{4}\pi\varepsilon_0 L} \Rightarrow$$

$$V_0^2 = \frac{2}{m}\,\frac{e^2}{\pi\varepsilon_0 L} \Rightarrow V_0 = \sqrt{\frac{2e^2}{L\pi\varepsilon_0 m}}$$

$$V_0 = \sqrt{\frac{2e^2}{\pi\varepsilon_0 m L}}$$

ΑΣΚΗΣΗ 24.51



α) $U = ?$

β) $\alpha = ?$

γ) $\upsilon_A = ?$ $\quad \upsilon_B = ?$

$r \to \infty$

$t \to \infty$

$U_\infty = 0$

$m_A = 5g$

$m_B = 10g$

$q = 5\mu C$

α) $\Rightarrow U = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d}$ ....

β) $\Rightarrow F = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d^2} \Rightarrow \alpha_A = \frac{F}{m_A}$

$\alpha_B = \frac{F}{m_B}$

γ) $KE = 0$

ΔΙΑΤ. ΕΝΕΡΓ. $\rightarrow \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d} = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

ΔΙΑΤ. ΟΡΜΗΣ $\rightarrow 0 = m_A V_A + m_B V_B$

$\upsilon_A = 7.75\ m/s$

$\upsilon_B = -3.87\ m/s$