ΗΛΕΚΤΡΙΚΟ ΠΕΔΙΟ ΑΠΟ ΓΡΑΜΜΙΚΟ ΑΓΩΓΟ (ΑΠΕΙΡΟ) 9.3.21 ①
ΜΕ ΓΡΑΜΜΙΚΗ ΠΥΚΝΟΤΗΤΑ λ

λ = ΦΟΡΤΙΟ ΑΝΑ ΜΟΝΑΔΑ ΜΗΚΟΥΣ $\rightarrow \lambda = \Delta Q / \Delta x$, $[\lambda] = C/m$ > 0

$E_x = 0$

$E_y \neq 0$

διαφορικό

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{H^2}\cos\theta$$

$$\cos\theta = \frac{h}{H}$$

$$H = \sqrt{x^2+h^2}$$

$\Rightarrow$

$$\Rightarrow dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2+h^2)}\frac{h}{(x^2+h^2)^{1/2}} = \frac{\lambda h}{4\pi\varepsilon_0}\frac{dx}{(x^2+h^2)^{3/2}} \Rightarrow \int_{-\infty}^{+\infty} dE_y = \frac{\lambda h}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{dx}{(x^2+h^2)^{3/2}} \Rightarrow$$

$$E_y = \frac{\lambda h}{4\pi\varepsilon_0}\left\{\int_0^{\infty}\frac{dx}{(x^2+h^2)^{3/2}} + \underbrace{\int_{-\infty}^{0}\frac{dx}{(x^2+h^2)^{3/2}}}_{= -\int_{+\infty}^{0}\frac{dx}{(x^2+h^2)^{3/2}} = \int_0^{\infty}\frac{dx}{(x^2+h^2)^{3/2}}}\right\} = \frac{2\lambda h}{4\pi\varepsilon_0}\int_0^{\infty}\frac{dx}{(x^2+h^2)^{3/2}}$$

$x \rightarrow -x$

$y = -x \quad (-\infty, 0) \rightarrow (\infty, 0)$

$dy = -dx$

ΔΙΔΕΤΑΙ :

$$\int_0^{\infty}\frac{dx}{(x^2+h^2)^{3/2}} = \frac{x}{h^2\sqrt{x^2+h^2}}$$

$$\rightarrow E_y = \frac{2\lambda h}{4\pi\varepsilon_0}\left[\frac{x}{h^2\sqrt{h^2+x^2}}\right]_0^{\infty}$$

$$\boxed{E_y = \frac{2\lambda \cancel{h}}{4\pi\varepsilon_0}\cdot\frac{1}{h^{\cancel{2}}} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{h}}$$

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$$\lim_{x\to\infty} \frac{x}{h^2(x^2+h^2)^{1/2}} \approx \frac{x}{h^2\,(x^2)^{1/2}} = \frac{\cancel{x}}{h^2\,\cancel{x}} = \frac{1}{h^2}$$

ΗΛΕΚΤΡΙΚΟ ΠΕΔΙΟ ΦΟΡΤΙΣΜΕΝΟΥ ΔΑΚΤΥΛΙΟΥ 9.3.21

②

$$\boxed{dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{z^2+R^2}\cos\theta} \longrightarrow dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{z^2+R^2}\frac{z}{(z^2+R^2)^{1/2}} \Rightarrow$$

$$\int_0^{2\pi} dE_z = \int_0^{2\pi}\frac{1}{4\pi\varepsilon_0}\frac{\lambda R\, d\phi}{z^2+R^2}\frac{z}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R z}{(z^2+R^2)^{3/2}}\int_0^{2\pi} d\phi = \frac{1}{4\pi\varepsilon_0}\frac{\overbrace{2\pi R\lambda}^{Q}\, z}{(z^2+R^2)^{3/2}}$$

$$\boxed{E_z = \frac{Q}{4\pi\varepsilon_0}\frac{z}{[z^2+R^2]^{3/2}}}$$

$$\boxed{E_z(z\gg) = \frac{Q}{4\pi\varepsilon_0 z^2}}$$

σημειακό φορτίο

$$\lambda = \frac{Q}{2\pi R} \Rightarrow Q = 2\pi R\lambda$$

$$\frac{z}{[z^2+R^2]^{3/2}} = \frac{\cancel{z}}{z^{\cancel{3}\,2}\left[1+\frac{R^2}{z^2}\right]^{3/2}} = \frac{1}{z^2}\left[1+\frac{R^2}{z^2}\right]^{-3/2} \approx \frac{1}{z^2}\left[1-\frac{3}{2}\frac{R^2}{z^2}\right]$$

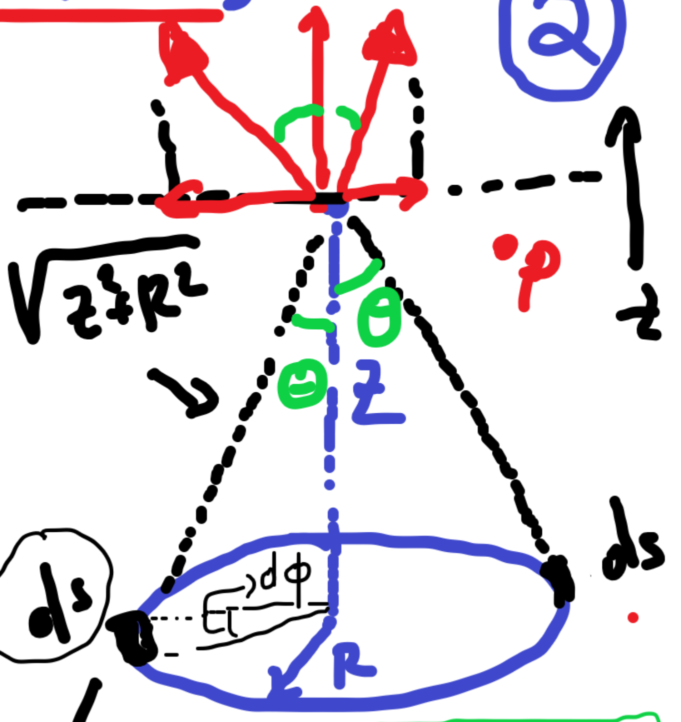
πώς ΑΛΛΑΖΕΙ ΟΤΑΝ $z \gg R$

$$(1+x)^N \simeq 1+Nx$$

$$E(z\gg R) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{z^2} - \left[\frac{Q}{4\pi\varepsilon_0}\frac{3}{2}\frac{R^2}{z^4}\right]$$

$$\boxed{ds = R\cdot d\phi}$$

rad



$$Q \rightarrow \boxed{\lambda = \frac{Q}{2\pi R}}$$

$$\lambda = \frac{dQ}{ds}$$

$$\boxed{d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\hat{r}}$$

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$$dE_z = \frac{dQ}{4\pi\varepsilon_0}\,\frac{z}{(z^2+r^2)^{3/2}}$$

$$dQ = dA\cdot\sigma = 2\pi r\cdot dr\cdot\sigma$$

ΔΑΚΤΥΛΙΟΣ

$$dE_z = \frac{\cancel{2}\pi r\,dr\cdot\sigma}{\cancel{4}_{2}\,\pi\varepsilon_0}\,\frac{z}{(z^2+r^2)^{3/2}} \Rightarrow$$

Επιφανειακή πυκνότητα φορτίου $\sigma = \frac{Q}{\pi R^2}$

$[\sigma] = \frac{C}{m^2}$

Επιφάνεια κύκλου

$$\boxed{dr^2 = 2r\,dr}$$

$$\int_0^R dE_z = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\,\frac{1}{2}\int_0^R \frac{2r\,dr}{(z^2+r^2)^{3/2}} \Rightarrow$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

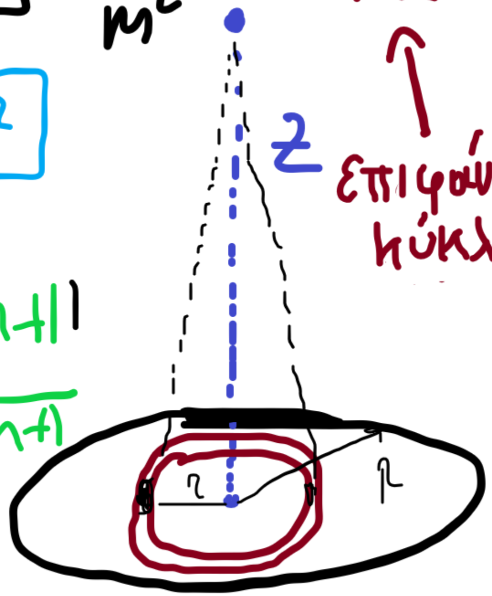
$x = z^2+R^2$

$d(z+x) = dx$

$$E_z = \frac{\sigma z}{4\varepsilon_0}\int_0^R \frac{dR^2}{(z^2+R^2)^{3/2}} = \frac{\sigma z}{4\varepsilon_0}\int_0^R \frac{d(z^2+R^2)}{(z^2+r^2)^{3/2}} \Rightarrow$$

$$E_z = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2+R^2)^{-3/2+1}}{\underbrace{-3/2+1}_{-1/2}}\right]_0^R = \frac{\sigma z}{4\varepsilon_0}(-2)\left[\frac{1}{\sqrt{z^2+R^2}}\right]_0^R$$

$$= -2\,\frac{\sigma z}{4\varepsilon_0}\left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{z}\right]$$



$dA = 2\pi r\,dr$

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$$E_z = (-2)\,\frac{\sigma\cdot z}{4\varepsilon_0}\left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{z}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

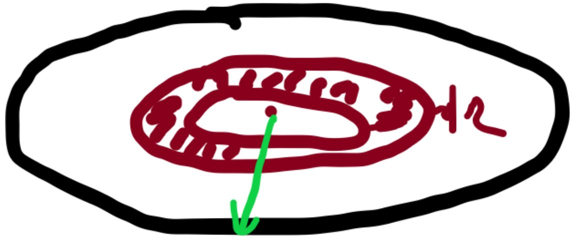
$$\boxed{E_z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]}$$

$\Delta S = R\,d\phi$

$dA = dr\cdot r\cdot d\phi$

$= \pi R^2$

ΕΜΒΑΔΟ ΚΥΚΛΟΥ



$$\underbrace{\int_0^R 2\pi \varrho\, d\varrho}_{} = 2\pi \frac{\varrho^2}{2}\Big|_0^R = \underbrace{\pi R^2}_{\text{επιφάνεια κύκλου}}$$

$$\sum_i \underbrace{2\pi R_i\, dR_i}_{dA_i \text{ (δακτύλιος)}}$$

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$$E_z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{\cancel{z}}{\cancel{z}\left[1+\frac{R^2}{z^2}\right]^{1/2}}\right]$$

$E_z\,(z \gg R)$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \left[1 + \frac{R^2}{z^2}\right]^{-1/2}\right]$$

$$\underbrace{z \gg R \rightarrow \left(\frac{z}{R}\right) \ll 1}$$

ΠΩΣ "ΦΑΙΝΕΤΑΙ ΤΟ ΠΕΔΙΟ ΓΙΑ ΠΟΛΥ ΜΕΓΑΛΟ z??

$$\Rightarrow E_z \cong \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 - \frac{R^2}{2z^2}\right)\right] \Rightarrow$$

$$E_z \simeq \frac{\sigma}{2\varepsilon_0}\,\frac{R^2}{2z^2} = \frac{\sigma R^2 \pi}{4\varepsilon_0 z^2 \pi} = \underbrace{\frac{1}{4\pi\varepsilon_0}\,\frac{Q^2}{z^2}}_{\text{ΗΛΕΚΤΡΙΚΟ ΠΕΔΙΟ ΣΗΜΕΙΑΚΟΥ ΦΟΡΤΙΟΥ}}$$