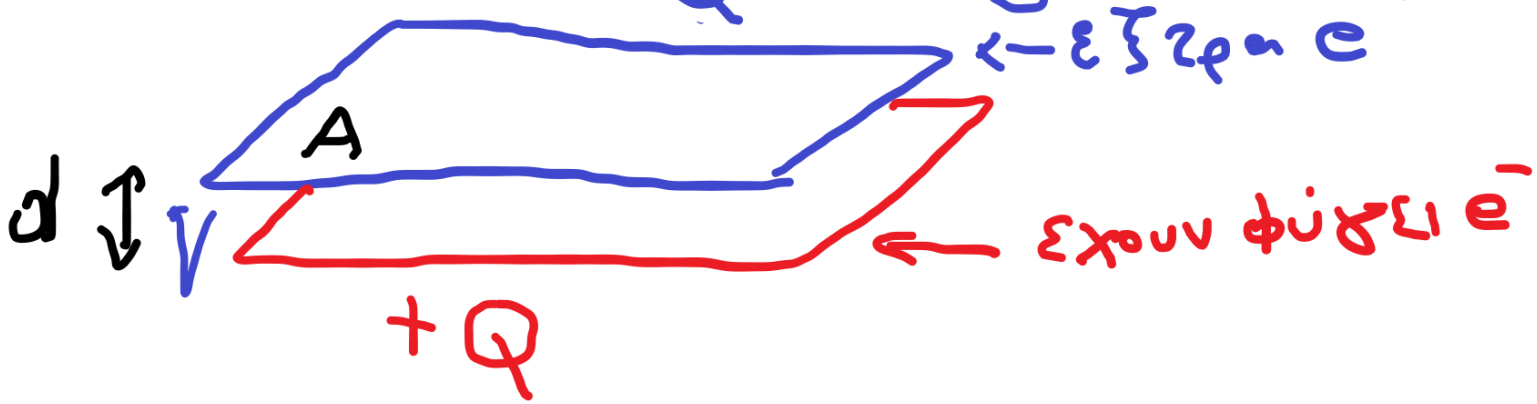
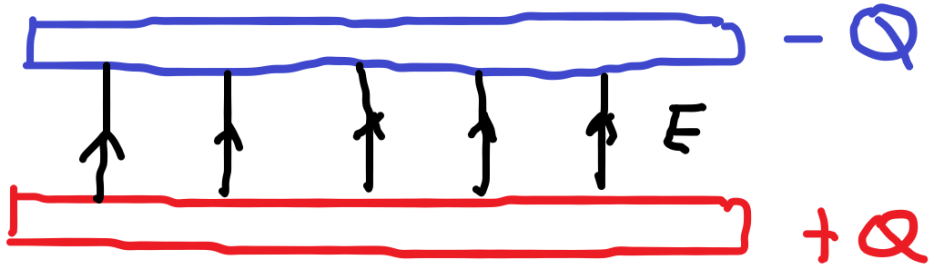
ΚΕΦΑΛΑΙΟ 25 - ΧΩΡΗΤΙΚΟΤΗΤΑ 7.4.20 ①

↳ CAPACITANCE

ΠΥΚΝΩΤΕΣ :

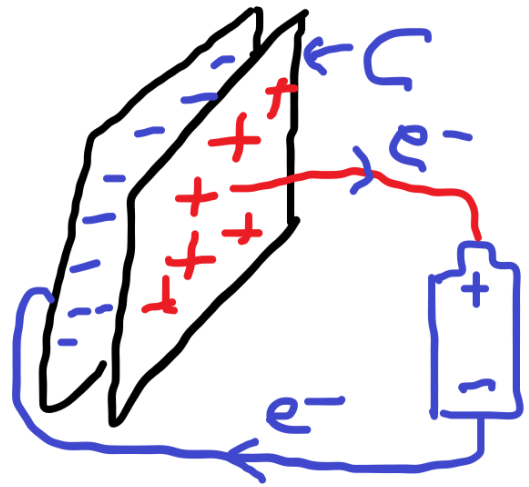


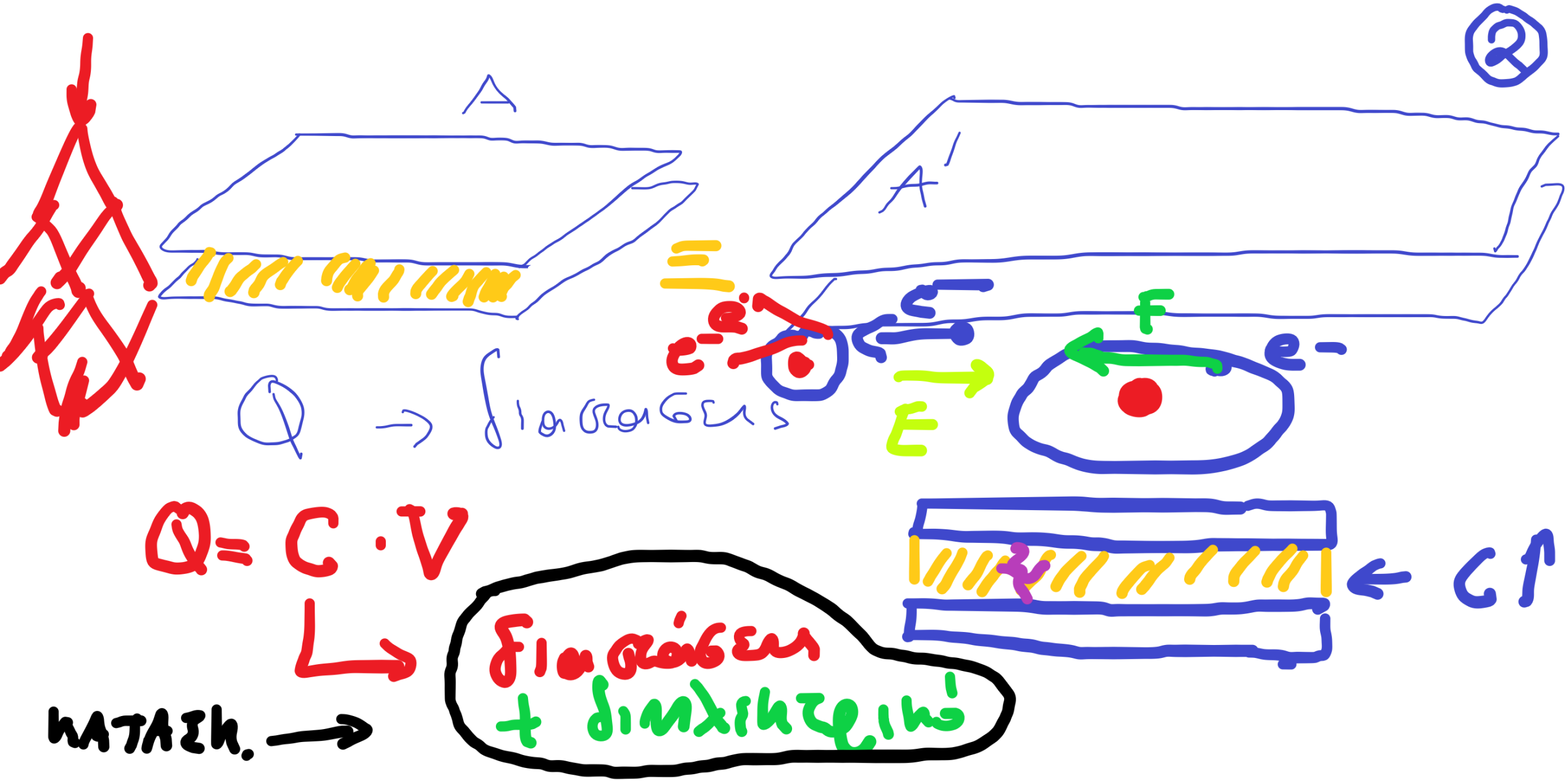


$$Q = C \cdot V$$

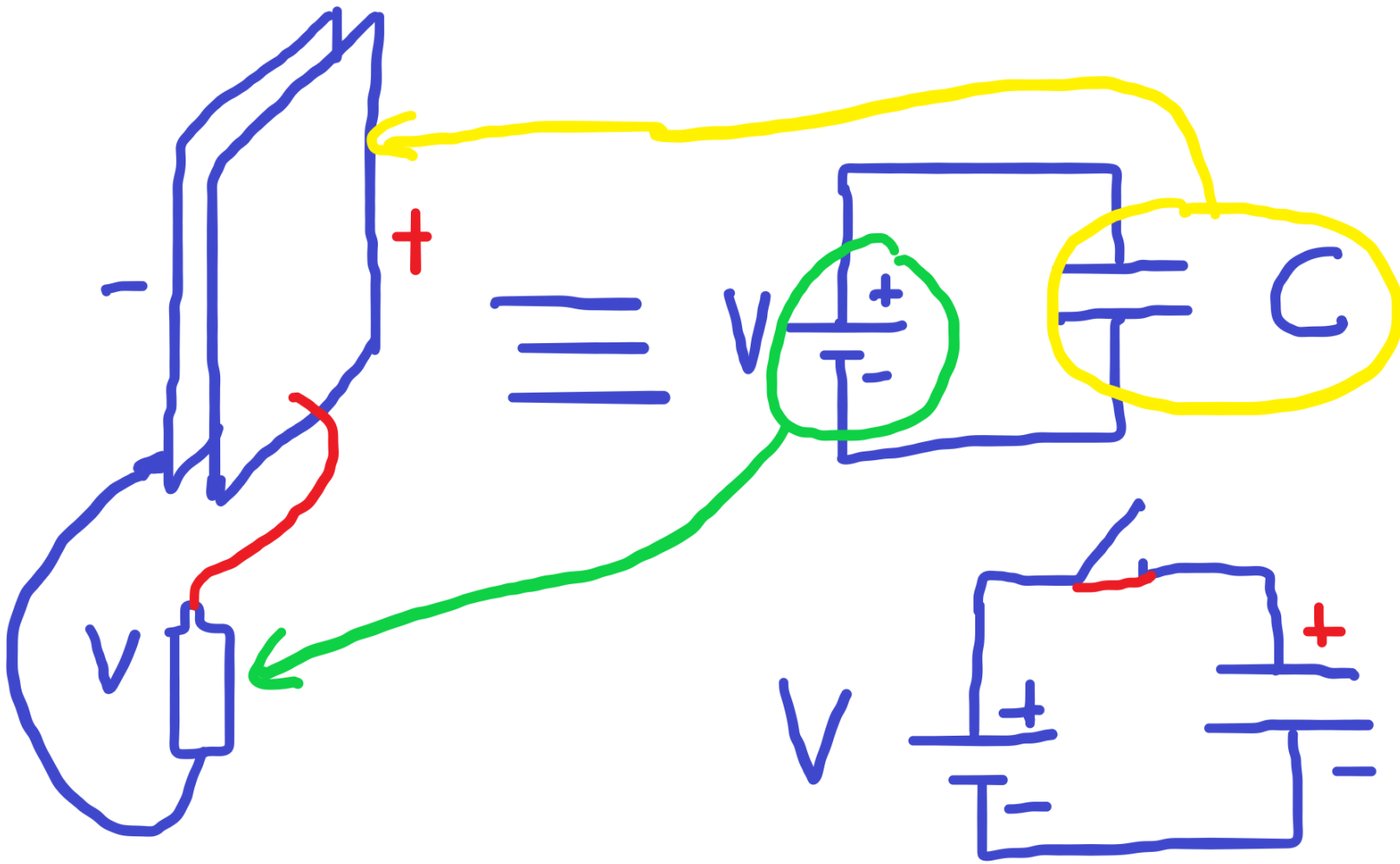
Q: C
C: FARAD
V: VOLT

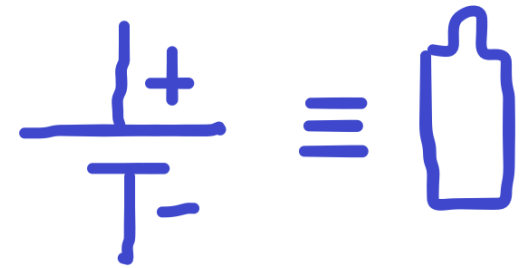
1 farad - $1F = \frac{C}{V}$




A
A'
=
Q → διαστάσεις
F
e⁻
E
Q = C · V
διαστάσεις
+ διηλεκτρικό
ΚΑΤΑΣΚ.
C ↑

7.4.20 ③





$$Q = C \cdot V$$

ΥΠΟΛΟΓΙΣΜΟΣ ΧΩΡΗΤΙΚΟΤΗΤΑΣ

7/4/20 (4)

$$C = \varepsilon_0 \frac{A}{d}$$

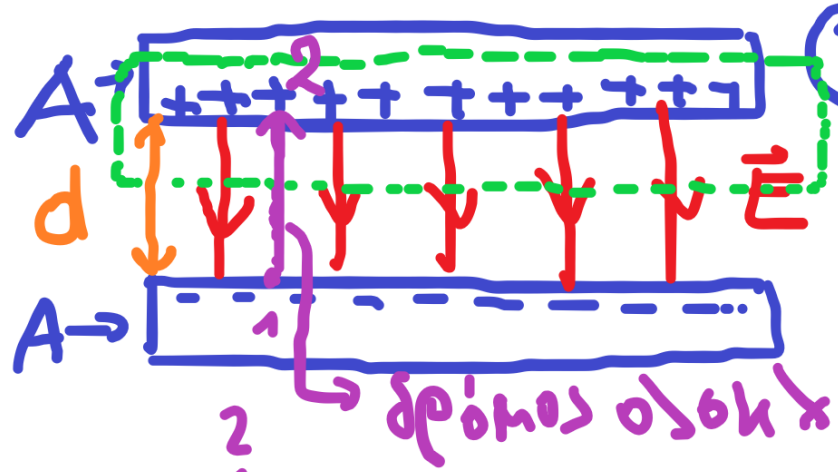
1 Υποθέτουμε φορτίο Q

2 Υπολογίζουμε το $\vec{E}$ με GAUSS $\quad \varepsilon_0 \oint \vec{E} \cdot d\vec{A}$

3 $V_{21} = -\int_1^2 \vec{E} \cdot d\vec{l}$

4 $C = Q/V$

ΠΑΡΑΔΕΙΓΜΑ: ΕΠΙΠΕΔΟΣ ΠΥΚΝΩΤΗΣ



1) Q

2) $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{IN} = Q \Rightarrow$

$\varepsilon_0 E \cdot A = Q \Rightarrow$

$$E = \frac{Q}{\varepsilon_0 A}$$

3) $V = -\int_1^2 \vec{E}\, d\vec{\ell} =$

$$= -\int_1^2 \frac{Q}{\varepsilon_0 A}(-\hat{n}) \cdot dz\,\hat{n} = \frac{Q}{\varepsilon_0 A} d \Rightarrow$$

$$V = \frac{Q d}{\varepsilon_0 A}$$

4) $Q = C \cdot V$

$$Q = \left[\frac{\varepsilon_0 A}{d}\right] V$$

7.420 (5)

$\rightarrow Q = \varepsilon_0 \underbrace{\frac{A}{d}} V$

$C = \varepsilon_0 A/d$

διηλεκτρ. ↓

$\varepsilon_0 \rightarrow \varepsilon_0 \varepsilon$

ε_0 ΔΙΗΛΕΚΤΡΙΚΗ ΣΤΑΘΕΡΑ ΤΟΥ ΚΕΝΟΥ

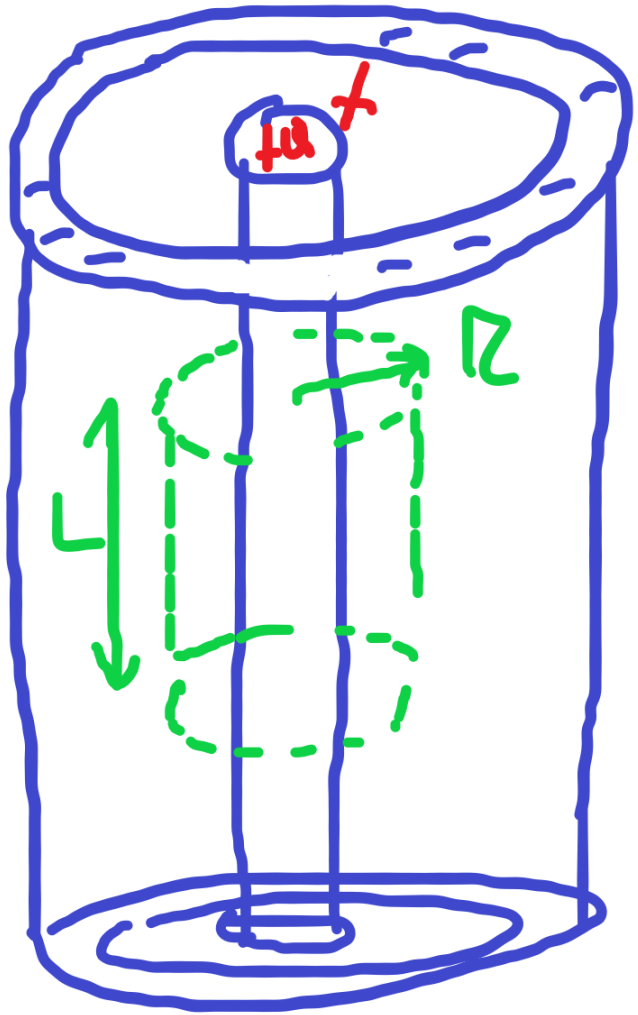
$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} = 8.85 \times 10^{-12} \frac{F}{m}$

$Q = \varepsilon_0 \frac{A}{d}$ (A: m^2, d: m, Q: F)

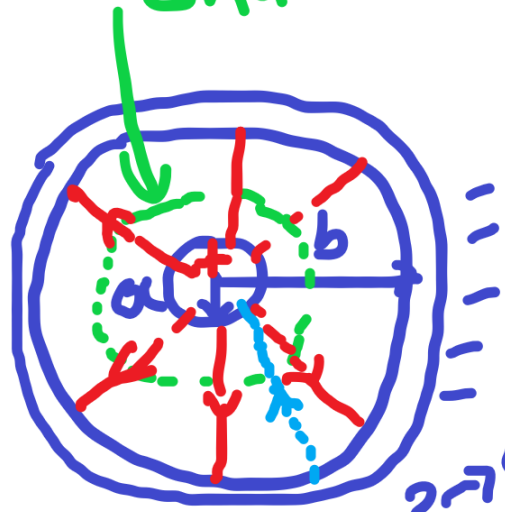
$\frac{C^2}{N m^2} = \frac{C^2}{\underbrace{N \cdot m \cdot m}_{\text{Joule} = C V}} = \frac{C^2}{C \cdot V \cdot m} = \frac{C}{V} \cdot \frac{1}{m} = \frac{F}{m}$

Παράδειγμα: ΚΥΛΙΝΔΡΙΚΟΣ ΠΥΚΝΩΤΗΣ

7.4.20 (6)



$C = ?$



2) $\varepsilon_0 \oint_S \vec{E}\cdot d\vec{A} = Q$

$\varepsilon_0 E\, 2\pi r \cdot L = Q \Rightarrow$

$$E = \frac{Q}{2\pi\varepsilon_0 r L}$$

3) $V = -\int_{\infty}^{b} \vec{E}\cdot d\vec{r} = -\int_{b}^{a} \vec{E}\cdot d\vec{r} = -\frac{Q}{2\pi\varepsilon_0 L}\int_{b}^{a}\frac{dr}{r}$

$V = -\frac{Q}{2\pi\varepsilon_0 L}\left[\ln r\right]_b^a = -\frac{Q}{2\pi\varepsilon_0 L}\left[\ln a - \ln b\right]$

$\ln(a/b)$

$\infty \to b \to a$

C

$$\rightarrow \quad V = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(b/a\right)$$

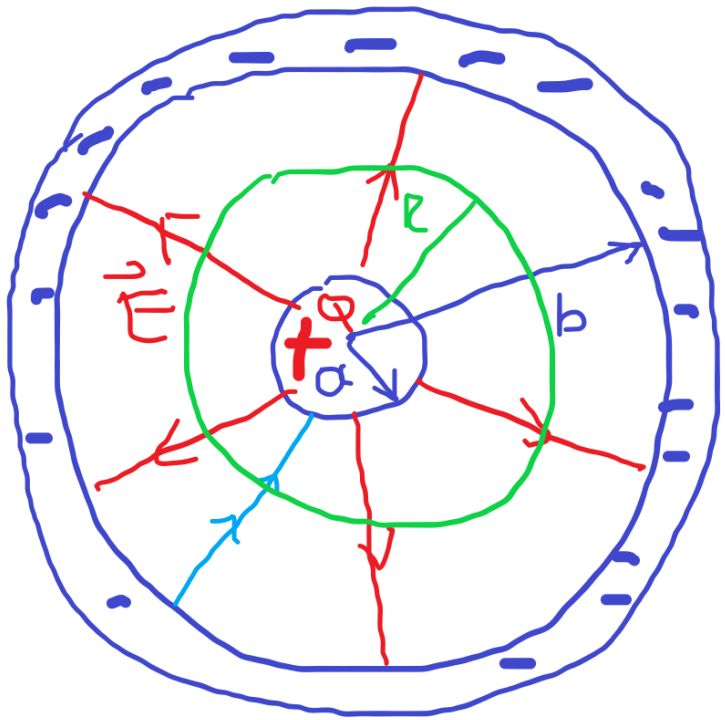
4) $$Q = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} V$$

$$Q = C V$$

$$\Rightarrow \boxed{C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}}$$

ΣΦΑΙΡΙΚΟΣ ΠΥΚΝΩΤΗΣ

7.4.20 (8)



2) $\varepsilon_0 E\, 4\pi r^2 = Q \rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

3) $V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{\ell} = -\frac{Q}{4\pi\varepsilon_0}\int_{b}^{a}\frac{dr}{r^2} \rightarrow$

$$V = -\frac{Q}{4\pi\varepsilon_0}(-1)\left[\frac{1}{r}\right]_b^a \rightarrow$$

$$\left[\frac{r^{-2+1}}{-2+1}\right]_b^a$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{Q}{4\pi\varepsilon_0}\frac{b-a}{a\cdot b}$$

4) $Q = \underbrace{\left[4\pi\varepsilon_0\frac{a\cdot b}{b-a}\right]}_{C} V \quad \therefore$

$$\boxed{C = 4\pi\varepsilon_0\frac{a\cdot b}{b-a}}$$

7.4.20 (9)

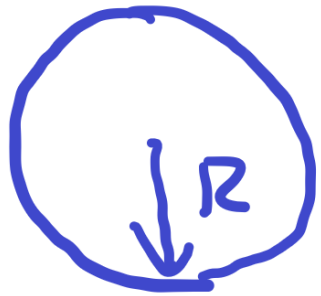
$\rightarrow \quad C = 4\pi\varepsilon_0 \frac{a \cdot b}{b-a}$

$\leftarrow \quad b \rightarrow \infty$

$$\lim_{b\to\infty} C = 4\pi\varepsilon_0 \lim_{b\to\infty} \frac{a \cdot b}{\underbrace{b-a}_{\approx b}} = 4\pi\varepsilon_0 a$$

ΧΩΡΗΤΙΚΟΤΗΤΑ ΣΦΑΙΡΑΣ

$$\boxed{C = 4\pi\varepsilon_0 R}$$



Παράδειγμα

Υπολογίστε την επιφάνεια επίπεδου πυκνωτή με $C = 1F$ και $d = 1mm$

$$C = \varepsilon_0 \frac{A}{d} \Rightarrow A = \frac{C \cdot d}{\varepsilon_0}$$

$$A = \frac{1F \cdot 10^{-3} m}{8.85\,10^{-12} F/m} = 1.1\,10^{8}\,m^2$$

ΠΑΡΑΔΕΙΓΜΑ: Υπολογίστε τη χωρητικότητα της Γης. (10)

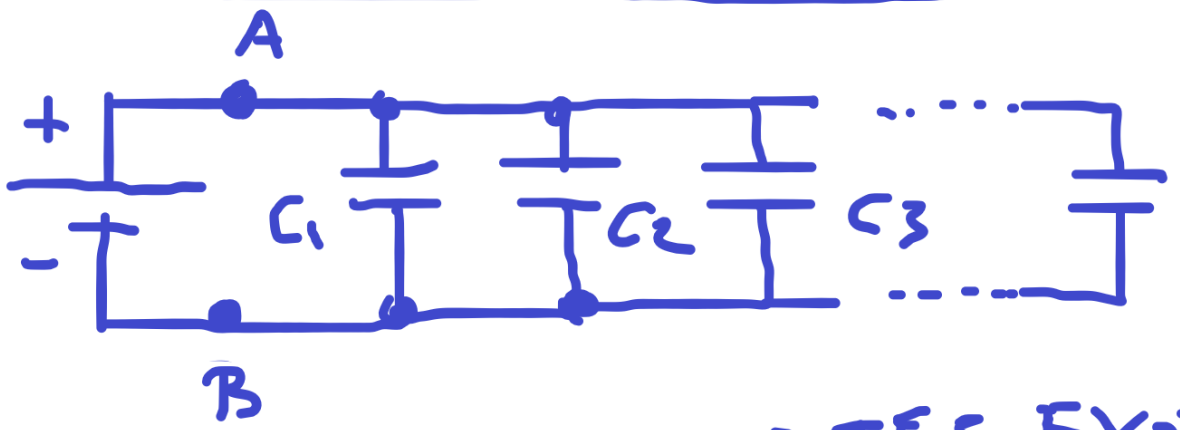
$R = 6370\ km$

ΛΥΣΗ: $C = 4\pi\varepsilon_0 R = 4 \cdot 3.14 \cdot 8.85\,10^{-12}\ \frac{F}{m} \cdot 6370\,10^{3}\ m$

$C = 710\ \mu F$

ΠΥΚΝΩΤΕΣ ΣΥΝΔΕΔΕΜΕΝΟΙ ΠΑΡΑΛΛΗΛΑ

7.4.70



$C_{TOT} = ?$

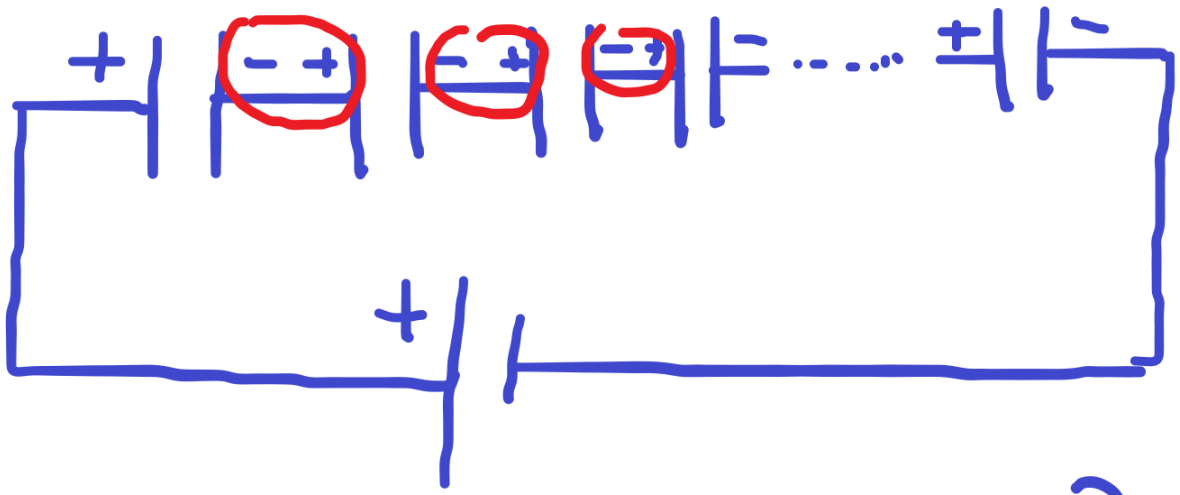
- ΟΛΟΙ ΟΙ ΠΥΚΝΩΤΕΣ ΕΧΟΥΝ ΤΟ ΙΔΙΟ $V = V_1 = V_2 = \cdots V_N$
- $Q_{TOT} = Q_1 + Q_2 + Q_3 + \cdots + Q_N$

$$C_{TOT} = \frac{Q_{TOT}}{V} = \frac{Q_1 + Q_2 + \cdots + Q_N}{V} = \underbrace{\frac{Q_1}{V}}_{C_1} + \underbrace{\frac{Q_2}{V}}_{C_2} + \cdots + \underbrace{\frac{Q_N}{V}}_{C_N}$$

$$C_{TOT} = C_1 + C_2 + \cdots + C_N$$

ΠΥΚΝΩΤΕΣ ΣΕ ΣΕΙΡΑ

7/4/20 (12)



$C_{ΟΛ} = ?$

- $Q_{ΟΛ} = Q_1 = Q_2 = \cdots = Q_N$
- $V_{ΟΛ} = V_1 + V_2 + V_3 + \cdots + V_N$

$$\frac{Q_{ΟΛ}}{C_{ΟΛ}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \cdots + \frac{Q_N}{C_N}$$

$\Rightarrow$

$$\boxed{\frac{1}{C_{ΟΛ}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}}$$