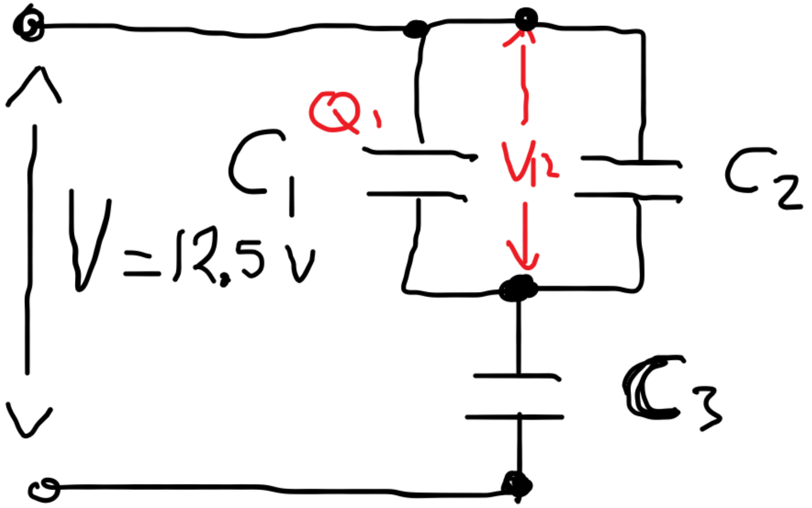
ΠΑΡΑΔΕΙΓΜΑ 25-2

10.4.20 (1)

$C_1 = 12\,\mu F$, $C_2 = 5.3\,\mu F$, $C_3 = 4.5\,\mu F$



$V_{12} = ?$

$Q_1 = ?$

$Q_1 = C_1 \cdot V_{12} \Rightarrow$

$Q = 12\,\mu F \times 2.58\,V$

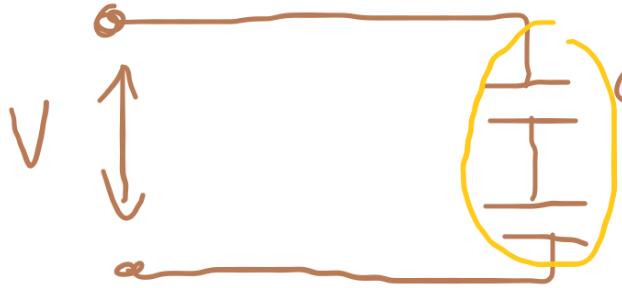
$= 31\,\mu C$

$Q_{123} = V \cdot C_{123} = 12.5\,V \cdot 3.57\,\mu F$

$Q_{123} = 44.6\,\mu C$

$Q_{123} = Q_{12} = C_{12} V_{12} \rightarrow V_{12} = \dfrac{Q_{12}}{C_{12}} = 2.58\,V$

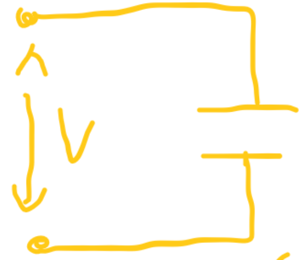
$C_{123} = 3.57\,\mu F$



$C_{12} = C_1 + C_2 = 17.3\,\mu F$

$C_3 = 4.5\,\mu F$

=



$1/C_{123} = 1/C_{12} + 1/C_3 \Rightarrow C_{123} = \dfrac{C_{12} \cdot C_3}{C_{12} + C_3}$

ΕΝ ΣΕΙΡΑ

$\dfrac{1}{C_{ολ}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dots + \dfrac{1}{C_n}$

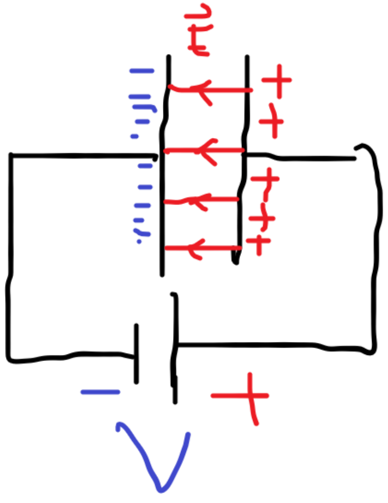
ΠΑΡΑΛΛΗΛΑ

$C_{ολ} = C_1 + C_2 + \dots C_n$

ΕΝΕΡΓΕΙΑ ΠΥΚΝΩΤΗ

10.4.20 (2)

ΠΟΣΗ ΕΝΕΡΓΕΙΑ ΑΠΑΙΤΕΙΤΑΙ ΓΙΑ ΝΑ ΦΟΡΤΙΣΟΥΜΕ ΠΥΚΝΩΤΗ ΜΕ ΧΩΡΗΤΙΚΟΤΗΤΑ C ΣΕ ΤΑΣΗ V



$$\int_0^{Q} dW = \int_0^{Q} dq \cdot V(q) = \int_0^{Q} dq\, \frac{q}{C} = \frac{1}{C}\int_0^{Q} q\, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^{Q} = \frac{Q^2}{2C}$$

ΔΕΝ ΕΙΝΑΙ ΣΤΑΘΕΡΟ ($V(q)$)

$$U_c = \frac{Q^2}{2C} = \frac{C^2V^2}{2C} = \frac{1}{2}CV^2$$

$Q = CV$

$Q = CV$

$$U_c = \frac{Q \cdot Q}{2C} = \frac{1}{2}QV$$

$$U_c = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

ΕΝΕΡΓΕΙΑ ΤΟΥ ΗΛΕΚΤΡΙΚΟΥ ΠΕΔΙΟΥ ΜΕΣΑ ΣΤΟΝ ΠΥΚΝΩΤΗ.

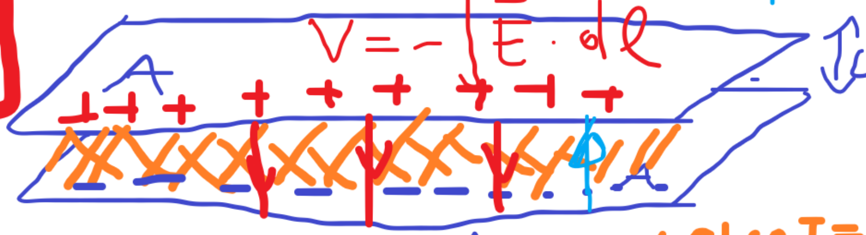
ΠΥΚΝΟΤΗΤΑ ΕΝΕΡΓΕΙΑΣ ΠΕΔΙΟΥ

$$u = \frac{1}{2}\varepsilon_0 E^2$$

$$u = U/(A\cdot d) = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(\varepsilon_0 \frac{A}{d})V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

$E = \frac{V}{d}$

$V = -\int \vec{E}\cdot d\vec{l}$



$C = \varepsilon_0 A/d$

όγκος $= A\cdot d$

ΠΡΟΒΛΗΜΑ 25-5 10.4.20 (3)

ΑΓΩΓΙΜΗ ΣΦΑΙΡΑ

$R = 6.85\ cm$

$q = 1.25\ \mu C$

$\mu \rightarrow 10^{-9}$

$U = ?$

$C_0 = 4\pi\varepsilon_0 R$

$$U = \frac{q^2}{2C} = \frac{(1.25\,10^{-9}C)^2}{2\cdot 4\cdot\pi\varepsilon_0\, 0.0685\ m}$$

$$U = 103\ \mu J$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)^2 = \frac{Q^2}{32\pi^2\varepsilon_0 R^4}$$

↑ στην επιφάνεια της σφαίρας

$$\approx 25.4\ \frac{\mu J}{m^3}$$

ΠΥΚΝΩΤΕΣ ΜΕ ΔΙΗΛΕΚΤΡΙΚΟ

10.4.20 (4)

C, V_{max}

$$C_0 = \varepsilon_0 \frac{A}{d}$$

$$Q = CV$$

K → ΔΙΗΛΕΚΤΡΙΚΗ ΣΤΑΘΕΡΑ

$$C = K C_0$$

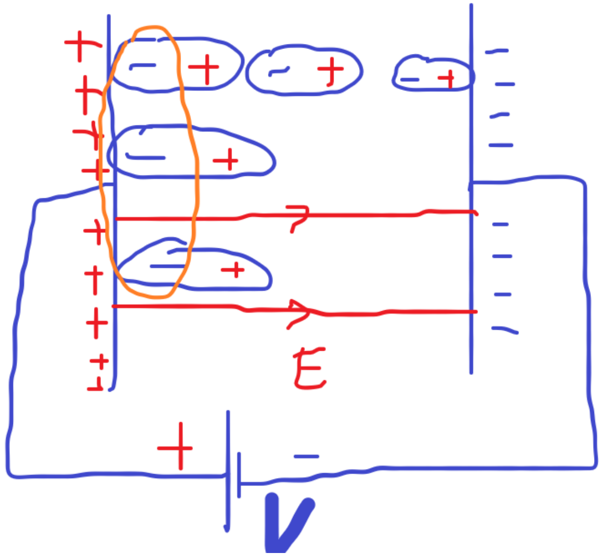
C: ΧΩΡΗΤΙΚΟΤΗΤΑ ΜΕ ΔΙΗΛΕΚΤΡΙΚΟ

C_0: ΧΩΡΗΤΙΚΟΤΗΤΑ ΧΩΡΙΣ ΔΙΗΛΕΚΤΡΙΚΟ

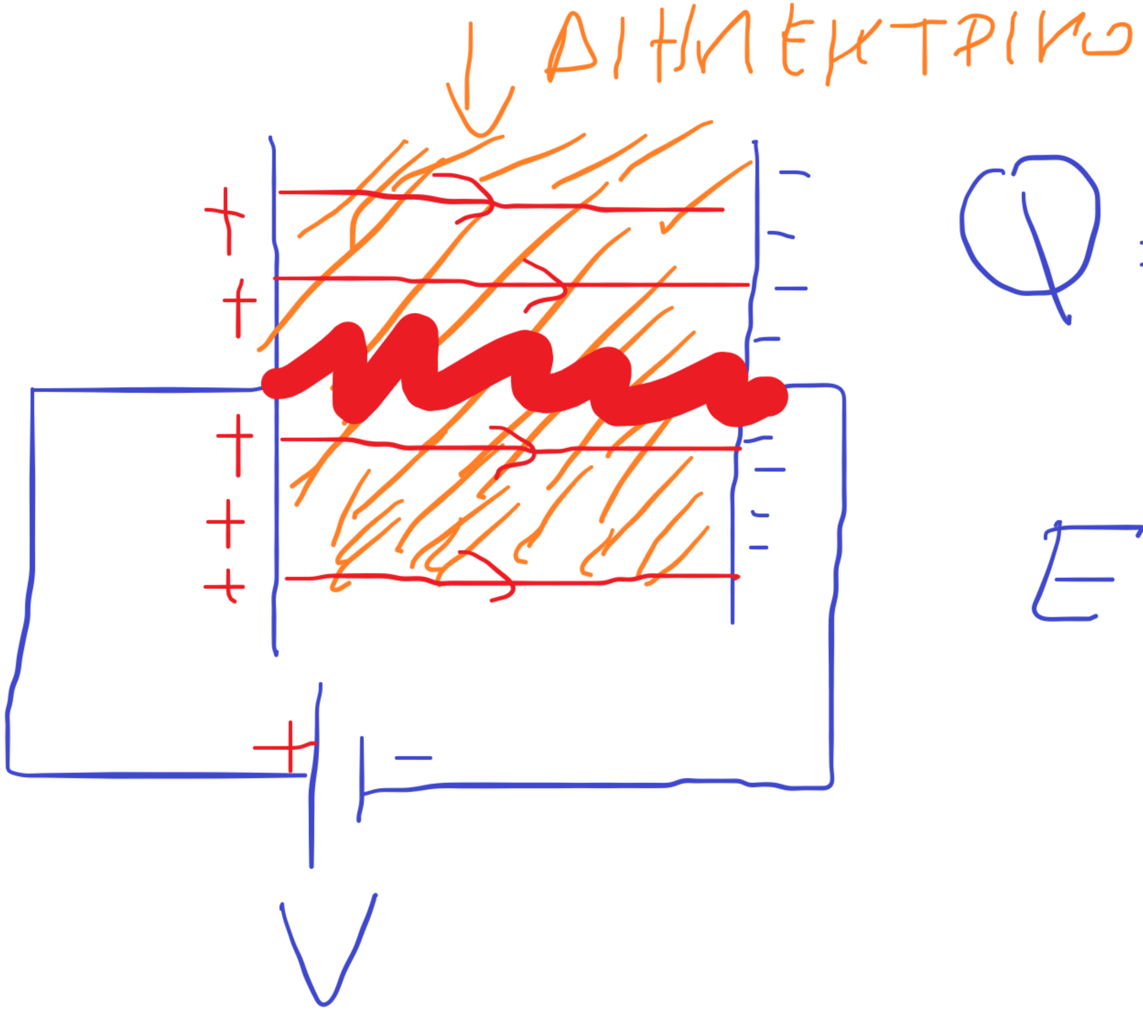
1837 FARADAY

"ΤΟ ΦΟΡΤΙΟ ΠΥΚΝΩΤΗ ΑΥΞΑΝΕΙ ΟΤΑΝ ΓΕΜΙΖΟΥΜΕ ΤΟΝ ΠΥΚΝΩΤΗ ΜΕ ΕΝΑ ΔΙΗΛΕΚΤΡΙΚΟ ΥΛΙΚΟ" (ΥΠΟ ΣΤΑΘΕΡΗ ΤΑΣΗ)

ΥΛΙΚΟ	K
ΑΕΡΑΣ	1.00054
ΠΟΛΥΣΤΕΡΙΝΗ	2.6
ΧΑΡΤΙ	3.5
ΤΙΤΑΝΙΟΥΧΟ ΣΤΡΟΝΤΙΟ	310



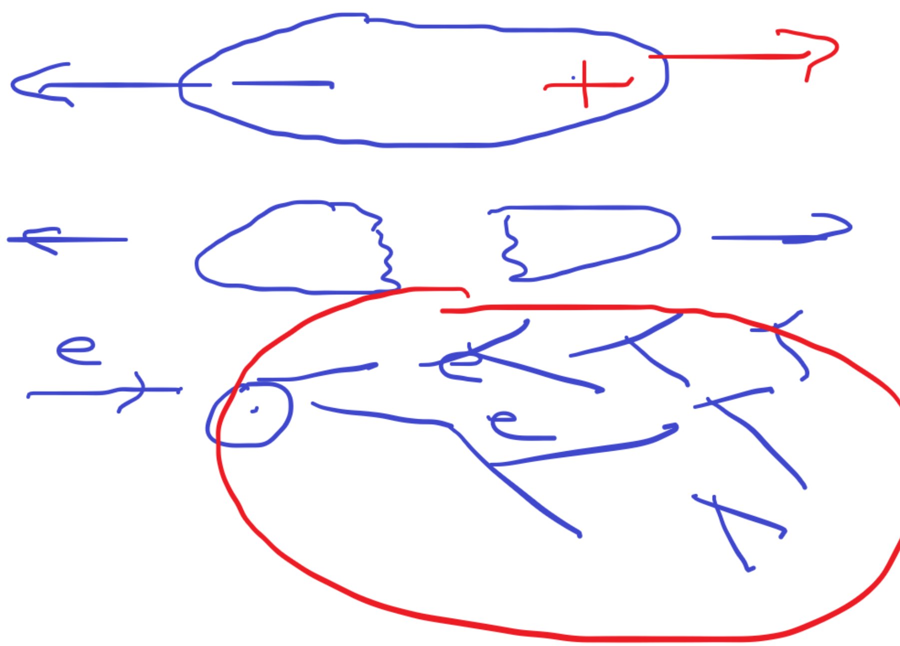
10.4.20 (5)



$$Q = \underbrace{K C_0}_{C} V$$

$$E = \frac{V}{d}$$

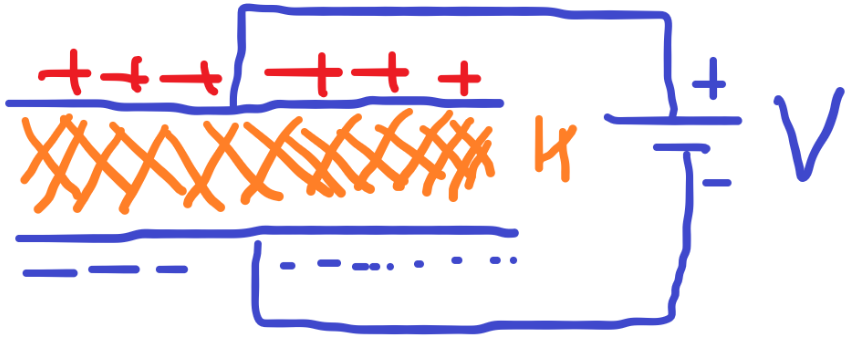
$\vec{E}$



Επινόηση ??

ΠΕΙΡΑΜΑ 1: ΦΟΡΤΙΣΗ ΠΥΚΝΩΤΗ ΥΠΟ ΣΤΑΘΕΡΗ ΤΑΣΗ

10.4.20 (6)



(1) $Q_0 = C_0 V$ ΧΩΡΙΣ ΔΙΗΛΕΚΤΡΙΚΟ

(2) $Q = \kappa C_0 V$

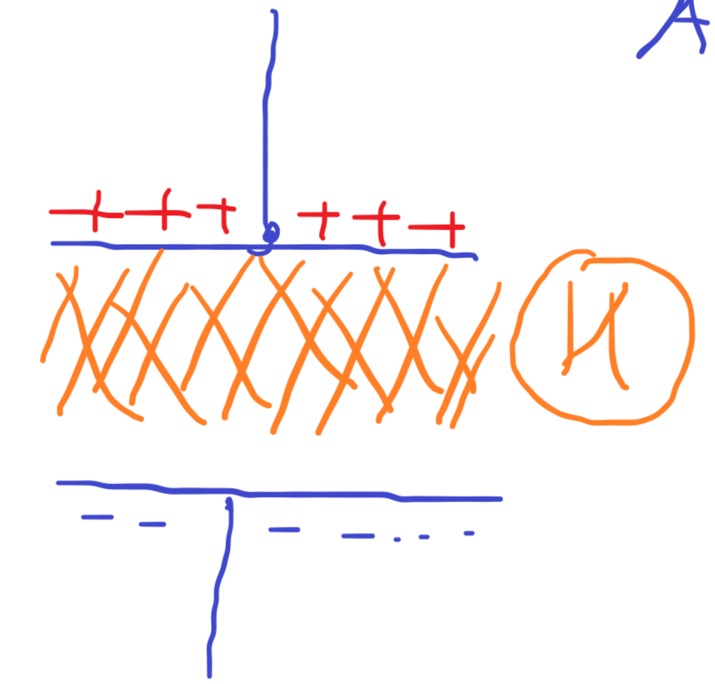
$$\frac{Q}{Q_0} = \frac{\kappa C_0 V}{C_0 V} = \kappa \Rightarrow \boxed{Q = \kappa Q_0}$$

$Q = CV$ ← σταθερό!

Ο: ΥΠΟ ΣΤΑΘΕΡΗ ΤΑΣΗ Η ΧΩΡΗΤΙΚΟΤΗΤΑ ΑΥΞΑΝΕΙ ΚΑΙ ΤΟ ΦΟΡΤΙΟ **ΑΥΞΑΝΕΙ**

ΠΕΙΡΑΜΑ 2: ΔΙΗΛΕΚΤΡΙΚΟ ΜΕ ΣΤΑΘΕΡΟ ΦΟΡΤΙΟ

10.4.20 (7)



ΑΡΧΙΚΑ $Q_0 = C_0 \cdot V_0$

ΜΕ ΔΙΗΛΕΚΤΡΙΚΟ → $C = K C_0$

$Q = Q_0$

$Q_0 = C_0 V_0 = K C_0 V$

$V = \frac{V_0}{K}$

10.4.20 (8)

ΟΛΟΙ ΟΙ ΤΥΠΟΙ ΑΛΛΑΖΟΥΝ:

$$\varepsilon_0 \to K\varepsilon_0$$

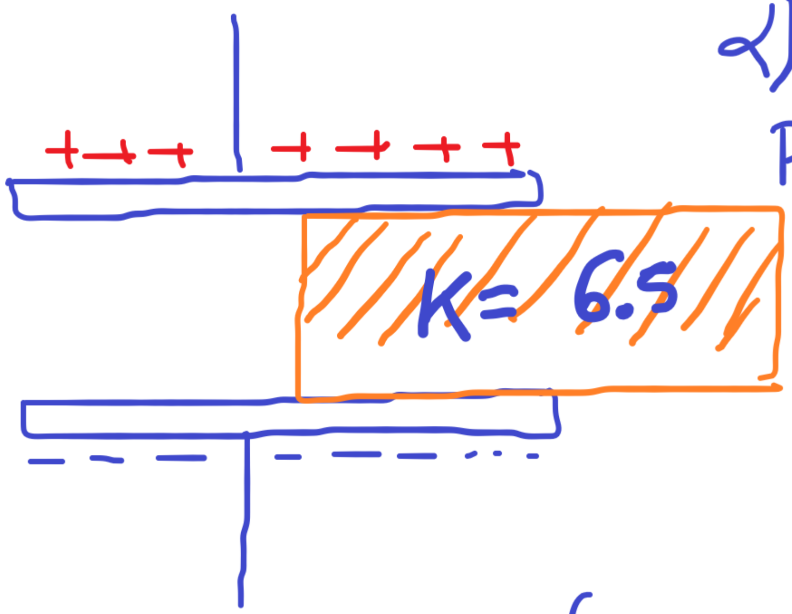
ΕΠΙΠΕΔΟΣ ΠΥΚΝΩΤΗΣ: $C = K\varepsilon_0 \dfrac{A}{d}$

Coulomb: $\vec{E} = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{Q}{r^2}$

$$E = \frac{\sigma}{\varepsilon_0} \to \frac{\sigma}{K\varepsilon_0}$$

Gauss: $K\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = Q$

ΠΡΟΒΛΗΜΑ 25-6



α) $U_{ΠΡΙΝ} = ?$

β) $U_{ΜΕΤΑ} = ?$

Η ΕΝΕΡΓΕΙΑ ΕΛΑΤΤΩΝΕΤΑΙ

ΠΟΥ ΠΑΕΙ Η ΕΝΕΡΓΕΙΑ ΠΟΥ ΧΑΘΗΚΕ??

$C_0 = 13.5\,pF$

$p \to 10^{-12}$

$V_0 = 12.5\,V$

ΛΥΣΗ

α) $U_{ΠΡΙΝ} = \frac{1}{2} C_0 V_0^2 = 1.055 \cdot 10^{-9}\,J$

β) $U_{ΜΕΤΑ} = \frac{q^2}{2KC_0} = \frac{(C_0 V_0)^2}{2KC_0} = \frac{C_0 V_0^2}{2K}$

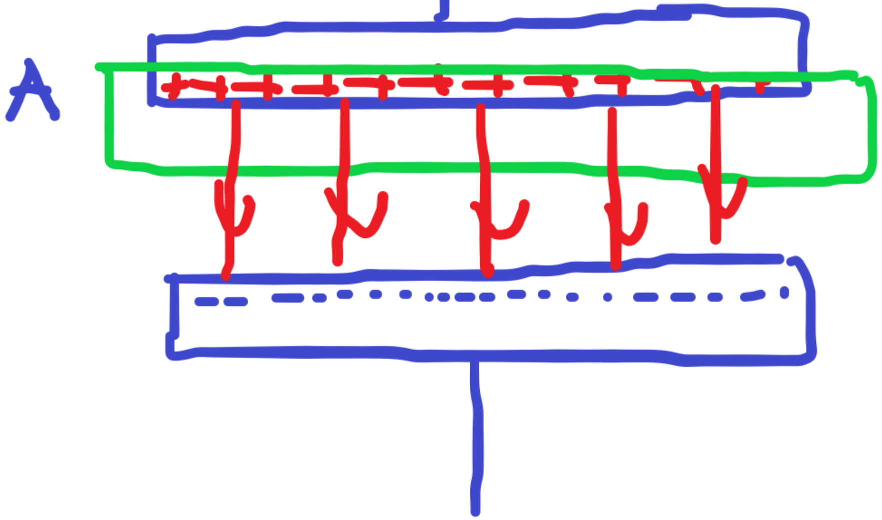
$U_{ΜΕΤΑ} = \frac{U_{ΠΡΙΝ}}{K} = \frac{1.055 \cdot 10^{-9}\,J}{6.5}$

$U_{ΜΕΤΑ} = 0.162 \cdot 10^{-9}\,J$!!!

ΔΙΗΛΕΚΤΡΙΚΑ ΚΑΙ ΝΟΜΟΣ GAUSS

10.4.20 (10)

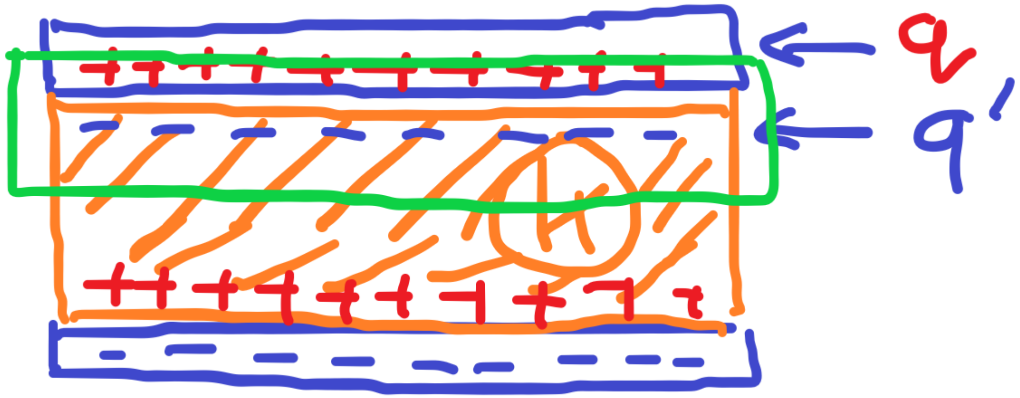
ΧΩΡΙΣ ΔΙΗΛΕΚΤΡΙΚΟ



$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E\cdot A = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{A\varepsilon_0}$$

ΜΕ ΔΙΗΛΕΚΤΡΙΚΟ



α) $\oint \vec{E}\cdot d\vec{A} = \frac{q-q'}{\varepsilon_0} \Rightarrow E\cdot A = \frac{q-q'}{\varepsilon_0}$

$$E = \frac{q-q'}{\varepsilon_0 A} \quad (1)$$

β) $\varepsilon_0 K \oint \vec{E}\cdot d\vec{A} = q \rightarrow E\cdot A = \frac{q}{\varepsilon_0 K} \rightarrow$

$$E = q/(\varepsilon_0 K A) \quad (2)$$

10.4.20 (11)

(1) $E = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$

(2) $E = \frac{q}{\varepsilon_0 \kappa A}$

$$\frac{q}{\varepsilon_0 \kappa A} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A} \Rightarrow$$

$$\frac{q}{\kappa} = q - q' \Rightarrow$$

$$q' = q - \frac{q}{\kappa} = q\left(1 - \frac{1}{\kappa}\right)$$

$$q' = q\left(1 - \frac{1}{\kappa}\right)$$

Φ.Υ.Ι

10.4.20

12

$$\vec{D} = \varepsilon_0 \kappa \vec{E}$$

ΗΛΕΚΤΡΙΚΗ ΜΕΤΑΤΟΠΙΣΗ

$$\oint \vec{D} \cdot d\vec{A} = Q$$

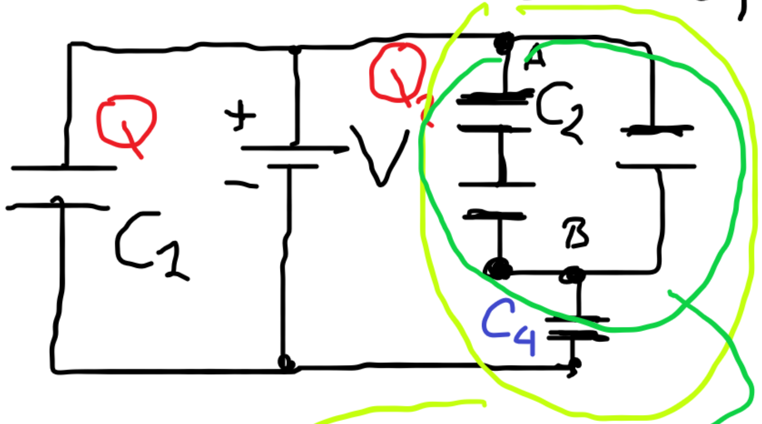
25.12

13.4.20 (1)

$V = 10V$

$C = 10\,\mu F$

α) $Q_1 = ?$ β) $Q_2 = ?$



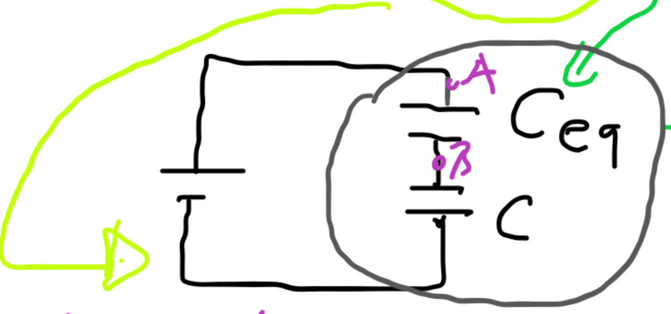
ΛΥΣΗ:

α) $Q_1 = CV = 10\,\mu F \cdot 10\,V = 100\,\mu C$

β) $Q_2 = C\,V_2$ → V_{AB} ΑΥΤΟ ΠΡΕΠΕΙ ΝΑ ΒΡΩ

↑ (1)

$V_2 = V_{AB}/2$ .... $Q_2 = V_2 C$



$\frac{1}{C_{AB}} = \frac{1}{C} + \frac{1}{C} = \frac{2C}{C^2} \rightarrow C_{AB} = \frac{C}{2}$

$C_{eq} = C + \frac{C}{2} = \frac{3}{2}C = 15\,\mu F$ (1)

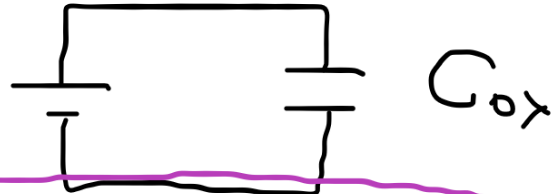
$Q_{eq} = Q_{o\lambda} = Q_4$

$C_{eq} V_{AB} = \frac{3}{5} C \cdot V$

$\frac{3}{2} C V_{AB} = \frac{3}{5} C V$

$V_{AB} = \frac{2}{5} V \rightarrow 4V$

$V_2 = \frac{V_{AB}}{2} = 2V$



$\frac{1}{C_{o\lambda}} = \frac{1}{C} + \frac{1}{C_{eq}} \rightarrow \frac{1}{C_{o\lambda}} = \frac{C_{eq} + C}{C \cdot C_{eq}} \Rightarrow$

$C_{o\lambda} = \frac{C \cdot C_{eq}}{C + C_{eq}} = \frac{3/2\,C^2}{5/2\,C} = \frac{3}{5} C \Rightarrow Q = \frac{3}{5} C \cdot V$ (2)

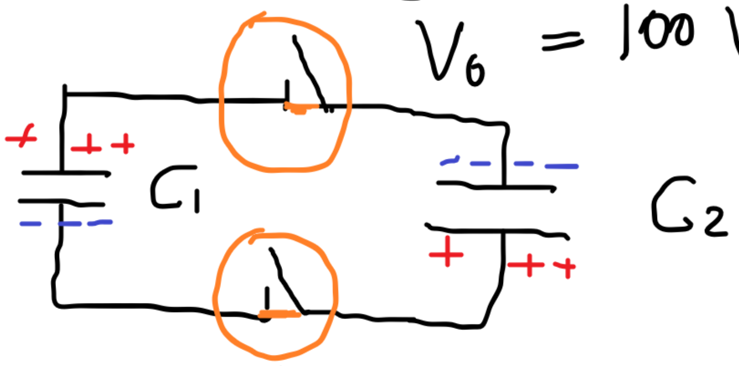
$Q_2 = 2V \times 10\,\mu F = 20\,\mu C$

25.19

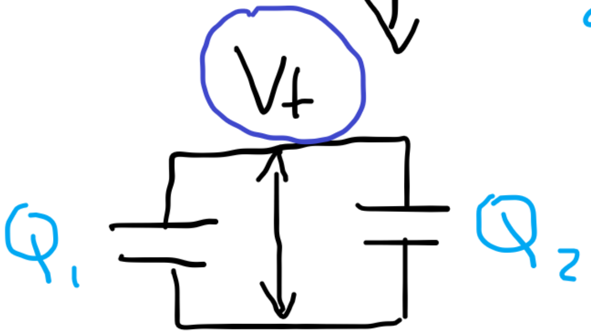
$C_1 = 1.0\,\mu F$

$C_2 = 3.0\,\mu F$

$V_0 = 100\,V$



ΛΥΣΗ



13.4.20 (2)

$Q_0 = \underbrace{100V \cdot 3\mu F}_{Q_2} - \underbrace{100V \cdot 1\mu F}_{Q_1}$

$Q_0 = 200\,\mu C$ ✓

α) $V_f = ?$ β) $Q_1 = ?$ γ) $Q_2 = ?$

α) $Q_0 = Q_1 + Q_2$

$Q_0 = Q = V_f C_1 + V_f C_2 \Rightarrow$

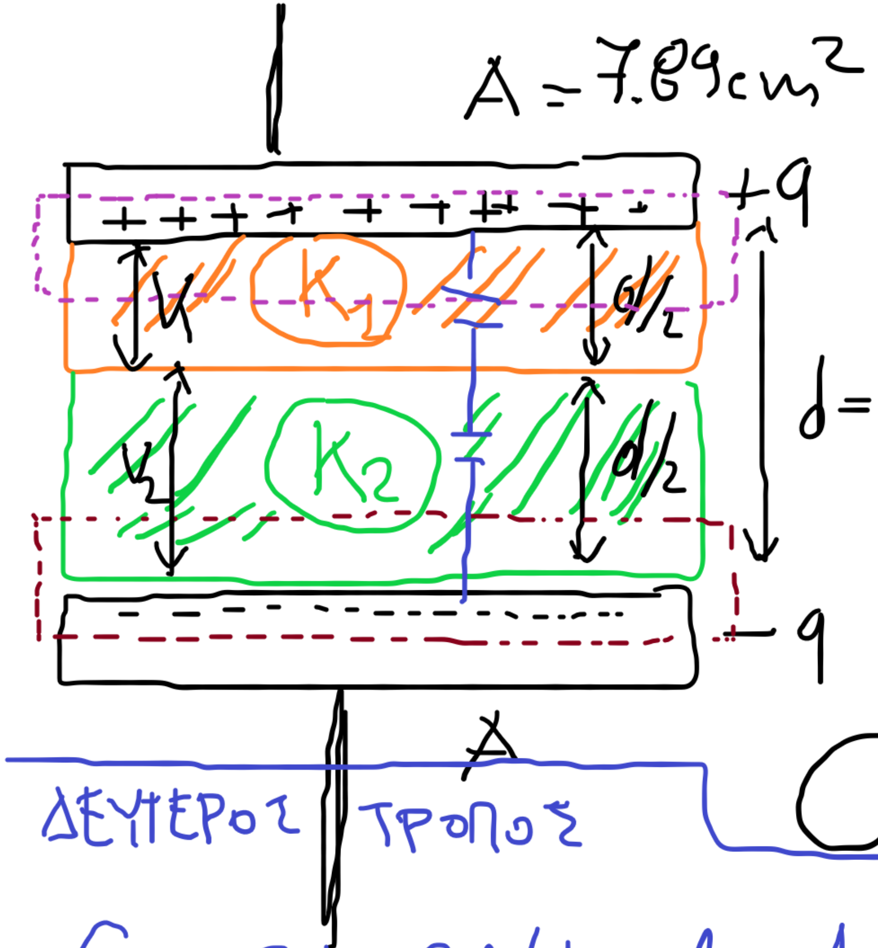
$V_f = \dfrac{Q_0}{C_1 + C_2} = 50$ VOLT

β) $Q_1 = C_1 \cdot V_f = 50\,\mu C$

γ) $Q_2 = C_2 V_f = 150\,\mu C$

25.49

13.4.20 (3)



$$C = \varepsilon_0 \frac{A}{d}$$

$C = ?$

1) $\varepsilon_0 K_1 \oint \vec{E}_1 \cdot d\vec{A} = q \rightarrow$

$\varepsilon_0 K_1 E_1 A = q \rightarrow E_1 = \dfrac{q}{\varepsilon_0 K_1 A}$

2) $\varepsilon_0 K_2 E_2 A = q \rightarrow E_2 = \dfrac{q}{\varepsilon_0 K_2 A}$

3) $V = V_1 + V_2 = E_1 \cdot \frac{d}{2} + E_2 \frac{d}{2} \Rightarrow$

$$V = \frac{d}{2}\left(\frac{q}{\varepsilon_0 K_1 A} + \frac{q}{\varepsilon_0 K_2 A}\right) = \frac{dq}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$V = \frac{dq}{2\varepsilon_0 A} \frac{K_1 + K_2}{K_1 K_2} \Rightarrow q = \left[\frac{2\varepsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2}\right] V$$

C

ΔΕΥΤΕΡΟΣ ΤΡΟΠΟΣ

$C_1 = \varepsilon_0 K_1 \, 2A/d$

$C_2 = \varepsilon_0 K_2 \, 2A/d$

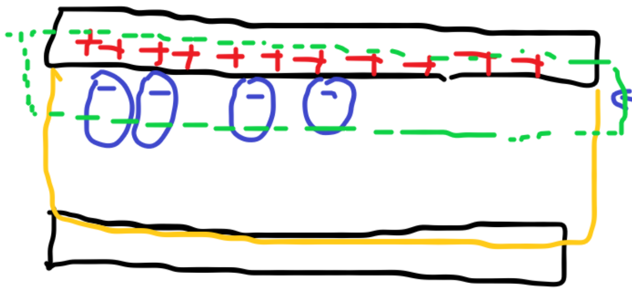
$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2}$

25.54

13.4.20 (4)

$A = 100\ cm^2$

$Q = 8.9\ 10^{-7}\ C$



$\vec{E} = 1.4\ 10^{6}\ V/m$

α) $K = ?$

β) $q^{επαγόμενο} = ?$

$\varepsilon_0 K \oint \vec{E} \cdot d\vec{A} = q$ ← φορτίο ηγκηθέν

$\varepsilon_0 K\ E \cdot A = Q$

$K = \frac{Q}{\varepsilon_0 E A} = 7.2$

β) $\varepsilon_0 \oint \vec{E}\, dA = q - q' \Rightarrow \varepsilon_0 E \cdot A = q - q'$

$E = \frac{q - q'}{\varepsilon_0 A}$ (1)

$\varepsilon_0 K \oint \vec{E}\, d\vec{A} = Q \Rightarrow E = \frac{q}{\varepsilon_0 K A}$ (2)

(1)(2) $\Rightarrow \frac{q - q'}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 K A} \Rightarrow \frac{q}{K} = q - q'$

$q' = q\left(1 - 1/K\right) = 8.9\ 10^{-7}\ C\left(1 - \frac{1}{7.2}\right)$

$= 7.7\ 10^{-7}\ C$