Advanced Particle Physics 04/05
Dr Gavin Davies - Problem Sheet 4

Questions 1, 2 and if possible 3 should be handed in by 4pm on 22/02/04 for rapid feedback

1. This question looks at three different particle identification methods. Electrons and muons can be distinguished from each other and from hadrons by their different behaviour in electromagnetic and hadronic calorimeters. However, to separate out the different types of hadrons is more difficult. The usual requirement is to distinguish charged pions (mass 139.6 MeV) from kaons (mass 493.7 MeV), where the momentum of the particle has already been measured in a tracking detector. All three methods are based on measuring the velocity; show that if the velocity β and the momentum p for a particle are known, then the mass m can be calculated from

$$m = \frac{p\sqrt{1-\beta^2}}{\beta}$$

- In a time-of-flight system the velocity is measured directly by finding the time the particle takes to travel a known distance. The timing is normally measured using plastic scintillator. If the time for the particle to go a distance l is measured as t, then show the expected value of t is

$$\langle t \rangle = l\sqrt{1 + \frac{m^2}{p^2}}$$

  Roughly sketch the shape of $\langle t \rangle$ as a function of p for two different masses.

  The CLEO detector time-of-flight system is mounted outside the tracking chamber and so forms a cylinder of radius 90 cm around the beamline. The time resolution of the system is $\sigma_t = 150$ ps. Show that the expected flight times for a pion and a kaon are significantly different (where significant is defined to be $3\sigma_t$) for momenta less than approximately 830 MeV.

- The amount of energy deposited in the gas of a tracking chamber depends on the velocity of the particle involved. Ignoring the relativistic rise effect, the energy deposited per unit length can be approximated to

$$-\frac{dE}{dx} \approx \frac{A}{\beta^2}$$

  where A is a constant for a given gas. Hence, show the expected value of the energy deposited per unit length is given by

$$-\left\langle \frac{dE}{dx} \right\rangle = A\left(1 + \frac{m^2}{p^2}\right)$$

  and roughly sketch the shape of this as a function of p for two different masses.

  The Aleph tracking detector gas gives a value of $A = 0.9$ KeV/cm and has a resolution on the energy deposited per unit length of $\sigma_E = 0.08$ KeV/cm. Find the highest momentum for which pions and kaons can be significantly distinguished.

  This energy loss by ionisation is used in several different ways to detect high energy charged particles. Outline why the energy deposited per unit length falls off as $1/\beta^2$ as stated above.

- Cherenkov radiation occurs when a charged particle travels though a material at a speed faster than the speed of light in the material. The waves emitted from the particle must add coherently; show that the angle of emission θ of the Cherenkov radiation with respect to the particle direction, in a medium of refractive index n, is given by

$$\cos\theta = \frac{1}{\beta n}$$

and hence show the momentum required for Cherenkov radiation to occur is given by

$$p > \frac{m}{\sqrt{n^2 - 1}}$$

Show for particles above threshold that the expected angle of emission is given by

$$\langle\theta\rangle = \cos^{-1}\left(\frac{1}{n}\sqrt{1 + \frac{m^2}{p^2}}\right)$$

Sketch the dependence of the angle on the momentum for two different masses.

The Delphi ring-imaging Cherenkov detector incorporates two Cherenkov radiator materials, liquid C_6F_{14} with a refractive index $n_l = 1.2718$ and gas C_5F_{12} with $n_g = 1.00194$. The angle of the Cherenkov ring can be measured with $\sigma_\theta = 4$ mrad. Find the maximum Cherenkov angle in the two radiators.

Calculate the threshold momentum values for pions and kaons in the two radiators and hence show that pions and kaons can be distinguished for all momenta between approximately 180 MeV and 13 GeV.

2. Electrons and photons give characteristic showers in electromagnetic calorimeters. This question gives a crude model for these showers.

Consider a calorimeter made of many thin sheets of lead with plastic scintillator between the sheets to detect the charged particles in the shower. Assume the probability in each sheet of an incoming photon converting to an e^+e^- pair or of an incoming electron or positron radiating a bremsstrahlung photon is p, which is small enough that multiple interactions in each sheet are negligible. Neglect the possibility of interactions in the scintillator.

Consider a shower started by a photon entering the front of the calorimeter. By considering the numbers of photons, electron and positrons entering layer $n+1$ compared with layer n, show that the average total number of particles entering each layer increases by $1+p$. Assuming that the average fraction of particles which are photons tends to a constant value for large n, show this fraction is $1/3$. What fraction of the particles are electrons? Would your calculation change if the shower was initiated by an electron rather than a photon?

Take the energy of the original photon as E_0. Assuming the energy of the interacting particle is evenly split between the outgoing particles in all cases, then show the average energy of the particles in layer n is $E_0/(1+p)^n$. The shower stops when the energy of the particles falls below a critical energy E_c. Below this energy, the photons no longer convert but are absorbed through photoelectric and Compton processes and the electrons and positrons no longer bremsstrahlung but lose energy through atomic ionisation. Hence, find the average layer at which the shower stops.

The Aleph calorimeter has lead sheets of thickness 2 mm. The radiation length of lead is 5.6 mm and its critical energy is $E_c = 7$ MeV. Find the total thickness of lead required on average to contain a shower from a photon of the highest possible energy, 104 GeV. The actual calorimeter is 57 sheets thick. What is the chance of the photon passing through the whole calorimeter without interacting?

3. Exam question: Part of 2001 question 5.

   An important interaction process for high energy charged particles in material is ionisation energy loss.

   A common form of detector used in particle physics experiments is a silicon vertex detector. Thin silicon wafers of reverse-biased diodes are used to measure the position of the charge deposited when a particle passes through the wafer.

   (iv) A typical arrangement has many flat silicon wafers arranged in a regular polygon around a beam collision region, forming a roughly cylindrical shape. A complete detector would consist of several such layers to make multiple measurements of the trajectory of each particle. Consider the case of a high energy particle emitted in a direction close to the horizontal x axis, which therefore passes through silicon wafers which are oriented parallel to the y axis. In the absence of a magnetic field, the particle trajectory will be a straight line. If the first two layers of silicon wafers are located at x_1 and x_2 and the charge is detected at positions y_1 and y_2, then show that the estimated intersection of the trajectory with the y axis is given by

   $$y_0 = \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1}.$$

   What is the error on this intersection point if each y position is measured to an accuracy of σ?

   (v) The Aleph experiment at the CERN laboratory has the first two silicon layers at radii of 6.3 cm and 10.8 cm and each measures the particle position with an error of 12 μm. Deduce the error on the extrapolated track intersection point and compare your answer with the typical decay distance of B mesons ($\tau_B \approx 1.6 \times 10^{-12}$ s). Briefly discuss why this resolution might not be obtained in practice.

4. Exam question: 2000 question 3.

   The $J^P = 0^-$ family of lightest mesons includes the pion states, $\pi^\pm$ and π^0, and the kaon states $K^\pm$, K^0 and $\overline{K}^0$.

   (i) Write down the composition of these mesons within the quark model and explain why there is no explicit $\overline{\pi}^0$ state.

   (ii) The masses of the $\pi^\pm$ are 140 MeV, the π^0 is 135 MeV, the $K^\pm$ are 494 MeV and the K^0 and $\overline{K}^0$ are 498 MeV. What is the significance of approximate equality of the masses of the three pion states and also of the four kaon states?

   (iii) Neglecting the small differences in mass, assume the "average" pion mass is 138 MeV and kaon mass is 496 MeV. The $J^P = 1^-$ excited states of the pions and kaons are the ρ and K^* states which have masses of 768 MeV and 892 MeV respectively. The two major contributions to the mass of all these particle states can be assumed to be due to a) the masses of the quarks and b) the QCD binding energy, which is dominated by a spin-dependent term (the colour equivalent of the QED magnetic splitting in atomic levels). This spin-dependent term results in a mass shift to the meson given by

   $$\Delta m = \frac{B}{m_i m_j} \langle \boldsymbol{s}_i . \boldsymbol{s}_j \rangle$$

for a meson made of quarks with masses m_i and m_j and spins $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$. The quantity B is constant for all mesons. Obtain an expression for the total angular momentum squared, $\boldsymbol{J}^2$, in terms of $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$, assuming there is no orbital angular momentum in these states. Hence, show that the expectation value of the spin term above is given by

$$\langle \boldsymbol{s}_i.\boldsymbol{s}_j \rangle = \frac{1}{2}J(J+1) - \frac{3}{4}$$

(iv) Obtain relationships between the masses of the π, K, ρ and K^*, assuming $m_u \approx m_d$. Evaluate the level of agreement in MeV of the masses under these assumptions.

(v) The small differences in mass within these groups are due to the difference between the u and d quark masses and to electromagnetic forces. Assuming the quarks in a kaon are on average 0.3 fm apart, calculate the Coulomb energy contribution to the K^+ and K^0 masses. The value of the fine structure constant is $\alpha = e^2/4\pi\epsilon_0 = 1/137$. What can be deduced about the difference in the u and d quark masses?

5. The explicit construction of the spin state symmetries of the baryons in this question shows which are allowed and which not under the Pauli exclusion principle. **This is the question referred to in the lectures and is very much optional!**

Consider a baryon made of three quarks. By taking the four possible combinations of $S_z = +1/2$ (spin-up $\uparrow$) and $S_z = -1/2$ (spin-down $\downarrow$) for the first two quarks, explicitly construct the four states of their combined spin, namely $S = 1$, with $S_z = +1$, 0 and -1, and $S = 0$.

Now add the third quark to the $S = 0$ state to make the $S = 1/2$ states with $S_z = +1/2$ and $-1/2$. Also add the third quark to the $S = 1$ states to make another set of $S = 1/2$ states with $S_z = +1/2$ and $-1/2$ and a set of $S = 3/2$ states with $S_z = +3/2$, $+1/2$, $-1/2$ and $-3/2$. There are eight states in all, corresponding to the $2^3 = 8$ possible combinations of the original three quark spins. For the last part, you need the Clebsch-Gordon coefficients for combining the spins; for a state $|S, S_z\rangle$, these are

$$\begin{aligned} |1/2,+1/2\rangle &= \sqrt{\frac{2}{3}}|1,+1\rangle|1/2,-1/2\rangle - \sqrt{\frac{1}{3}}|1,\ \ 0\rangle|1/2,+1/2\rangle \\ |1/2,-1/2\rangle &= \sqrt{\frac{1}{3}}|1,\ \ 0\rangle|1/2,-1/2\rangle - \sqrt{\frac{2}{3}}|1,-1\rangle|1/2,+1/2\rangle \end{aligned}$$

and

$$\begin{aligned} |3/2,+3/2\rangle &= |1,+1\rangle|1/2,+1/2\rangle \\ |3/2,+1/2\rangle &= \sqrt{\frac{1}{3}}|1,+1\rangle|1/2,-1/2\rangle + \sqrt{\frac{2}{3}}|1,\ \ 0\rangle|1/2,+1/2\rangle \\ |3/2,-1/2\rangle &= \sqrt{\frac{2}{3}}|1,\ \ 0\rangle|1/2,-1/2\rangle + \sqrt{\frac{1}{3}}|1,-1\rangle|1/2,+1/2\rangle \\ |3/2,-3/2\rangle &= |1,-1\rangle|1/2,-1/2\rangle \end{aligned}$$

Under quark interchange, for instance exchanging $1 \leftrightarrow 2$, any particular state can be in one of three categories: symmetric, meaning $|q_2q_1q_3\rangle = |q_1q_2q_3\rangle$; antisymmetric, meaning $|q_2q_1q_3\rangle = -|q_1q_2q_3\rangle$; or not have a definite symmetry, meaning $|q_2q_1q_3\rangle \neq a|q_1q_2q_3\rangle$ for any a. List the three-quark states you calculated above in terms of these categories for all three possible quark interchanges.

The Pauli exclusion principle requires identical fermions to be in antisymmetric states; note this is a stronger requirement than requiring them not to be in a symmetric state as it also excludes the states without a definite symmetry. If the quarks were not coloured, which of the states would be allowed for all three quarks different (e.g. uds), for two identical quarks (e.g. uud) and for all three identical (e.g. uuu)? How do these compare with the known baryons (listed on the handout)?

The QCD colourless combination for a system containing three colours is the completely antisymmetric combination of r, b and g; specifically

$$\sqrt{\frac{1}{6}}\,(r_1 b_2 g_3 - r_1 g_2 b_3 + g_1 r_2 b_3 - g_1 b_2 r_3 + b_1 g_2 r_3 - b_1 r_2 g_3)$$

Check that this is indeed antisymmetric under any of the three possible interchanges. If this colour factor is included, which states are now allowed and which not under the Pauli exclusion principle? How do these now compare with the known baryons?