Advanced Particle Physics 04/05
Dr Gavin Davies - Problem Sheet 1

Questions 1, 2 and 3 (if possible) should be handed in by 4pm on 11/01/05 for rapid feedback

1. This question *should* be revision of the relativistic formulæ for energy and momentum.

   Energy and momentum form a four-vector, $(E, \boldsymbol{p})$, in the same way as time and space, $(t, \boldsymbol{r})$. This means the Lorentz transformations for $(E, \boldsymbol{p})$ are exactly equivalent to those for $(t, \boldsymbol{r})$.

   For a particle at rest, the total energy is clearly just due to the mass, so $E = m$. Show by an explicit Lorentz transformation that, for a particle moving with speed β, the energy and momentum magnitude are given by

   $$E = \gamma m, \qquad p = \gamma m \beta$$

   where

   $$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

   Invert these equations to give β and γ in terms of m, p and E.

   Show that the quantity $E^2 - p^2$ is an invariant under a Lorentz transformation; to what physical quantity does it correspond?

2. This question goes through the calculation of the kinematics for a two-body decay, as given in the lectures.

   Consider the decay of a particle X to two particles c and d in the rest frame of X. Using only energy and momentum conservation, show that

   $$E_c = \frac{m_X^2 + m_c^2 - m_d^2}{2m_X}$$

   and similarly for E_d. Check energy is conserved by these formulæ. What is the value of the energy if the final state particles are the same (or antiparticles of each other)?

   The general expression for the magnitude of the momentum of c and d is more complicated. Show that

   $$p_c = \frac{\sqrt{m_X^4 + m_c^4 + m_d^4 - 2m_X^2 m_c^2 - 2m_X^2 m_d^2 - 2m_c^2 m_d^2}}{2m_X}$$

   What is the value of p_d? What is the value of the momentum if the final state particles are the same (or antiparticles of each other)?

   Show that if one of the final state particles is massless, e.g. $m_d = 0$, then the expression for the momentum simplifies to

   $$p_c = \frac{m_X^2 - m_c^2}{2m_X}$$

3. Exam question: 2001 question 8 (non-essay parts). (Note, Feynman diagrams (section (i)) have not yet been covered in this course but you saw the diagram for this decay last year in the NPP course)

   The main decay of the neutral pion (π^0) meson, mass $m_\pi = 135.0$ MeV, is to two photons. Its lifetime is $\tau_\pi = 8.4 \times 10^{-17}$ s.

(i) Draw the lowest order Feynman diagram for this decay. What are the energies of the two emitted photons in the π^0 rest frame?

(ii) In an experiment such as BaBar, π^0 mesons are often produced from the decays of heavier particles, like B mesons. A typical energy for the π^0 mesons produced by these decays at BaBar is around 0.5 GeV. Calculate the maximum energy of a photon which could be observed from the decay of a 0.5 GeV π^0. What is the energy of the second photon in the decay?

(iii) The existence of the π^0 meson in such a B decay is deduced by combining the energies and momenta of the two observed photons and showing the invariant mass is close to that of a π^0 meson. Obtain an expression for the invariant mass in terms of the two observed energies of the photons for the specific case described in section (ii) above.

(iv) The BaBar calorimeter has approximately 2% energy resolution, so it can be treated as having $\sigma_E/E = 0.02$. What would be the expected resolution, in MeV, of the π^0 mass due to the calorimeter resolution? What would be the order of magnitude of the spread of the mass due to the natural width of the π^0 meson? Comment on the values you find.

(v) The maximum energy for a π^0 meson originating from a B decay in BaBar is around 4 GeV. How far would such a π^0 travel on average before it decays? Consider the case of the two photons being emitted in the π^0 rest frame perpendicular to the π^0 direction of motion in the laboratory. Calculate the angle between the two photons in the laboratory frame for a 4 GeV π^0 meson. Discuss this in relation to the dimensions of the BaBar calorimeter, which is approximately a cylinder of radius 100 cm, made of crystals of dimensions 5 cm $\times$ 5 cm on their front face.

4. * The decaying particle of question 2 is rarely at rest, so this question looks at decays from a moving frame, for some special cases.

Consider the decay of $X \to c + d$ above and for simplicity consider the case where $m_c = m_d$. When the particle c is emitted directly along the z axis, do an explicit Lorentz transformation along the z axis by speed $-\beta$ to find its momentum in the frame where the particle X is moving by β. Under what condition is the boosted momentum of particle c bigger than that of particle X?

Now consider the simpler case where $m_c = m_d = 0$. For the case when the particle c is emitted at an angle θ_c to the z axis in the rest frame, do an explicit Lorentz transformation, again along the z axis by speed $-\beta$, to find its energy in the boosted frame. For an unpolarised decay, there is no preferred direction in space so in the rest frame, the particles are emitted equally throughout the solid angle. This means they have a probability distribution in $\cos\theta_c$ of

$$P(\cos\theta_c)d(\cos\theta_c) = \frac{1}{2}d(\cos\theta_c)$$

Calculate the distribution of the energy of the particle c in the boosted frame.