Particle Physics Homework Assignment 9

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Problem 1: Write down the spinor for a positive energy neutrino of negative helicity and show explicitly that it is en eigenvector for both helicity and handedness.

Problem 2: If ν_L is a spinor describing neutrinos of negative helicity show that

$$\frac{1-\gamma_5}{2}\nu_L = \nu_L \quad \text{and} \quad \frac{1+\gamma_5}{2}\nu_L = 0$$

Problem 3: Prove the identity:

$$\left(1 - \frac{\vec{\sigma}\,\vec{p}}{E+M}\right) = \frac{1}{2}\left(1 - \frac{p}{E+M}\right)(1 + \vec{\sigma}\,\vec{p}) + \frac{1}{2}\left(1+\frac{p}{E+M}\right)(1 - \vec{\sigma}\,\vec{p})$$

where $p = |\vec{p}|$

Problem 4: Show that the operator $\frac{(1 - \gamma_5)}{2}$ acting on a spinor describing a negative energy spin-half massive fermion will result to a spinor with both positive helicity (right handed) and negative helicity (left handed) components. Further more show that the left handed component is suppressed by the ratio $\frac{M}{E}$. Hence if the particle is massless it results to a pure right handed component.

Problem 5: Consider a massive chiral fermion given by:

$$\Psi_L = \frac{(1 - \gamma_5)}{2}\Psi$$

Define the polarization of the chiral massive fermion to be:

$$P = \frac{|\alpha_{RH}|^2 - |\alpha_{LH}|^2}{|\alpha_{RH}|^2 + |\alpha_{LH}|^2}$$

Where $|\alpha_{RH}|^2$, $|\alpha_{LH}|^2$ are the positive and negative helicity amplitudes of the fermion

Show that $P = -\frac{p}{E} = -\beta$ where p, E are the momentum and energy of the fermion.