Particle Physics Homework Assignment 7

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Problem 1: Consider $\Psi = \begin{pmatrix} u_A \\ u_B \end{pmatrix}$ to be solution of the Dirac equation where u_A, u_B are two-component spinors. Show that in the non-relativistic limit $u_B \sim \beta = v/c$.

Problem 2: Show that at the non-relativistic limit the motion of a spin half fermion of charge e at the presence of an electromagnetic field $A^\mu = (A^0; \vec{A})$ is described by:

$$\left[\frac{(\vec{p} - e\vec{A})^2}{2m} - \frac{e}{2m} \vec{\sigma} \cdot \vec{B} + eA^0 \right] \chi = E\chi$$

where $\vec{B}$ is the magnetic field, σ^i are the Pauli matrices and $E = p^0 - m$. Identify the g-factor of the fermion and show that the Dirac equation predicts the correct gyromagnetic ratio for the fermion. To write down the Dirac equation at the presence of an electromagnetic field substitute: $p^\mu \rightarrow p^\mu - eA^\mu$.

Problem 3: Show that:

(a) $\bar{\Psi} \gamma_5 \Psi$ is a pseudoscalar.
(b) $\bar{\Psi} \gamma_5 \gamma^\mu \Psi$ is an axial vector.

Comment on the Lorentz and parity properties of the quantities:

(a) $\bar{\Psi} \gamma_5 \gamma^\mu \Psi \bar{\Psi} \gamma_\mu \Psi$
(b) $\bar{\Psi} \gamma_5 \Psi \bar{\Psi} \gamma_5 \Psi$
(c) $\bar{\Psi} \Psi \bar{\Psi} \gamma_5 \Psi$
(d) $\bar{\Psi} \gamma_5 \gamma^\mu \Psi \bar{\Psi} \gamma_5 \gamma_\mu \Psi$
(e) $\bar{\Psi} \gamma^\mu \Psi \bar{\Psi} \gamma_\mu \Psi$