Particle Physics Homework Assignment 6

Dr. Costas Foudas 8/3/07

Problem 1: Show that: $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) \;=\; \vec{a}\cdot\vec{b}+i\,\vec{\sigma}\,(\vec{a}\times\vec{b})$

Problem 2: Positive energy solutions of the Dirac Equation correspond to the 4-vectror current: $J^{\mu} \;=\; 2\,p^{\mu} \;=\; 2(E\,;\vec{p})\,;\quad E>0$. Show that negative energy solutions correspond to the current $J^{\mu} \;=\; -2(E\,;\vec{p}) \;=\; 2(|E|\,;-\vec{p}) \;=\; -2\,p^{\mu}\,;\quad E<0$.

Problem 3: (1) Show that the helicity operator commutes with the Hamiltonian:

$$[\vec{\Sigma}\cdot\hat{p}\,,H\,] \;=\; 0$$

(2) Show explicitly that the solutions of the Dirac equation are eigenvectors of the helicity operator:

$$[\vec{\Sigma}\cdot\hat{p}\,]\varPsi \;=\; \pm\varPsi$$