APP - Lecture 18 - Weak force and charged current weak interactions

18.1 Introduction

We believe the EM and strong interactions are determined by gauge symmetries of the Lagrangian density and so it is natural for us to look for a similar symmetry for the weak interactions. This will turn out to look like a bad idea as the symmetry appears to not exist but in fact, it *is* believed the weak interactions are fundamentally due to an SU(2) symmetry.

18.2 SU(2) gauge invariance

It would be nice at this point to write down the equivalent in SU(2) to what we did for SU(3) in the QCD case and then say that's the weak interaction. Although things will not be that simple, what would we get? We need a pair (rather than a trio) of fermions of the same mass and, as we will see, these need to be the electron and the electron neutrino so we write

$$\Psi = \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix}$$

Hence, e and ν_e are supposed to be like u_r, u_b and u_g, i.e. they differ only in their weak (or strong) charge type. Note, we have obviously already hit a problem as we know these do not have the same mass; in fact the neutrino is massless in the Standard Model.

Ignoring this "minor" problem, then we write down the general SU(2) special unitary 2×2 matrix. As before, it can be expressed as the exponent of a set of Hermitian matrices but, instead of eight parameters, there are now only three

$$U = e^{-i\alpha_i \sigma_i}, \qquad i = 1, 3$$

where the σ_i are the three Pauli matrices rather than the eight λ_i Gell-Mann matrices. There will be only three conserved currents rather than eight

$$J_i^\mu = \overline{\Psi}\gamma^\mu \sigma_i \Psi$$

It is worth looking at the structure of these explicitly; the first is

$$J_1^\mu = \overline{\Psi}\gamma^\mu \sigma_1 \Psi = \overline{\psi}_e \gamma^\mu \psi_\nu + \overline{\psi}_\nu \gamma^\mu \psi_e$$

the second is

$$J_2^\mu = \overline{\Psi}\gamma^\mu \sigma_2 \Psi = i\overline{\psi}_e \gamma^\mu \psi_\nu - i\overline{\psi}_\nu \gamma^\mu \psi_e$$

and the third is

$$J_3^\mu = \overline{\Psi}\gamma^\mu \sigma_3 \Psi = \overline{\psi}_\nu \gamma^\mu \psi_\nu - \overline{\psi}_e \gamma^\mu \psi_e$$

We now go to local gauge invariance by coupling three fields A_i^μ to these three currents in an interaction term

$$-g_W A_{i\mu} \overline{\Psi}\gamma^\mu \sigma_i \Psi$$

with some coupling strength g_W. Note, the interaction terms then correspond to two fields transforming e into ν_e and vice versa while the third is a ee or $\nu_e\nu_e$ interaction.

The field gauge transformations and field tensors are then uniquely defined by the gauge invariance requirement as before. Again, the A_i^μ fields are self-interacting ("non-Abelian") and a mass term $m^2 A_i^\mu A_{i\mu}/2$ is not allowed as it cannot be made gauge invariant.

How well does this correspond to what we know about the weak interactions?

1. As already stated, $m_e \neq m_\nu$ so this obviously fails.
2. There would be three fields, two of which convert between e and ν_e while the other does not. We know the weak bosons are the $W^\pm$ and the Z, so this is correct.
3. The W and Z boson masses would have to be zero; this is not the case.
4. The currents would be $\overline{\psi}\gamma^\mu\psi$, but we think the weak ones are actually $\overline{\psi}\gamma^\mu(1-\gamma^5)\psi/2$.
5. There would be WWZ, $WWZZ$ and $WWWW$ interactions as the fields self-interact; this is thought to be correct.
6. Since g_W, like g_S, appears in the field tensor definition, then it must be a universal charge magnitude, so all particles must have the same weak charge. This is true for the $W^\pm$ interactions, but not the Z ones.
7. P and C would be conserved as the Lagrangian density is invariant to these operations. This is not correct.

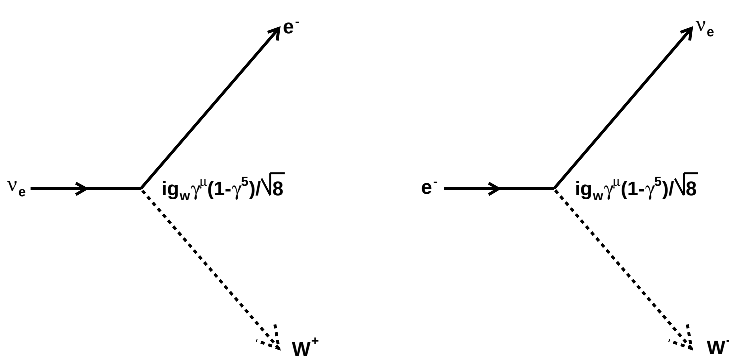
This appears to be about half right and it turns out we will need to add two different extra pieces to fix it all up correctly. We need to shelve this for a while.

18.3 Charged current weak interactions

We will abandon local gauge invariance for now and write down the Lagrangian interaction term which we think describes the weak interactions for at least the $W^\pm$. Reactions involving the $W^\pm$ are called "charged current", to distinguish them from the "neutral current" interactions mediated by the neutral Z. The correct term is thought to be

$$-\frac{g_W}{\sqrt{2}}\left[W_\mu^+\overline{\psi}_e\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\psi_\nu + W_\mu^-\overline{\psi}_\nu\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\psi_e\right]$$

which gives Feynman diagram vertices as follows.



The $W^\pm$ have non-zero masses, so their propagators are more complicated than for the photon; the latter is

$$-i\frac{g^{\mu\nu}}{q^2}$$

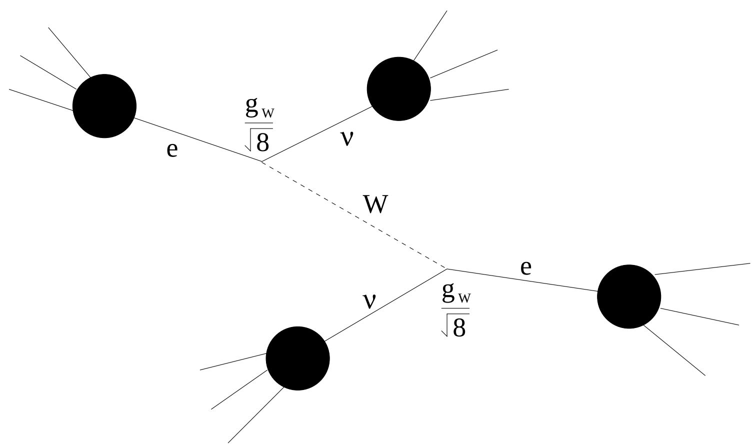
while for the $W^\pm$, we must use

$$-i\frac{g^{\mu\nu} - q^\mu q^\nu / M_W^2}{q^2 - M_W^2}$$

Note, this does *not* go smoothly to the photon propagator as $M_W \to 0$. The W is $S = 1$ and so has three possible spin states, while the photon only has two; hence there is a discontinuity at $M_W = 0$. For low energies, where $q^2 \ll M_W^2$, then the propagator is approximately

$$i\frac{g^{\mu\nu}}{M_W^2}$$

which is a propagator for an infinitely short range force. Let's consider any arbitrary process containing a virtual W



In the limit of low energies, there will be terms from both vertices and the propagator, giving an overall factor of

$$\left(\frac{g_W}{\sqrt{8}}\right)^2 \frac{1}{M_W^2} = \frac{1}{8}\left(\frac{g_W}{M_W}\right)^2$$

This combination is what is actually measured in all low energy weak interactions and so historically g_W and M_W could not be separately determined. In fact, this combination was measured very accurately in many decay modes so the effective coupling constant was G_F, where

$$\frac{G_F}{\sqrt{2}} = \frac{1}{8}\left(\frac{g_W}{M_W}\right)^2$$

The Fermi coupling constant G_F was found to be

$$G_F = (1.16639 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

It is because this is so small that the weak force was called "weak".

However, we can now do experiments at high energies and so can see the q^2 dependence of the propagator and hence separate out the g_W and M_W terms. The W mass has been measured to be

$$M_W = 80.42 \pm 0.05\ \mathrm{GeV}$$

and so we can find the size of g_W from the above to be

$$g_W = 0.6532 \pm 0.0004$$

This gives the equivalent of the fine structure constant to be

$$\alpha_W = \frac{g_W^2}{4\pi} = 0.03396 \pm 0.00004 \sim \frac{1}{29}$$

Hence, the weak coupling α_W is about five times *stronger* than the EM coupling α! This shows the weak force is only apparently weak at low energies because the W mass is very large. At high energies, meaning bigger than M_W, its full force becomes apparent and it dominates over the EM force at these energies.

18.4 The interaction term structure

Let's consider the form of the interaction again. Each piece goes like

$$\overline{\psi}\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right)\psi = \frac{1}{2}\left(\overline{\psi}\gamma^\mu\psi - \overline{\psi}\gamma^\mu\gamma^5\psi\right)$$

There are three things to observe here

1. We know from the Dirac equation that $\overline{\psi}\gamma^\mu\psi$ is conserved, but what about $\overline{\psi}\gamma^\mu\gamma^5\psi$? It is straightforward to show that

   $$\partial_\mu\left(\overline{\psi}\gamma^\mu\gamma^5\psi\right) = 2im\overline{\psi}\gamma^5\psi$$

   and so is not conserved unless the mass is zero. Hence, for this to be usable for local gauge invariance, not only do the e and ν_e have to have the same mass, but that mass has to be zero. This holds for all particles, as all the quarks and leptons all undergo weak interactions, so they "should" all be zero mass particles.

2. We already saw $\overline{\psi}\gamma^\mu\psi$ is a polar vector and $\overline{\psi}\gamma^\mu\gamma^5\psi$ is an axial vector. This term is sometimes called "$V-A$" because of this. Irrespective of what parity properties the W has, we are adding two terms here with opposite parity properties. This is going to lead directly to parity violation. In fact, the same terms also lead to C violation too. Again, the terms are effectively the same size; this is not 0.99 of one added to 0.01 of the other, so we would indeed expect maximal parity violation.

3. What is γ^5? In the standard Dirac matrix representation, this is

   $$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   It is generally an Hermitian matrix and so could correspond to an observable in some sense. In fact, $\gamma^5/2$ is called the "handedness" operator and has eigenvalues of $\pm 1/2$. The nomenclature often used is that the $+1/2$ eigenstate is called "right handed" (R) and the $-1/2$ eigenstate is "left handed" (L). What do these variables correspond to physically? A good question; they have no classical analogue. In the high energy limit, they become identical with helicity states, but they are not the same thing. To be specific; in the limit of $\beta \to 1$ or $m \to 0$, the R state becomes an helicity $+1/2$ state and the L state becomes an helicity $-1/2$ state. However, it is important to distinguish these two as they are not the same thing; helicity always refers to the relative directions of the momentum and spin, while handedness refers to some other property which only shares eigenstates with helicity in the high energy limit. However, for a general case of a non-zero mass particle, the handedness eigenstate is not an eigenstate of the Dirac equation (and hence of helicity either). Unless $\beta = 1$, then a free particle is in a mixture of R and L handedness.