Particle Physics Homework Assignment 3

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Problem 1: The HERA accelerator collides 30 GeV electrons with 920 GeV protons.

I. Compute the centre of mass energy (total available energy at the electron proton centre of mass) assuming that the angle between the proton and the electron beam momenta is 180^0 (head on collision).

II. Compute the boost, $\vec{\beta_{CM}}$, of the electron-proton centre of mass frame relative to the laboratory frame.

III. What should be the energy of an electron beam colliding with protons at rest if the centre of mass energy were to be the same with HERA ? This type of experiment is called *fixed target experiment* to distinguish it with the previous which is called a *collider experiment.*

Problem 2: Deduce an expression for the energy of a γ-ray from the decay of the neutral pion, $\pi^0 \rightarrow \gamma\gamma$, in terms of the mass m, energy E, and velocity βc of the pion and the angle of emission ϑ^* in the pion rest frame. Show that if the pion has spin zero, so that the angular distribution is isotropic, the laboratory energy spectrum for the γ-rays will be flat extending from E(1+β)/2 to E(1-β)/2. For relativistic pions, find an expression for the disparity D (the ratio of energies) for the γ-rays and show that D>3 is half <u>the decays</u> and D>7 is one quarter of them.(Perkins p33, 4th edition)

Problem 3: **A** high energy electron collides with an atomic electron which can be considered at rest. What is the threshold (the minimum kinetic energy of the incident electron) for producing and electron positron pair?

Problem 4: An experiment which measured parity violation in weak interactions as well as the μ^+ magnetic moment and the Lande' g factor, used a μ^+ beam which was produced from the decay in flight of a π^+ beam ($\pi^+ \rightarrow \mu^+ \nu_\mu$). The kinetic energy of the π^+ beam was **85 MeV**. The π^+ beam was produced by colliding a proton beam from an accelerator with a target and selecting the positively charged π^+ from the negatively charged π^- using a magnetic field. Positive pions of a certain direction and momentum were selected by passing the π^+ beam through an appropriately positioned concrete block with a straight hole through it.

Questions regarding the beam:

I. Compute the decay length of the pions in the laboratory frame. The lifetime of the π^+ is 26 ns and the π^+ mass is about **140 MeV**. The incident beam to the

experiment was a mixture of 10% μ^+ and 90% π^+. How far was the experiment from the point where the pions were produced ?

II. Compute the pion mean free path in carbon assuming that the pion carbon cross section at the relevant energy is 10 mb.

III. Compute the muon and neutrino energies in the rest frame of the pion. Assume that the neutrino is massless and that the mass of the muon is $m_\mu = 106 \, MeV$.

IV. Compute the maximum and minimum energy of the muon in the lab frame.

V. The authors of the paper used a carbon block 20 cm long. Justify this given that pions and muons of this energy lose in carbon approximately 4.5 MeV/cm due to ionization.
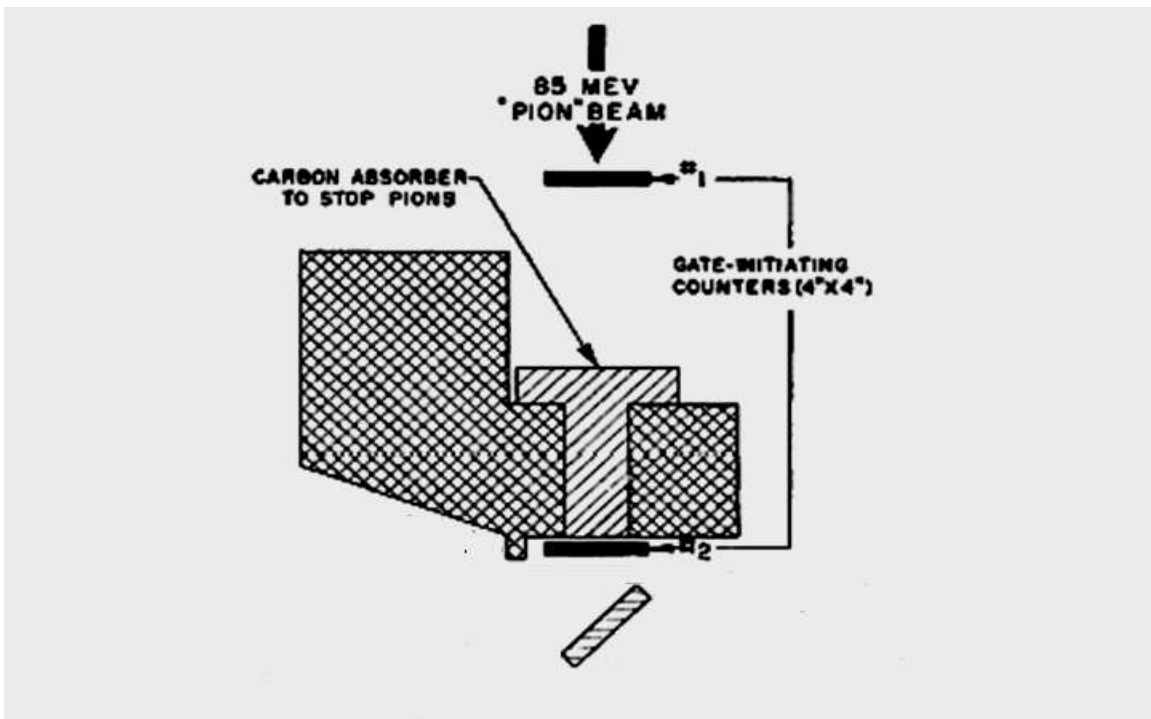
Questions Regarding the Apparatus:

Figure 1: *Part of the apparatus of the Garwin-Lederman-Weinrich experiment Phys. Rev., 105:1415-1417, 1957. The 85 MeV NEVIS pion beam is seen entering from the top. The carbon absorber length has been chosen so that only muons exit the carbon block whilst the pions stop in the block. Hence a signal coincidence from the counters #1 and #2 indicates that a muon has gone through. If the Carbon length is chosen correctly is also means that it will most likely stop on the carbon target below.*

The experimental apparatus is shown in Fig. 1. A carbon block has been used to separate the pions from the muons. The carbon block was introduced to for two reasons: (1) to stop the pions (2) to slow down the muons in such a way so that when they exit the carbon block they have very little kinetic energy left and thus are captured for study by the carbon target shown tilted at the bottom of Fig. 4.

Two counters #1 and #2 have been placed at the path of the beam before and after the carbon block. Counters are devices made of liquid or plastic scintillator. When a charged particle goes through a counter it excites the scintillator material and produces light which can be collected and converted by photo-tubes to charge. Detection of this charge using electronic circuits provides information which indicates that a particle has gone through the scintillator material as well as allows a measurement of the time that the particle went through.

The signals from Counters #1 and #2 in this experiment are used in coincidence to provide an experimental trigger. This means that if both counters detected that a particle went though at the same time[1] (coincidence trigger) then that meant that a muon went through the carbon and exited from the other side albeit with very little kinetic energy so that it most likely stopped on the target. If the trigger coincidence condition was fulfilled then detectors around the target were activated appropriately and recorded the data from the muon decays.

VI. Compute the maximum possible angle of the decay muons from pions in the Laboratory frame. Use the results from (V) and explain how does the carbon block constrain this angle? Do the angles of the muons which are capable to penetrate the block extend up to this angle ?

VII. Given that this experiment does not record data unless if *there is a trigger* compute and discuss how does the distance between the two counters affects the muon energy spectrum detected in this experiment. Assume that the beam enters at the center of the fist counter.

Hence, this part of the apparatus has been designed to collect muons which stop at the target (for sometime since the muon lives only 2.16 µsec) to study their properties. The rest of this experiment as well as the results of it will be discussed later in the course when we discuss parity violation in weak interactions and and about the muon magnetic moment.

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[1] The time difference introduced by the muon travelling between the two counters is very small and the experiment is not sensitive to it.

Problem 5: When the muon was originally discovered back in the 30s, people thought that this was the Yukawa particle. The Yukawa particle was considered then to be the mediator of the strong interaction. This incorrect interpretation of the nature of the muon was due to the fact that the muon mass (106 MeV) was not very different from the expected mass of the particle predicted by Yukawa. Later on it turned out that the Yukawa particle was the pion which was discovered in the 40s at Bristol.

This problem[2] relates to the calculations done by Tomonaga and Araki who predicted that if the muon was the mediator of the strong interaction then negative muons passing through matter would be more likely to be captured by the nuclei rather than decay.

(a) Show that a negative muon captured in an S-state by a nucleus of charge Ze and mass A will spend a fraction $f \simeq 0.25 A(Z/137)^3$ of its time inside the nuclear matter and that in time t it will travel a total distance $fct\, 0.25(Z/137)$ in the nuclear matter. The hydrogen atom ground state wave function can be used in these calculations with modifications to account for the fact that the muon mass is of the order of 200 times larger than the electron mass:

$$\Psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{\alpha_0}\right)^{3/2} e^{-\frac{Zr}{\alpha_0}} \quad \text{where} \quad \alpha_0 = \frac{\hbar^2}{M_R e^2} \quad \text{and} \quad M_R = \frac{m_p m_\mu}{m_p + m_\mu} \quad \text{is the}$$

reduced mass of the proton muon system. The proton and muon masses are $m_p = 938\, MeV$ and $m_\mu = 106\, MeV$.

(b) The law of radioactive decay of free muons is $dN/dt = -\Gamma_d N$, where $\Gamma = 1/\tau$ is the decay constant (width) and the lifetime is $\tau = 2.16\,\mu\, sec$. For a negative muon captured in an atom Z the decay constant is $\Gamma_{TOT} = \Gamma_d + \Gamma_c$, where Γ_c is the width for nuclear capture i.e. the probability per unit time of nuclear capture. For aluminium $(Z=13, A=27)$ the mean lifetime of negative muons is $\tau = 0.88\,\mu\, sec$. Calculate Γ_c and using he expression for f in (a), compute the interaction mean free path Λ for a muon in nuclear matter.

(c) From the magnitude of Λ estimate the magnitude of the coupling constant of the interaction that caused the nuclear capture $\mu^- + p \rightarrow n + \nu$ given that the strong interaction coupling constant is α_S and corresponds to a mean free path of $1\, fm$.

Conversi, Pancini and Piccioni[3] did experiments in Rome in the 40s to test Tomonaga's and Araki's hypothesis and found that positive muons traversing different materials

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[2] This is a modified version of problem 1.9 in p34 in Perkins 4th edition.
[3] Conversi, Pancini and Piccioni, Phys. Rev., 71. No 3, 1 Feb. 1947.

always decay rather than being captured (not surprising). They also found that negative muons undergo nuclear capture in iron. However, in carbon negative muons decay and do not get captured by the nucleus in direct contradiction of the Tomonaga-Araki hypothesis. Hence, the muons have nothing to do with the strong interaction.