Particle Physics Homework Assignment 8

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Problem 1: Consider an experiment where negative pions, π^- , at rest are being captured by a Hydrogen nuclei.

1) Draw conclusions about the spin of the π^- given that the reaction $\pi^- p \to n\gamma$ occurs.
2) Can the reaction $\pi^- p \to n$ occur if the initial state proton is free (not bound in a nucleus) ? What happens if the proton is bound to a nucleus ?
3) Draw conclusions about the parity of the π^0 and π^- provided that the reaction $\pi^- p \to n\pi^0$ also occurs.
4) Draw conclusions about the π^0 spin provided that it decays into two photons: $\pi^0 \to \gamma\gamma$

Solution:

1) The photon is a spin-1 particle and one can conclude this from simple arguments: Photons come in two polarizations (left and right-handed states) therefore they cannot have spin 0 which would require them to appear only in one state. In addition they must satisfy the Maxwell equations and have infinite range. Therefore they must be massless spin one bosons (classical polar vectors). However spin-one particles have 3 states (2S+1=3) and photons have only two. This is due to the fact that photons are massless (local gauge invariance) and thus they are deprived from longitudinal polarization. Hence, photons are spin-one particle which appear in nature in two transverse polarizations. Protons an neutrons are spin-half fermions. Based on these facts the total angular momentum in the right-hand side of $\pi^- p \to n\gamma$ should be equal to $J = 1/2,\ 3/2$ whilst the total angular momentum at the z-direction should be equal to $J_Z = \pm 1/2,\ \pm 3/2$. Angular momentum is conserved. **Hence, the negative pion must be a boson with possible spin assignment s = 0, 1, 2**.
2) If the pion and the proton are at rest then the neutron must be at rest (conservation of momentum) and conservation of energy would require that the mass of the negative pion is equal to the neutron-proton mass difference. **This is clearly not true and the reaction cannot occur for this reason.** If on the other hand the proton is bound in a nucleus, the nucleus can easily balance energy and momentum by recoiling against the neutron. Hence, energy and momentum conservation is saved this way. **Parity conservation would also prevent this reaction from happening via the strong interaction** because the baryon parity cancels whatever it might be (this is the reason that the proton parity is a matter of convention) and the pion has negative parity as we have seen. Hence, parity is violated. However, if the proton is bound to a nucleus, the final state nucleus can be exited to a state with an appropriate orbital angular momentum, **L**, so that the parity of the nucleus wave function is proportional to $\mathbf{(-1)^L = -1}$ which saves parity form being violated. Obviously this only happens for odd values of **L**.

3) Since the photon has two polarizations and spin equal to 1 the reaction $\pi^0 \to \gamma\gamma$ implies that the neutral pion is a boson with spin either 0 or 2. The value of one is excluded also because the wave function of the photon-photon system must be symmetric (photons are bosons). However, under particle exchange, which in this case is identical to parity, the photon-photon wavefunction accuires a factor of $(-1)^L$ where **L** is the total anglular momentum. Therefore, if **L = 1** then the photon-photon wavefunciton would be anti-symmetric under particle exchange. Hence, **L = 1** is ruled out. At the LHC the Higgs particle was discovered in 2012 via the reaction $H^0 \to \gamma\gamma$ and the same conclusion was drawn, namely that the Higgs spin must be either 0 or 2. Further measuremetns since then have established that the Higgs has spin 0.

Problem 2: The ρ^0 is a vector boson, that is an 1^- state. Explain why the decay $\rho^0 \to \pi^+\pi^-$ is allowed and why the decay $\rho^0 \to \pi^0\pi^0$ is forbidden.

Solution:

Fact 1: The neutral pions in $\rho^0 \to \pi^0\pi^0$ are of course identical bosons and their wave function must be symmetric under interchange of particles.

Fact 2: Parity must be conserved here (it is not a weak process to violate parity) and since the two neutral pions have an overall positive intrinsic parity, $(-1)\times(-1) = +1$, then it must be that the orbital part of the two-pion wave function must have a dependence of the type: $(-1)^L$ where L is odd. Since the pions have spin zero and the total angular momentum is conserved we conclude that **L=1.** However, for a two body system of spin-0 particles this implies that the wave function is anti-symmetric under particle interchange.

Facts 1, 2 are inconsistent and the reaction cannot occur.

The reaction $\rho^0 \to \pi^+\pi^-$ can proceed in nature because the two pions of different charge are not identical particles.

Problem 3: The η meson is a 0^- state. Explain why the decay $\eta \to \pi^-\pi^+$ is forbidden whilst the decay $\eta \to \pi^-\pi^+\pi^0$ is allowed via the electromagnetic interaction.

Solution:

Consider the reaction $\eta \to \pi^-\pi^+$ in the η meson rest frame. In this frame the total angular momentum is zero and parity is negative. As this is not a weak process, parity is conserved. Hence, the parity of the two charged pion system must be:

$$P(\pi^+\pi^-) = (-1)\times(-1)\times(-1)^L \Rightarrow L = 1,\ 3,\ 5,\ ....$$

This is not consistent with the total angular momentum being zero and the reaction $\eta \rightarrow \pi^- \pi^+$ cannot proceed.

As before the reaction $\eta \rightarrow \pi^- \pi^+ \pi^0$ must have zero total angular momentum in the final state. However, we have here three pions each having a negative intrinsic parity. Therefore, the parity of the final system of three particles is:

$$P(\pi^+\pi^-) = (-1)\times(-1)\times(-1)\times(-1)^L \Rightarrow L = 0, 2, 4, \ldots$$

The choice of **L = 0** satisfies both angular momentum and parity conservation and the reaction can occur in nature.

Problem 4:

The decay ${}^{20}Ne(1^+) \rightarrow {}^{16}O(0^+) + \alpha(0^+)$ proceeds via the strong interaction. The spin and parity assigments of the particles are given in parenthesis. Is this decay allowed or forbidden ?

Solution:

The Ne nucleus has spin 1 and positive parity. The oxygen nucleus and the alpha particle have both zero spin and positive parity. Therefore, to conserve angular momentum the oxygen nucleus and the alhpa particle must have total angular momentum equal to 1. However, this would mean that the parity on the right-hand side for the decay must be equal to

$$P = (+1)(+1)(-1)^1 = -1$$

and this would mean that parity is not conserved. However, strong interactions conserve parity therefore this decay is ruled out.

Problem 5:

The decay ${}^{20}Ne(1^+) \rightarrow {}^{16}O(3^-) + \alpha(0^+)$ proceeds via the strong interaction. The spin and parity assigments of the particles are given in parenthesis. Is this decay allowed or forbidden ?

Solution:

As seen here the total angular momentum of the decay products must be equal to 1. However, the spin of the Oxygen nucleus is 3. Therefore the total orbital angular momentum of the decay products can only be 2, 3, 4. Only these values added to 3 can give total angular momentum equal to 1.

Since, this is also a decay that proceeds via the strong interaction which conserves parity we should now investigate under which conditions parity is conserved. The Ne nucleus has positive parity. Therefore if this decay should happen it must be that the parity of the oxygen-alpha system as also positive parity. The parity of the decay products is

$$\boldsymbol{P = (-1)(+1)(-1)^{L} = (-1)^{L+1}}$$

Therefore if the only possible values of L are L = 2, 3, 4 then the decay can proceed via L = 3 only because this conserves parity.