Helicity and Chirality

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Helicity: As we have seen before the helicity operator is defined as:

$$\vec{\Sigma}\cdot\hat{p} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix} \qquad (9.1)$$

where $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ are the 2x2 Pauli matrices and $\hat{p} = \vec{p}/|\vec{p}|$ where $\vec{p}$ is momentum operator and $|\vec{p}|$ is the eigenvalue of $\vec{p}$. As seen from (1) the **helicity eigenvalues represent the projection of the particle spin at the direction of motion**. It is easy to show that the helicity operator commutes with the Dirac hamiltonian:

$$[\vec{\Sigma}\cdot\hat{p}, H] = 0 \qquad (9.2)$$

Hence, the Dirac hamiltonian and helicity have a common set of eigenvectors. This is also the reason for the two-fold degeneracy found for every energy eigenstate of the Dirac hamiltonian. It is easy to show explicitly that the solutions of the Dirac equation are indeed eigenvectors of the helicity operator:

Consider the first two positive solutions of the Dirac Equation:

$$\Psi^{(s)}(x) = N\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\chi^s e^{-ipx} \quad \text{where} \quad s=1,\ 2 \qquad \chi^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

by applying the helicity operator we have:

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot(-i\vec{\nabla})}{p} & 0 \\ 0 & \dfrac{\vec{\sigma}\cdot(-i\vec{\nabla})}{p} \end{pmatrix}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\chi^s e^{-ipx} \quad \Rightarrow$$

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\chi^s e^{-ipx} \quad \Rightarrow$$

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix} \vec{\sigma}\cdot\hat{p} \\ \vec{\sigma}\cdot\hat{p}\,\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\chi^s e^{-ipx} \quad \Rightarrow$$

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\vec{\sigma}\cdot\hat{p}\chi^{s}e^{-ipx} \quad \Rightarrow$$

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}(\pm 1)\,\chi^{s}e^{-ipx} \quad \Rightarrow$$

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(s)}(x) = (\pm 1)\sqrt{E+M}\begin{pmatrix} 1 \\ \dfrac{(\vec{\sigma}\cdot\vec{p})}{(E+m)} \end{pmatrix}\chi^{s}e^{-ipx} \quad \Rightarrow$$

$$(\vec{\Sigma}\cdot\hat{p})\Psi^{(1,2)}(x) = (\pm 1)\,\Psi^{(1,2)}(x)$$

where the plus corresponds to s = 1 and the minus to s = 2.

Hence, we have shown that the positive energy eigenvectors of the Dirac hamiltonian are also eigenvectors of the helicity operator. In the last step we have used the relationship $\vec{\sigma}\cdot\hat{p}\,\chi^{s} = (\pm 1)\,\chi^{s}$ which can easily be proven by selecting the unit vector at the direction of the z-axis. The same is true for the negative energy solutions albeit a key but subtle difference which we will address in the next Lecture which is about anti-particles.

It is important to notice that the solutions of the Dirac equation are eigenvectors of the helicity operator and in general not eigenvectors of the spin operator:

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

except at the particle rest frame where the momentum is zero. Why should they be anyway ? The spin operator does not commute with the hamiltonian as we have seen before. Therefore the helicity operator has the following properties:

(a) **Helicity is a good quantum number:** The helicity is conserved always because it commutes with the Hamiltonian. That is, its value does not change with time within a given reference frame. As we have seen before (2) is valid for both massive and massless fermions. Hence, helicity is conserved for both massive and massless particles.

(b) **Helicity is not Lorentz invariant:** This is obvious since helicity is a product of a 3-vector with an axial vector. One can think of a *gedanken* experiment where a spin ½ fermion is moving relative to an inertial observer and has positive helicity. A second observer who is moving faster than the fermion overcomes the fermion and for him the fermion momentum has flipped its sign whilst its spin remains the same. Hence, the second observer measures the opposite helicity than the first. This *gedanken* experiment demonstrates why helicity is not a Lorentz invariant quantity.

Chirality or Handedness:

Consider now the chirality/handedness operator in the Pauli-Dirac representation:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

which satisfies:

$$\{\gamma_5, \gamma^\mu\} = 0$$

This anti-commutation relationship is true in any Dirac matrix representation. Lets evaluate the commutator of the chirality operator with the Dirac Hamiltonian:

$$[\gamma_5, H] = [\gamma_5, \vec{\alpha}\cdot\vec{p} + m\beta] = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} m & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m \end{pmatrix} - \begin{pmatrix} m & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m \end{pmatrix}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \Rightarrow$$

$$[\gamma_5, H] = \begin{pmatrix} \vec{\sigma}\cdot\vec{p} & -m \\ m & \vec{\sigma}\cdot\vec{p} \end{pmatrix} - \begin{pmatrix} \vec{\sigma}\cdot\vec{p} & m \\ -m & \vec{\sigma}\cdot\vec{p} \end{pmatrix} \Rightarrow$$

$$[\gamma_5, H] = 2m\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

So the chirality/handedness operator does not commute with the hamiltonian unless if the mass is zero. **Hence, although we don't know yet the physical observable which is associated with this operator we do know that it is conserved and corresponds to a good quantum number only if the mass is zero or can be neglected as is the case when the particle energy is far larger than its mass.**

Next we will study certain properties of the chirality operator by means of solving exercises which demonstrate these properties.

Exercise 1: Consider the Dirac Hamiltonian for massless fermions $H = \vec{\alpha}\cdot\vec{p}$. Use the anti-commutation relationships $\{\gamma_5, \gamma^\mu\} = 0$ which are representation independent to show that the result $[\gamma_5, H] = 0$ is true in any representation provided that the mass is zero.

Solution:

$$[H,\gamma_5] = [\gamma^0\gamma^i p^i,\gamma_5] = p^i\{\gamma^0[\gamma^i,\gamma_5]+[\gamma^0,\gamma_5]\gamma^i\} \Rightarrow$$

$$[H,\gamma_5] = [\gamma^0\gamma^i p^i,\gamma_5] = p^i\{\gamma^0(\gamma^i\gamma_5-\gamma_5\gamma^i)+(\gamma^0\gamma_5-\gamma_5\gamma^0)\gamma^i\} \Rightarrow$$

$$[H,\gamma_5] = [\gamma^0\gamma^i p^i,\gamma_5] = p^i\{\gamma^0(\gamma^i\gamma_5+\gamma_5\gamma^i)-(\gamma^0\gamma_5+\gamma_5\gamma^0)\gamma^i\} \Rightarrow$$

$$[H,\gamma_5] = 0$$

If you try adding a mass term you get :

$$[H,\gamma_5] = [\vec{\alpha}\cdot\vec{p}+m\gamma^0,\gamma_5] = m(\gamma^0\gamma_5-\gamma_5\gamma^0)=-2m\gamma_5\gamma^0$$

Exercise 2: Show explicitly that for massless fermions the chirality and the Dirac hamiltonian have a common set of eigenfunctions which is expected because they commute.

Solution: This is easy to show: The eigenvectors of γ_5 are $\Psi^{\pm} = C\begin{pmatrix}1\\ \pm 1\end{pmatrix}$ with eigenvalues ± 1 respectively. Consider the positive energy solutions of the Dirac Equation:

$$\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix}1\\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\end{pmatrix}\chi^s e^{-ipx}$$

If the mass is zero we have that:

$$\Psi^{(s)}(x) = \sqrt{E+M}\begin{pmatrix}1\\ \vec{\sigma}\cdot\hat{p}\end{pmatrix}\chi^s e^{-ipx} = \sqrt{E+M}\begin{pmatrix}1\\ \pm 1\end{pmatrix}\chi^s e^{-ipx}$$

which is obviously an eigenfunction of the chirality operator.

Exercise 3: Show that for massless fermions if Ψ is a solution of the Dirac equation then $\gamma_5\Psi$ is also a solution of the same equation.

Solution: The Dirac equation,

$$[i\gamma^\mu\partial_\mu - m]\Psi(x) = 0$$

for massless fermions becomes:

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = 0.$$

Using $\{\gamma_{5,}\gamma^{\mu}\} = 0$ we get:

$$i\gamma^{\mu}\partial_{\mu}(\gamma_5\Psi(x)) = 0$$

Associating Chirality with an observable:

Lets now investigate the physical meaning of the chirality. Consider the massless Dirac equation:

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = 0$$

Let $\Psi(x) = u(\vec{p})e^{-ipx}$ be a solution of the Dirac equation. By substituting we get that:

$$(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{p})u(\vec{p}) = 0 \Rightarrow$$

$$\gamma^0 p_0 u(\vec{p}) = \vec{\gamma}\cdot\vec{p}\,u(\vec{p}) \Rightarrow$$

$$\gamma_5\gamma^0\gamma^0 p_0 u(\vec{p}) = \gamma_5\gamma^0\vec{\gamma}\cdot\vec{p}\,u(\vec{p}) \Rightarrow$$

$$p_0\gamma_5 u(\vec{p}) = \gamma_5\gamma^0\vec{\gamma}\cdot\vec{p}\,u(\vec{p}) \tag{9.3}$$

If this is a positive energy solution then we have that $p^0 > 0$ and (3) becomes:

$$\gamma_5 u(\vec{p}) = \gamma_5\gamma^0\vec{\gamma}\cdot\hat{p}\,u(\vec{p}) \tag{9.44}$$

If this is a negative energy solution then, $p^0 < 0$ and

$$\gamma_5 u(\vec{p}) = -\gamma_5\gamma^0\vec{\gamma}\cdot\hat{p}\,u(\vec{p}) \tag{9.5}$$

Lets compute the matrix product on the right side:

$$\gamma_5\gamma^0\vec{\gamma} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} = \vec{\Sigma} \Rightarrow$$

$$\vec{\Sigma} = \gamma_5\gamma^0\vec{\gamma} \tag{9.6}$$

and **this is the definition of the spin operator** in terms of the gamma matrices valid in any representation. From (9.4) (9.5) and (9.6) we have that for:

$$p^0 > 0 \quad \Rightarrow \quad \gamma_5 u(\vec{p}) = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix} u(\vec{p}) = \vec{\Sigma}\cdot\hat{p}\, u(\vec{p}) \tag{9.7}$$

and

$$p^0 < 0 \quad \Rightarrow \quad \gamma_5 u(\vec{p}) = -\begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix} u(\vec{p}) = -\vec{\Sigma}\cdot\hat{p}\, u(\vec{p}) \tag{9.8}$$

Using (9.7) and the fact that the Dirac spinors are eigenvectors of the helicity operator i.e.

$$[\vec{\Sigma}\cdot\hat{p}]u(\vec{p}) = \pm u(\vec{p})$$

we conclude that when acting on <u>positive energy solutions</u> the operators:

$$P_L = \frac{(1-\gamma_5)}{2} = \frac{(1-\vec{\Sigma}\cdot\hat{p})}{2}$$

and

$$P_R = \frac{(1+\gamma_5)}{2} = \frac{(1+\vec{\Sigma}\cdot\hat{p})}{2}$$

project to negative and positive helicity states respectively. Equivalently, using (9.8), when the above operators act on <u>negative energy solutions</u> they project to positive and negative helicity states respectively. Hence, we have the physical interpretation for the chirality operator: **The chirality or handedness is the same as the helicity operator (up to a sign) when the particle mass is zero or it can be neglected.** The operators P_L and P_R are commonly referred as left handed and right handed projection operators.

Projection Operator Summary:

In general if Ψ^+ is a positive energy spinor and Ψ^- is a negative energy spinor we have that:

$$\frac{1+\gamma_5}{2}\Psi^\pm = \pm\frac{N}{2}\begin{pmatrix} +1 \\ +1 \end{pmatrix}(1\pm\vec{\sigma}\cdot\hat{p})\chi^s$$

$$\frac{1-\gamma_5}{2}\Psi^\pm = \pm\frac{N}{2}\begin{pmatrix} +1 \\ -1 \end{pmatrix}(1\mp\vec{\sigma}\cdot\hat{p})\chi^s$$

or alternatively

$$\frac{1\pm\gamma_5}{2}\Psi^{+} = +\frac{N}{2}\begin{pmatrix}+1\\+1\end{pmatrix}(1\pm\vec{\sigma}\cdot\hat{p})\chi^{s}$$

$$\frac{1\pm\gamma_5}{2}\Psi^{-} = -\frac{N}{2}\begin{pmatrix}+1\\\pm1\end{pmatrix}(1\mp\vec{\sigma}\cdot\hat{p})\chi^{s}$$

As before $\chi^{s} = \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ for **s = 0, 1**.

<u>Conclusions on Chirality or Handedness:</u>

For **massless** particles the chirality or handedness operator has the following properties:

(a) **It is Lorentz invariant** (it is a constant matrix).

(b) **It is conserved.**

(c) **It has a common set of eigenvectors with the Dirac Hamiltonian.**

(d) **It has the same properties with the Helicity operator, which gives it a physical meaning.**

Helicity and Chirality for massive particles:

So far we considered chirality/handedness for massless fermions. However, the 'chirality properties' of massive fermions are also of interest eventhough, in this case, chirality is not a good quantum number,. The reason for this is that most particles are massive and the charged current weak interaction couples to <u>left handed</u> (negative chirality) spinors (V-A theory). Hence, we need a way to associate these left handed spinors with positive an negative helicity states.

Consider the identity:

$$1-\frac{\vec{\sigma}\vec{p}}{E+M} = \frac{1}{2}\left(1-\frac{|\vec{p}|}{E+M}\right)(1+\vec{\sigma}\cdot\hat{p})+\frac{1}{2}\left(1+\frac{|\vec{p}|}{E+M}\right)(1-\vec{\sigma}\cdot\hat{p}) \qquad (9.9)$$

where E, M, $\vec{p}$ are the energy, mass and momentum of a fermion respectively.

Next consider a left handed operator acting on a positive energy Dirac solution:

$$\Psi_L = \frac{(1-\gamma_5)}{2}\Psi(x) = \frac{N}{2}\begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+M} \end{pmatrix}\chi^s e^{-ip^\mu x_\mu} \Rightarrow$$

$$\Psi_L = \frac{N}{2}\begin{pmatrix} +1 \\ -1 \end{pmatrix}\left(1-\frac{\vec{\sigma}\cdot\vec{p}}{E+M}\right)\chi^s e^{-ip^\mu x_\mu} \quad \text{and using (9.9) we get that:}$$

$$\Psi_L = \frac{N}{2}\begin{pmatrix} +1 \\ -1 \end{pmatrix}\left[\frac{1}{2}(1-\frac{|\vec{p}|}{E+M})(1+\vec{\sigma}\cdot\hat{p})+\frac{1}{2}(1+\frac{|\vec{p}|}{E+M})(1-\vec{\sigma}\cdot\hat{p})\right]\chi^s e^{-ip^\mu x_\mu} \quad (9.10)$$

The first term in (9.10) projects to positive helicity states and the second term to negative helicity states. However, the coefficient of the positive helicity term vanishes at high energies where the particle mass can be neglected while the coefficient of the negative helicity term approaches the value of one at high energies.

One can show that at energies much larger than the particle mass these coefficients become:

$$1-\frac{|\vec{p}|}{E+M}=1-\frac{\sqrt{E^2-M^2}}{(E+M)}=1-\frac{\sqrt{1-M^2/E^2}}{(1+M/E)}=1-\left(1-\frac{M^2}{2E^2}+..\right)\left(1-\frac{M}{E}+..\right) \Rightarrow$$

$$1-\frac{|\vec{p}|}{E+M} \approx \frac{M}{E} \quad \text{(order of } \mathbf{M/E}\text{).}$$

and

$$1+\frac{|\vec{p}|}{E+M}=1+\frac{\sqrt{E^2-M^2}}{(E+M)}=1+\frac{\sqrt{1-M^2/E^2}}{(1+M/E)}=1+\left(1-\frac{M^2}{2E^2}+..\right)\left(1-\frac{M}{E}+..\right) \Rightarrow$$

$$1+\frac{|\vec{p}|}{E+M} \approx 2-\frac{M}{E} \quad \text{to order of } \mathbf{M/E.}$$

Hence,

$$\Psi_L \approx \frac{N}{2}\begin{pmatrix} +1 \\ -1 \end{pmatrix}\left[\left(\frac{M}{2E}\right)(1+\vec{\sigma}\cdot\hat{p})+\left(1-\frac{M}{2E}\right)(1-\vec{\sigma}\cdot\hat{p})\right]\chi^s e^{-ip^\mu x_\mu}$$

Therefore the left handed operator acting on positive energy states of the Dirac equation gives:

$$\Psi_L = \frac{(1-\gamma_5)}{2}\Psi(x) \approx \frac{N}{2}\begin{pmatrix} +1 \\ -1 \end{pmatrix}\left[\left(\frac{M}{2E}\right)(1+\vec{\sigma}\cdot\hat{p})+\left(1-\frac{M}{2E}\right)(1-\vec{\sigma}\cdot\hat{p})\right]\chi^s e^{-ip^\mu x_\mu}$$

As seen here the left handed positive energy spinor has contributions from both positive and negative helicity components. However the negative helicity component is dominant and becomes 100% in the case where the mass is much smaller than the energy and can be neglected. The positive helicity component decreases $\sim \boldsymbol{M/E}$ and approaches zero as the energy increases.

The Measurement of the Neutrino Helicity – The Goldhaber, Grodzins and Sunyar Experiment:

One of the most clever experiments in the history of physics was designed by Goldhaber, Grodzins and Sunyar[1] in 1957 to measured the helicity of the electron neutrino:

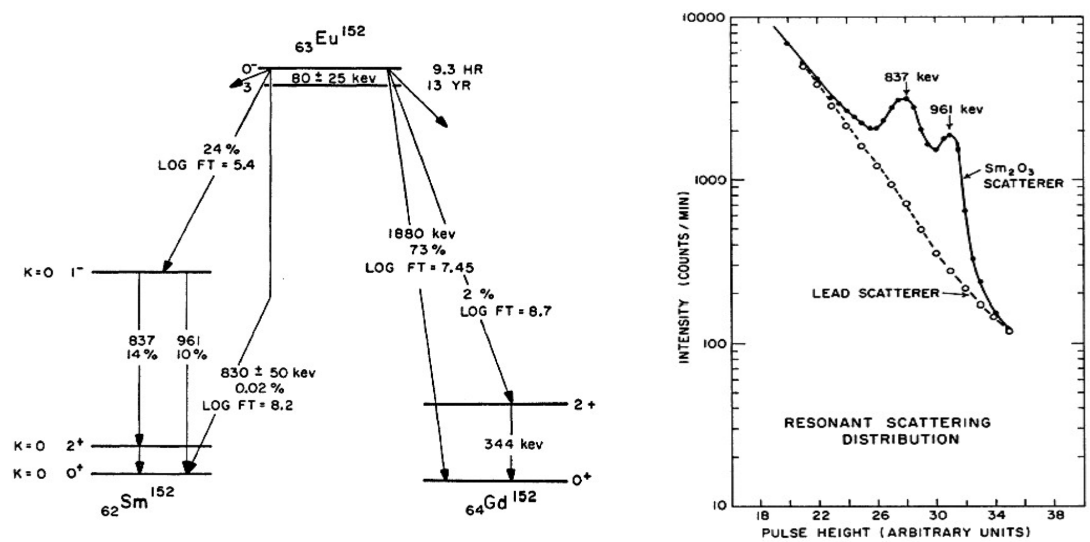


Figure 1*: Shown at the left is the decay chain of $_{63}Eu^{152}$ which is a 0^- state and undergoes electron capture to give first $_{62}Sm^{152*}$ and a 840 KeV electron neutrino. $_{62}Sm^{152*}$ is an 1^- excited state of $_{62}Sm^{152}$. $_{62}Sm^{152*}$ subsequently decays 10% of the time to a photon (961 KeV) and $_{62}Sm^{152}$ which is a 0^+ state and 14% of the time to a photon (837 KeV) and a 2^+ state which then decays to $_{62}Sm^{152}$.*

The experiment used the decay chain of the element Europium, $_{63}$Eu^{152} which had been studied earlier by L. Grodzins[2]. The decay chain is shown in Fig. 1 (left) and the resulting gamma-ray spectrum from the $_{62}$Sm^{*152} decay is shown in Fig. 1 (right). The initial nucleus of Europium is an $\mathbf{0^-}$ state which undergoes electron capture (charged current interaction) to give an excited Sm* nucleus which is a $\mathbf{1^-}$ and a neutrino. Since the Europium is a spin-zero state the sum of the Samarium and neutrino spins must be equal to the spin of the electron which was captured by the Europium nucleus as shown in Fig. 2.

1 M. Goldhaber, L. Grodzins, A.W. Sunyar, Phys. Rev. 109, 1014, (1958).
2 L. Grodzins, Phys. Rev. 109, 1014, (1958).

Hence, the polarization of the neutrino (left/right handed) is always the same as the polarization of the Sm*. In other words measuring the Sm* polarization is one and the same thing as measuring the neutrino polarization.

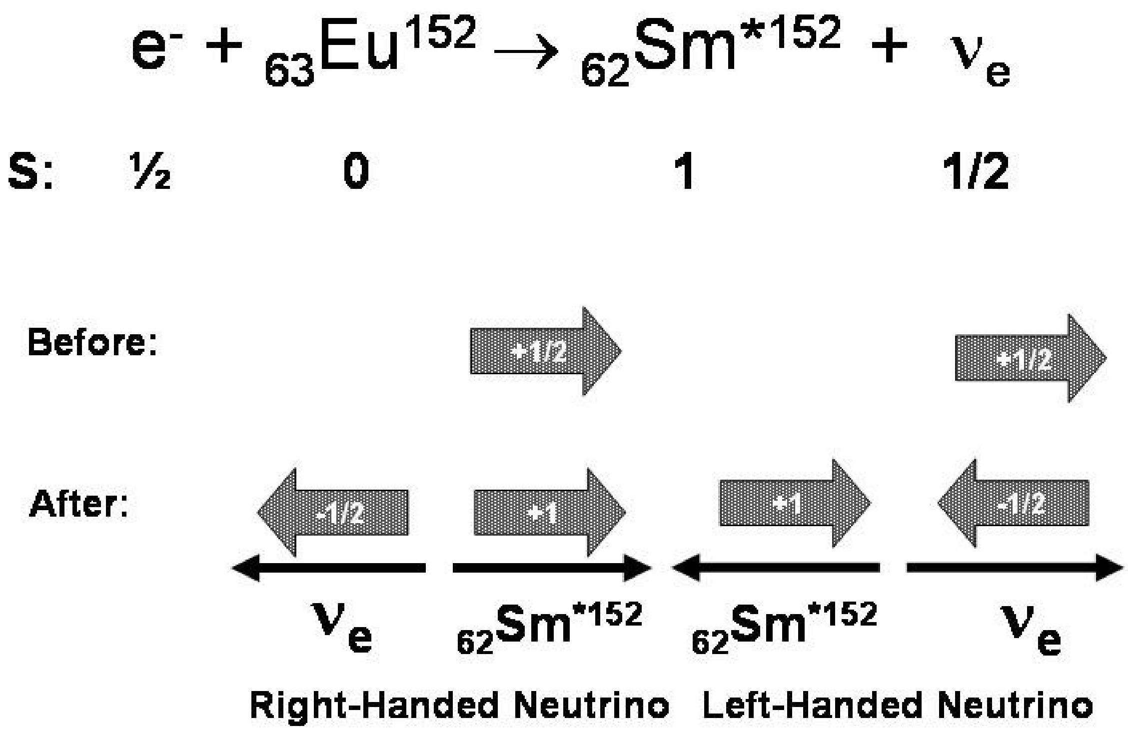


Figure 2: *Europium, $_{63}Eu^{152}$, undergoing electron capture from the K-Shell. The z-component of the electron angular momentum is ½ before the capture. Therefore the z-component of the total angular momentum should also be ½ after the capture. Hence, the spins of the $_{63}Sm^{152\,*}$ and the neutrino should always be opposite so that the result is always 1/2. However, the neutrino and $_{63}Sm^{152}$ move in opposite directions to conserve momentum. Therefore the neutrino and $_{63}Sm^{152\,*}$ polarizations must always be the same.*

However, measuring the $_{62}Sm^{*152}$ polarization seems a hard thing to do also until one realizes that it decays electromagnetically to $_{62}Sm^{152}$, a spin-zero state, and a photon as shown in Fig. 3. Conservation of angular momentum requires that the photon spin points to the same direction as the $_{62}Sm^{*152}$ spin (they are both $\mathbf{1}^-$ states). Hence, if the photon is emitted at the direction of motion of the $_{62}Sm^{*152}$ it will have the same polarization as the $_{62}Sm^{*152}$. **Therefore, photons emitted at the direction of $_{62}Sm^{*152}$ have the same polarization/helicity as the electron neutrinos** (up to a factor of ½) **. The experimental strategy was to select only those photons which are emitted at the direction of the $_{62}Sm^{*152}$ whose polarization is directly correlated to the electron neutrino polarization and measure their polarization.**

Goldhaber, Grodzins and Sunyar setup their experiment to measure two quantities: The photon direction relative to the $_2Sm^{*152}$ direction as well as the photon polarization. Both measurements rested on deep understanding of two physical processes: Compton Scattering and Resonant Scattering of gamma-ray photons.

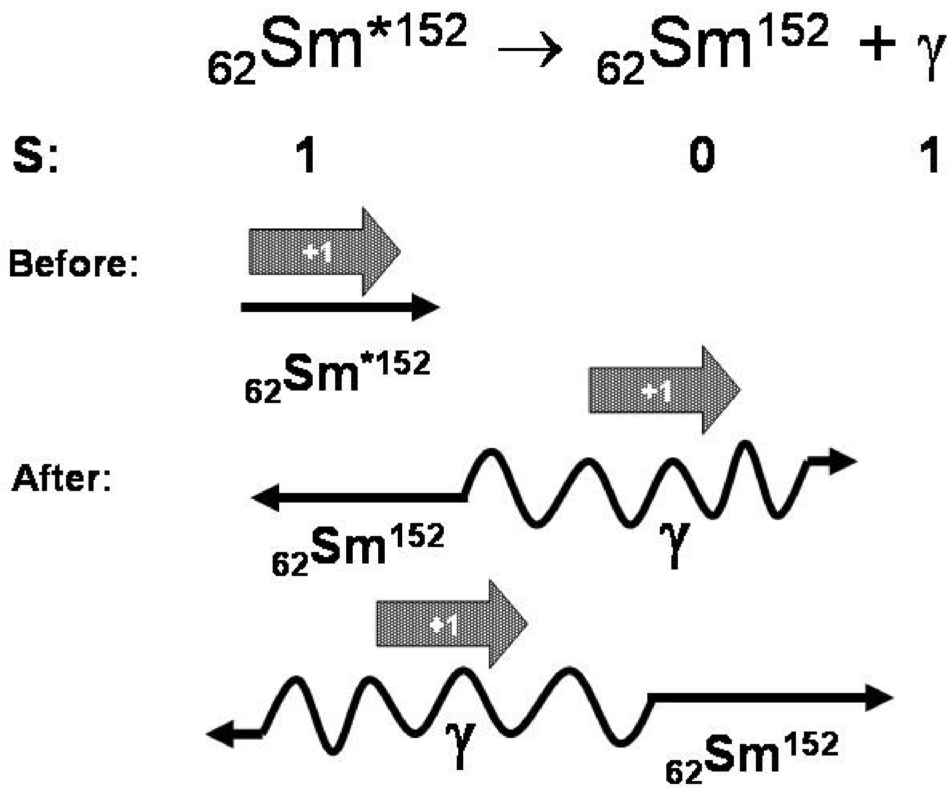


Figure 3: *The excited $_{62}Sm^{*152}$ state decays to Samarium and a photon. The photon spin is always the same as the direction of the Samarium spin to conserve angular momentum regardless which direction the photon is emitted (forward backward).*

The apparatus of Goldhaber, Grodzins and Sunyar is shown in Fig. 4 (left). The Europium source was placed in an analyzing magnet. The magnetic field was used to align the spins of the atomic electrons in the magnet material at a certain direction (up/down). In the magnet the $_{63}Eu^{152}$ nuclei decay according to the chain described in Fig. 1 (left) resulting to $_{62}Sm^{152}$ (scalar) and a photon (polar-vector). The polarization of the outgoing photon from the Samarium decay was measured using the properties of gamma-ray ***Compton Scattering*** by polarized atomic electrons which were previously studied by L. Page[3]. According to Pages's studies, which were both theoretical and experimental, the gamma-ray photons from the $_{62}Sm^{*152}$ decay interact with the atomic

[3] S.B Gunst and L.A. Page, Phys. Rev. 92, p970, Nov. 1953. L. A. Page, Phys. Rev. 106, p394, May 1957.

electrons differently depending upon the polarization of the atomic electrons in the magnet. As seen in Fig. 5, if the photon spin and the electron spin are at the same direction the photon cannot change the electron spin. Hence, it cannot be absorbed. If the electron spin is opposite to that of the photon then the photon can be absorbed to change the electron spin by one unit (½ -1 = - ½). In other words:

- **The photons which have their spins aligned with the spin of the atomic electrons will penetrate the magnet material and will exit from the other side.**

- **The photons which have their spins aligned opposite to the spin of the atomic electrons will be absorbed by the atomic electrons in the magnet and will not exit the magnet.**

Hence, by controlling the magnetic field one can chose the polarization of the outgoing photons.

To measure the direction of the photon relative to the Sm* motion they used a phenomenon called ***Resonant Scattering*[4]*:*** Consider a gamma-ray photon emitted from the electromagnetic nuclear transition of the ${}_{62}Sm^{*152}$ nucleus. In the reference frame of the emitting nucleus (${}_{62}Sm^{*152}$) the photon has a fixed energy which corresponds to the difference between the energies of the final and the initial states (up to a natural width).

However in computing the photon energy relative to the Lab frame one has to take in to account two phenomena:

1) As the ${}_{62}Sm^{*152}$ disintegrates to a ${}_{62}Sm^{152}$ ground state and a photon, part of the initial energy, $E_\gamma^2/2Mc^2$ is transferred to the ${}_{62}Sm^{152}$ nucleus as recoil energy (E_γ, M is the photon energy and the ${}_{62}Sm^{152}$ mass). Hence, the photon appears to have lower energy at the Lab frame.

2) The ${}_{62}Sm^{*152}$ nuclei are not at rest because they are the products of the electron capture reaction and they are recoiling against the neutrino which is commonly refereed to as *preceding radiation.* The motion of the parent, ${}_{62}Sm^{*152}$, nucleus will Doppler-shift the photon energy in a way that is of course depended upon the photon direction relative to the ${}_{62}Sm^{*152}$ direction.

The phenomenon of Resonant Scattering occurs if the gamma-rays emitted can be reabsorbed by ${}_{62}Sm^{152}$ to give ${}_{62}Sm^{*152*}$. Hence, the name Resonant Scattering. Obviously for this to happen we must have that the photon energy in the lab frame, where the ${}_{62}Sm^{152}$ absorber is located, should be equal to the quantum transition energy which in this case is **961 KeV**. This can only happen of the recoil effect described in (1) can be cancelled by the Doppler effect described in (2). Since the ${}_{62}Sm^{152}$ mass is known and the photon energy is **961 KeV** one can calculate that the recoil energy going to the ${}_{62}Sm^{152}$

[4] F.R. Metzger, Phys. Rev. 101, p286, Jan. 1956.

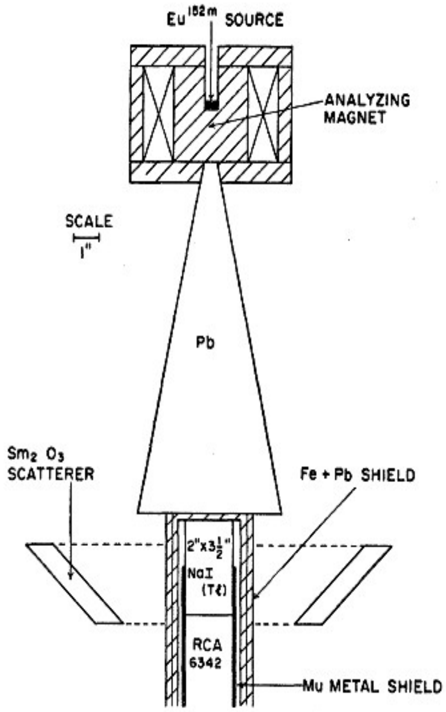
nucleus which is just **3.2 eV.** However small this is , it is much larger than the natural width of the transition which is only **0.023 eV.**

Hence, the photon recoil energy alone is enough to shift the photon energy such that no resonant scattering occurs. However, if the preceding radiation has enough energy, it can 'correct' this by transferring some of the neutrino-recoil momentum of the ${}_{62}Sm^{*152*}$ to the photon via Doppler. **The effect is maximum when the photon is emitted at the direction of the ${}_{62}Sm^{*152}$ motion.** The reader now should see another reason that this particular decay chain was selected: The **840 KeV** neutrino energy from the electron capture is comparable to the **961 KeV** photon energy. This is typical and a required condition for all decay chains which exhibit resonant scattering. Namely that the preceding radiation should have enough energy to be able to 'kick-back' the photon into resonance.

The photons exiting the magnet were directed to a ring where they could undergo resonant scattering if the conditions above were satisfied. Those that the did were measured using a NaI crystal attached to a phototube. Observing resonant scattering in this experiment means that the photons which were emitted at the ${}_{62}Sm^{*152*}$ direction have indeed been selected. Their polarization can be inferred by the direction of the magnetic field. Data runs were taken with the magnetic field pointing both up and down. As seen in Fig. 4 they did observe resonant scattering which indicated that their experiment selected mostly the photons whose polarization was correlated with the neutrino polarization. The number of scatterings with the magnetic field up (N_+) and down (N_-)were measured and compared. If the weak interaction produced electron neutrinos of left and right handed spices in equal numbers, the difference between the up and down scatters should have been zero. Instead they measured a non zero result. They observed more scatters when the magnetic field was pointing upwards:

$$\frac{N_+ - N_-}{\frac{1}{2}\left(N_+ + N_-\right)} = \mathbf{0.017 \pm 0.003}$$

The magnet had a length of about 3 mean free paths, from which using the Compton scattering cross section they could conclude that $68 \pm 14\%$ of the photons were polarized and that their helicity was negative. **They used this result to conclude that the electron neutrino helicity is also negative.**



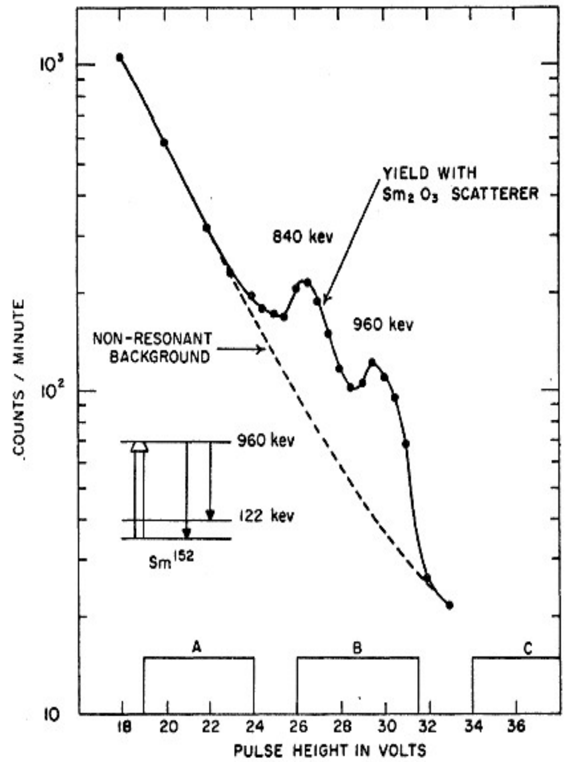


Figure 4: *Shown at the left is the apparatus of Goldhaber, Grodzins and Sunyar. The Europium source shown at the top in the analyzing magnet. The gamma-rays from the decay of the exited Sm* are scattered by the Samarium oxide ring provided that they are emitted at the direction of motion of the Sm* atom. The photons are detected by a NaI crystal using an RCA phototube. The photon spectrum that this experiment observed is shown at the right and this gives the proof that the experiment detected photons moving at the direction of the Sm* atom. The observation of this spectrum along with information on the direction of the magnetic field gave proof that the neutrino has negative helicity.*

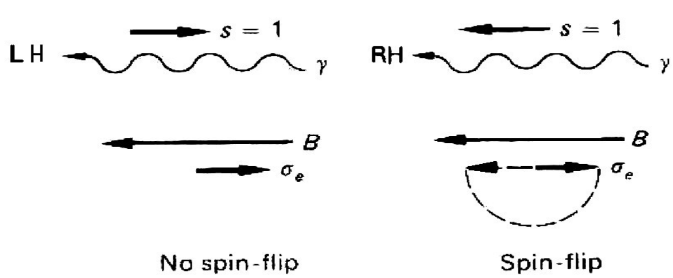


Figure 5: *Compton scattering by polarized atomic electrons. The case where the photon cannot be absorbed is shown at the left and the case where the photon can be absorbed by flipping the spin is shown to the right.*

Right-handed Neutrinos and Neutrino Mass

To this day no one has ever observed right-handed neutrinos (or left-handed antineutrinos). This could mean either that they don't exist at all or that they do exist but they are very heavy to be observed in current experiments or that they don't couple via the weak interaction. Introducing a hypothetical heavy right-handed neutrino would require a theoretical extension of the Standard Model since introducing neutrino mass using a simple Dirac mass term[5] like we do for the other massive leptons and quarks

$$m\bar{\Psi}\Psi = m\bar{\Psi}_L\Psi_R + m\bar{\Psi}_R\Psi_L \tag{9.11}$$

requires that the left-handed and right-handed particles have the same mass and this contradicts the fact the the left-handed partner is very light. Hence, the hypothesis of a heavy right-handed neutrino and a light left-handed neutrino requires the introduction of a new mass term which is different than the usual Dirac mass term as well as a new mechanism which results to this mass difference. However, if one wish to stick to the Dirac mass term as shown in (9.11) then the result would be that the right-handed neutrino has the same mass as the left-handed neutrino. In this case, the Standard Model symmetries dictate that the neutrino would not couple via the week interaction so it would be a **sterile neutrino** which would couple only via its mass to the gravitational field and threfore remains unobserved.

Within the context of the Standard Model, the fact that the observed neutrinos come only as left handed particles (and the anti-neutrinos as right-handed particles) is directly related to the Vector-minus-Axial-vector (V-A) nature of the weak current, whose direct consequence is that the weak interaction violates parity[6]:

$$J^{\mu}_{WEAK-CC} = \bar{\Psi}(l^-)\gamma^{\mu}(1-\gamma_5)\Psi(\nu_l) \quad , \quad J^{\mu}_{WEAK-NC} = \bar{\Psi}(\nu_l)\gamma^{\mu}(1-\gamma_5)\Psi(\nu_l)$$

Another way of seeing this is to notice that parity transforms left-handed particle spinors to a right-handed particle spinors. Hence, if an interaction is invariant under parity we should have in nature equal numbers of left-handed and right-handed particles. This is exactly what is observed for photons. The electromagnetic interaction is invariant under parity and this implies that we should have in nature equal numbers of left-handed and right-handed photons. It is well known that photons are transverse plane waves which means that we do indeed have equal numbers of left handed and right handed photons. On the contrary the fact that we don't have equal numbers of right-handed and left-handed neutrinos implies that parity is violated by the interaction which produces them, the weak interaction. One can say that the fact that we don't seem to have right-handed neutrinos at all implies that parity is violated maximally in the weak interaction.

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[5] The Dirac mass term will be discussed in Lecture 12.

[6] It is easy to show that the $(1-\gamma_5)$ term violates parity.