Lecture 10

Particle Physics, 4^{th} year undergraduate, University of Ioannina

Charge Conjugation and Anti-Particles

Prof. Costas Foudas 07/01/23

We still have not confronted the issue of what to do with the negative energy solutions of the Dirac equation. These solutions, as we have seen, correspond to positive probability. Therefore one is entitled to ask why are they there and if they actually represent something. Surely we cannot just ignore them. They are part of the Hilbert space defined by the eigenfunctions of the Dirac Hamiltonian and any spin-half fermion state can be expressed as a superposition of these eigenfunctions. Furthermore, shortly after Dirac published his equation, J. R. Oppenheimer computed the decay rate for particles in the positive energy levels to decay to negative energy levels and the rate was quite high. This means that all particles occupying the positive energy solutions could decay to the negative energy solutions very fast. Since the negative energy solutions extend to negative infinity, the same away as the positive solutions extend to positive infinity, it means that our world, us included, would decay rather fast to oblivion[1]. This clearly just does not happen. Hence, Dirac’s theory had a serious problem. Another problem can be seen by using the formula for relativistic energy:

$$E=\pm\sqrt{P^2+M^2}$$

If the energy is negative one comes to the conclusion that the more the particle decays and loses energy the higher its momentum becomes, which is of course nonsenses[2].

Dirac tried to solve these problems by proposing his Hole Theory. **He claimed that the negative energy levels must be fully occupied by electrons which have negative energy.** Electrons are identical spin-half fermions. According to Pauli’s exclusion principle only two electrons can possibly occupy a single energy state, one with spin up and the second with spin down. Adding a third electron would force it to have the same quantum numbers as one of the two already there and this is not allowed by Pauli's exclusion principle.

Hence, Dirac's claim explained the absence of decays from the positive to negative energy states and allowed him to make some predictions:

[1] In a model with just fermions this would not happen. However, as soon as one includes photons in the theory which couple to electrons, the electrons can emit photons and decay.

[2] Theorists are more tactful and call these run-away solutions.

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I. When a photon strikes a negative energy energy spin-up electron as seen in Fig. 1 (left) it gives it some energy and excites it to a positive energy state. The negative energy level, where the electron previously was, is left unoccupied and this is what Dirac referred to as hole. This hole represents the absence of a negative energy negative charge spin-up object. **Hence, the hole behaves as a positive energy, positive charge, spin-down particle**. This is the equivalent of the electron-positron pair creation process. This way Dirac predicted that **for very known charged spin-half fermion there must exist another spin-half fermion of opposite charge which is represented by a hole**. One can generalize this argument to read that for every spin-half fermion we need another spin-half fermion with opposite quantum numbers. These, hypothetical at the time, particles were called anti-particles.

II. Alternatively if a positive energy electron decays to an unoccupied negative energy level by emitting a photon and fills in a hole as shown in Fig. 1 (right) this represents the process of an electron positron pair annihilation to a photon.

III. Since particle and anti-particle are solutions of the same Dirac equation they must have the same mass.

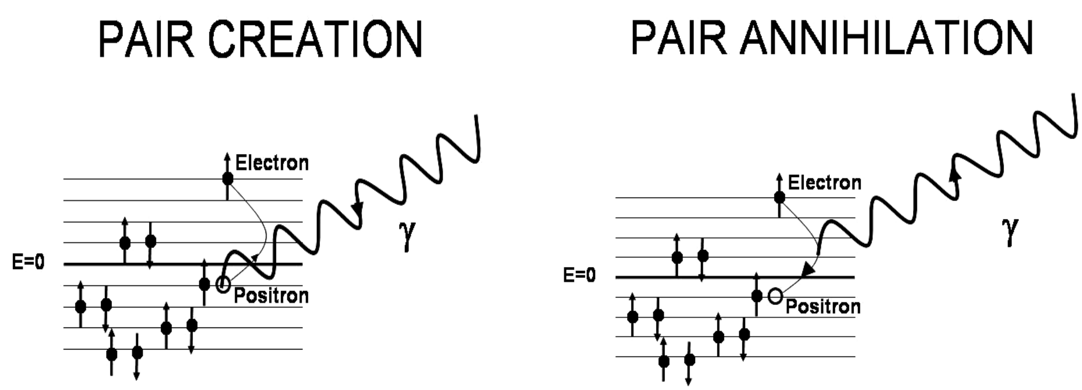

Figure 1: *Pair Creation (left) and pair annihilation (right) processes according to Dirac's Hole Theory.*

Summary of Dirac Hole Theory:

I. All negative energy levels predicted by the Dirac equation are fully occupied and no particle can decay to them.

II. Only positive energy particles exist in nature and these are either positive energy electrons or positive energy holes.

III.For every positive energy particle in nature we have a positive energy anti-particle, described by the a Hole, which is of equal mass and opposite quantum numbers.

The Discovery of the Positron:

It was not long after Dirac proposed his Hole Theory that Anderson[3] at CALTECH discovered the positron using a cloud chamber. The cloud chamber was immersed in a magentic field to enable momentum measurements and was instrumented with a **6 mm** lead sheet to identify the direction of the charge particle tracks. In **1300** exposures of his cloud chamber he found **15 tracks** corresponding to positively charged particles and the question was what were these particles. Where they protons (the only positively charged particle known at the time) or were they some kind of a new particle which had not seen before ?

One of C. Anderson's positron tracks is shown in Fig. 2. The upper part of the track has evidently less momentum than the lower part because it bends more in the magnetic field of the chamber. Hence, the particle in the picture is moving upwards because one would expect it to lose energy as it goes through the **6 mm** lead sheet which is the reason that the lead sheet was put there in the fist place. The magnitude and direction of the magnetic field was experimentally controlled. Hence, once the direction of the particle was known Anderson could easily conclude that it was a positively charged particle and could compute the momenta before entering the lead sheet and after exiting it.

<u>The obvious question Anderson had to answer first was whether this particle was a proton</u>. One can test this hypothesis using the theory of energy loss due to ionization. The idea is that if we can measure the momentum, which we can, then we can convert this to velocity by assuming the particle mass. If we know the velocity then we know the energy loss per unit length due to ionization and we can tell how far the particle should travel before it stops.

Using Andreson's picture and a ruler one can measure distances on the picture and can convert them to path-lengths in centimeters because it is given that the lead-sheet is 6mm thick. This way we can measure the radius of the upper track and from this to get a measure for the track momentum. Here is the calculation: The radius of the upper track is **R = 0.05 m** and the magnetic field was **B = 15000 Gauss**. Hence, assuming that we are dealing with a proton which has a unit charge we arrive to the conclusion that the momentum of the upper part of the track is:

$$\boldsymbol{p_0(GeV/c) = 0.3 \times B[Tesla] \times R[meters] = 0.3 \times 1.5 \times 0.05\,GeV \approx 23\,MeV/c}$$

[3] C. Anderson, Phys. Rev. 43, p491, March 1933.

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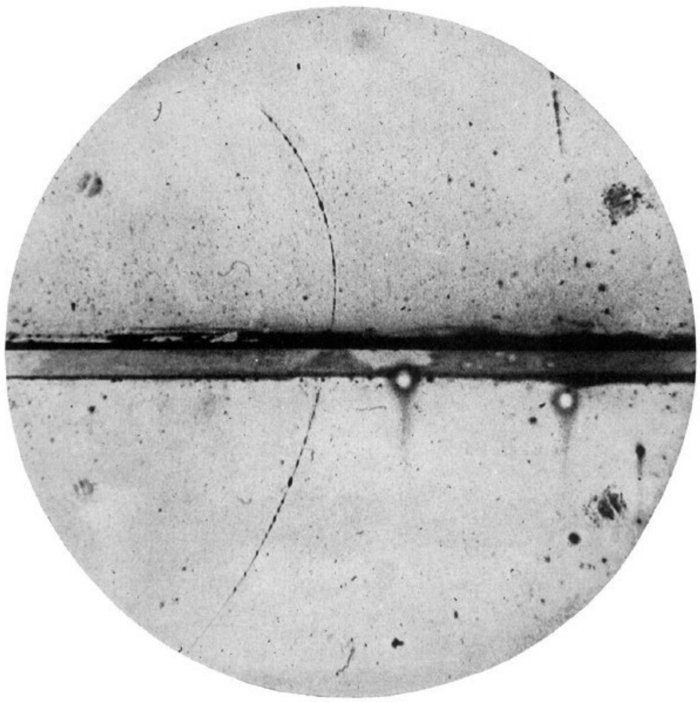

FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

Figure 2: *A positron event as seen in Andreson's cloud chamber. The magnetic field points into the picture and the particle moves upwards because the upper track has less momentum than the lower part due to loses suffered in the 6 mm lead plate shown in the middle of the picture.*

A proton with this momentum will be highly ionizing because it will have:

$$\beta\gamma = \frac{23\, MeV}{938\, MeV} = 24 \times 10^{-3}$$

For this value of **βγ** the ionization plot in Lecture 4 gives

$$\frac{dE}{dx} \approx 200 \frac{MeV\, cm^2}{g}$$

which using the air density $d = 1.2\, 10^{-3}\, g/cm^3$ can be converted to:

$$\frac{dE}{dx} \approx 240 \frac{KeV}{cm}$$

Using the track momentum one can estimate the kinetic energy of the hypothetical proton and find that it is ***KE = 282 KeV***. This can be computed using also the non-relativistic formula because protons at these momenta are slow. Anderson estimated it to be ***300 KeV*** which is not very far from our value. Hence, such a proton could not travel more than

$$L \approx \frac{282}{240} cm = 1.2\, cm$$

Anderson in his paper claims this range to be **0.5 cm** derived from data at the same energy range[4] which is more than a factor of two smaller than the figure above and most likely closer to the truth than our rough estimate. However, it does not matter because a close look at the picture shows that t**he track length is at least 5 cm** and this is only the visible part of it since the particle exits the chamber at the end. **Hence, the energy loss of this particle is not consistent with that of a proton. The track is not a proton track.** This particle must be faster than a proton because because its ionization energy loss is consistent with a particle of smaller mass which would make **βγ** larger and result to lower ionization energy loss. Hence, the mass, ***m***, of the unknown particle must be smaller than a proton and its charge, *z*, must be below a certain value otherwise the ionization energy loss will be too large.

The posibility that this was in fact a chance coincidence two different tracks was also ruled out based on probability grounds. Tracks of such momentum were observed at a rate of 1/500. Hence, the probability of observing two such tracks would have been $(1/500)^2$ times the probability that the two independent tracks started from the same space point which is negligible.

Next, Anderson compared roughly the ioniation loss of the track (droplet density) and with the ionization loss from electrons of similar momentum. He found that the track was losing energy via ionization at a rate which was less than four times that of an electron track at similar energy:

[4] Rutherford, Chadwick and Ellis, *Radiations from radioactive Substances,* p294.

$$\left(\frac{dE}{dx}\right)_{track} < 4 \times \left(\frac{dE}{dx}\right)_{e^-}$$

Since the ionization energy loss depends upon the square of the charge of the incident particle

$$\frac{dE}{dx} \sim \frac{z^2}{\beta^2}$$

he concluded that **the charge of the observed particle must be less than twice he absolute value of the charge of the electron and most probably equal to that of the electron[5].**

Anderson also checked if the energy loss in the lead sheet is consistent with that of the electron: The radiation length in lead is $X_0 = 5.6\,mm$ and the momentum of the particle below (assuming unit charge) is **63 MeV** which can be computed from the track curvature (or read off from Anderson's paper). Hence we expect that the particle above will have a momentum equal to:

$$E(upper) = E(lower)e^{-\frac{6\,mm}{5.6\,mm}} = 0.343\times 63\,MeV = 21.6\,MeV$$

which is rather close to the measured value of **23 MeV**. Hence, the particle loses energy like a electron. From this we also conclude that the upper and lower tracks belong to the same particle and are not two different tracks which happened to pass from the same point which was excluded also from probability arguments.

Alternatively, if one assumed that the particle had unit charge, one could calculate the exected energy loss due to ionization using the momentum measurement for different assumptions about the mass of the particle. Anderson compared the different predictions for dE/dx, each for a different particle mass, with the measured energy loss of the track. However, this track exhibited very low energy loss due to ionization and this resulted only to an u**pper limit for the mass of the new particle of 20 times the mass of an electron.**

[5] Anderson was somewhat lucky that back then people knew only 3 elementary particles, the proton, the neutron and the electron. So anything positively charged and lighter than the proton must have been a discovered new particle. Today, or even a couple of decades later he would have had more choices to make and it is not clear that he would have made the correct one using this apparatus.

Hence, Anderson claimed discovery of a new particle of positive charge which had a mass which was less than twenty times the mass of the electron. He got the Nobel price for this in 1936. P. Blackett and G. Occhialini confirmed Anderson's results in the Spring of 1933. They used their well known technique of a cloud chamber triggered using Geiger-Miller counters and made many pictures of positrons.

Final Comment on Dirac's Hole Theory:

Dirac's theory was radical at its time not only because it predicted the antiparticles[6] but also because it introduced, for first time in particle physics, mechanisms for creation and annihilation of particles. Hence, Dirac introduced this way the first ideas for a Relativistic Quantum Field Theory. Starting at about this time along with Dirac, Pauli, Heisenberg and others developed Relativistic Quantum Field Theories in an attempt to describe processes involing particle scattering and particle creation and annihilation. The final triumph of Reletivistic Quantum Field Theories was the Standard Model of Particle Physics developed by S. Weinberg (Harvard), A. Salam (Imperial College), ans S. Glashaw (Harvard).

As it always happens in physics, despite its success in predicting the anti-particles, Dirac's Hole theory looked most unnatural. It is hard to believe that half of the world is made of an infinite number of fermions whose only job is to occupy the negative energy levels so no particle can decay into them. Even if one is willing to accept this there is another argument against it: Bosons also have antiparticles and the Hole theory does not apply to them because it is based on the Pauli exclusion principle which bosons do no obey. Hence, we cannot explain using the Hole theory why charged pions should come in pairs of opposite charge or how they are created or annihilated. Clearly we need a theory of antiparticles which applies equally well for both bosons and fermions.

Constructing Positron Spinors

In this section we shall give heuristic arguments for constructing positron spinors and thus give an alternative interpretation to the negative energy solutions of the Dirac equation which is very different than Dirac's Hole Theory. This interpretation is due to Stückelberg (1941) and Feynman (1948) and has the advantage that it applies to fermions

[6] These days we have another radical theory, the Supersymmetry (SUSY) which predicts that for every known fermion there is a bosonic partner with which has a spin which is only half a unit different than the original particle. Several searches at the LHC for SUSY particles have provided so far no evidence for their existence and many physicists believe that evidence for SUSY will never be found. In this sense Dirac was fortunate to have experimental evidence for his predictions about a year after he published which unfortunately is not the case for the SUSY predictions.

as well as bosons. Lets start from the positive energy electron solutions which we derived before

$$\Psi^{(s)}(x) = \sqrt{(E+M)}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\chi^{s}\,\mathrm{e}^{-ip\,x} \qquad s=1,\ 2, \qquad \chi^{1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi^{2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and compute the corresponding electron current which as we have seen in Lecture 6 is

$$J^{\mu}_{EM}(e^{-}) = -e\,\bar{\Psi}^{(s)}\gamma^{\mu}\Psi^{(s)} = -2\,e\,P^{\mu} \quad \text{with} \qquad e>0,\ \ P^{\mu}=(E\,;\vec{P}),\ \ E>0$$

Consider now a positron e^{+} which is a particle with the same mass and spin as the electron but has charge opposite to that of the electron. The positron must also be described by a positive energy solution of the Dirac equation and we can write down the positron current as

$$J^{\mu}_{EM}(e^{+}) = +e\,\bar{\Psi}\,\gamma^{\mu}\,\Psi = +2\,e\,P^{\mu} \ \text{again with}\ e>0,\ \ P^{\mu} = (E\,;\vec{P}),\ \ E>0$$

where $\mathbf{\Psi}$ is the spinor that describes the positron which we will construct later in this lecture. Notice that this current can be written as

$$J^{\mu}_{EM}(e^{+}) = +2\,e\,P^{\mu} = -2\,e(-P^{\mu}) \tag{10.1}$$

which represents an electron current with negative energy an momentum $(-E\,;-\vec{p})$. The interpretation of this is that **negative energy electron solutions of the Dirac equation describe positive energy positron solutions.**

This can be verified, as shown in Lecture 6, by substituting the negative energy electron solutions

$$\Psi^{(s+2)}(x) = \sqrt{(|E|+m)}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E-m} \\ 1 \end{pmatrix}\chi^{(s)}\,\mathrm{e}^{-ip\,x} \qquad\qquad E<0,\ \ s=1,2$$

into

$$J^{\mu}(e^{+}) = +e\,\bar{\Psi}^{(s+2)}\gamma^{\mu}\,\Psi^{(s+2)} = -2\,e\,P^{\mu} = 2\,e(-E\,;-\vec{p}) = 2\,e(|E|\,;-\vec{p}) \tag{10.2}$$

This 4-vector has positive energy as it should because it describes a physical particle. However, the momentum is inverted. Finally by comparing 10.1 with 10.2 we arrive at the conclusion that indeed **we can use negative energy solutions to describe a positron provided that we substitute $\vec{p} \to -\vec{p}$ and $E \to -E$.**

Furthermore, this can also be demonstrated explicitly starting from from the negative energy solutions and re-expressing them as follows:

$$\Psi^{(s+2)}(x) = \sqrt{(|E|+m)} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E-m} \\ 1 \end{pmatrix} \chi^s e^{-ipx} \quad \Rightarrow$$

$$\Psi^{(s+2)}(x) = \sqrt{(|E|+m)} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{-|E|-m} \\ 1 \end{pmatrix} \chi^s e^{-ip^0 x_0 + i\vec{p}\cdot\vec{x}} \quad \Rightarrow$$

$$\Psi^{(s+2)}(x) = \sqrt{(|E|+m)} \begin{pmatrix} \frac{\vec{\sigma}\cdot(-\vec{p})}{|E|+m} \\ 1 \end{pmatrix} \chi^s e^{-iEx^0 + i\vec{p}\cdot\vec{x}} \quad \Rightarrow$$

$$\Psi^{(s+2)}(x) = \sqrt{(|E|+m)} \begin{pmatrix} \frac{\vec{\sigma}\cdot(-\vec{p})}{|E|+m} \\ 1 \end{pmatrix} \chi^s e^{+i|E|x^0 - i(-\vec{p})\cdot\vec{x}}$$

In conclusion, we started with a negative energy solution of the Dirac equation and we end up with a solution which has positive energy and reversed 3-momentum. Indeed, if we reverse the momentum 4-vector this is identical with one of the positive energy solutions of Lecture 6. Hence, by reversing the momentum 4-vector of the negative energy solutions we obtain solutions which describe antiparticles with positive energy (physical particles).

Another observation is that we started with a negative energy particle solution, which moves backward in time, since $p^0 = E < 0$, and we found that this is equivalent to an antiparticle solution which has positive energy and moves forward in time because $p^0 = |E| > 0$. In other words, negative energy particle solutions going backwards in time describe antiparticle solutions which have positive energy and move forward in time.

This is shown diagrammatically in Figure 3.

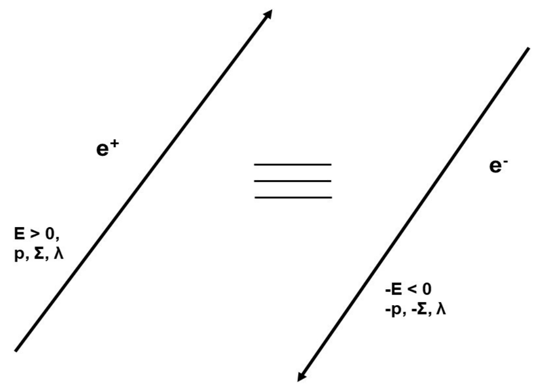

Figure 3: A p*ositive energy positron solution moving forward in time is equivalent to a negative energy electron solution moving backwards in time. E is the energy, p the momentum, Σ the spin and λ the helicity.*

Charge Conjugation

In the previous section we derived anti-particle solutions of the Dirac equation using heuristic arguments. Here we will derive them in a direct way which also demonstrates the charge conjugation symmetry of the electromagnetic interactions.

Consider the Dirac equation for an electron in the presence of an electromagnetic field[7].

$$[i\gamma^{\mu}(\partial_{\mu}-ie\,A_{\mu})-m]\Psi(x)=0 \qquad e>0 \qquad (10.3)$$

The corresponding equation for a positron in an electromagnetic field will be

$$[i\gamma^{\mu}(\partial_{\mu}+ie\,A_{\mu})-m]\Psi_{c}(x)=0 \qquad (10.4)$$

where $\Psi(x)$ are the electron spinor solutions which were derived in Lecture 6 and $\Psi_c(x)$ is the positron spinor which we will derive here in terms of $\Psi(x)$. We expect that this is true because the trajectory of a charged particle in an electromagnetic field should not change if one reverses the filed $(A_{\mu}\rightarrow -A_{\mu})$ and replaces the charged particle with its anti-particle $(\Psi(x)\rightarrow\Psi_c(x))$. This symmetry of the electromagnetic interactions is called **Charge Conjugation Symmetry.**

7 We will see why it takes this form in Lecture 12 when we discuss local gauge invariance.

So we start by assuming that

$$\boldsymbol{\Psi}_c = A\boldsymbol{\Psi}^* \quad \Rightarrow \quad \boldsymbol{\Psi}^* = A^{-1}\boldsymbol{\Psi}_c \tag{10.5}$$

and from (10.3) and (10.5) we have that

$$[-i\gamma^{\mu *}(\partial_\mu + ie A_\mu) - m] A^{-1}\boldsymbol{\Psi}_c(x) = 0 \quad \Rightarrow$$

$$[-i A\gamma^{\mu *} A^{-1}(\partial_\mu + ie A_\mu) - m]\boldsymbol{\Psi}_c(x) = 0 \tag{10.6}$$

By comparing 10.6 with 10.4 we conclude that

$$A \ \gamma^{\mu *} A^{-1} = -\gamma^\mu \quad \Rightarrow \tag{10.7}$$

$$A\gamma^\mu + \gamma^\mu A = 0 \qquad \mu = 0,\ 1,\ 3$$

and

$$A\gamma^\mu - \gamma^\mu A = 0 \qquad \mu = 2$$

This means that A anti-commutes with $\gamma^0,\ \gamma^1,\ \gamma^3$ and that A commutes with γ^2 . Hence, it must be that

$$A = i\gamma^2 \tag{10.8}$$

which means that

$$\boldsymbol{\Psi}_c = i\gamma^2\boldsymbol{\Psi}^*$$

In the literature the matrix $\boldsymbol{C}$ is used which is defined as

$$C = A\gamma^0 = i\gamma^2\gamma^0 \tag{10.9}$$

Hence,

$$\boldsymbol{\Psi}_c = C\gamma^0\boldsymbol{\Psi}^* = C\bar{\boldsymbol{\Psi}}^T$$

So $\boldsymbol{\Psi}_c$ is a positron spinor since we have demonstrated that the matrices **A, C** do exist.

Note that the exact form of the $\boldsymbol{A}$ and $\boldsymbol{C}$ matrices depends upon the representation of the gamma matrices that we use.

Exercise: Show that matrix $\boldsymbol{C}$ satisfies the relationship

$$\gamma^{\mu} = -C\gamma^{\mu T} C^{-1}$$

Solution:

Using 10.8 and 10.9 one can show that

$$A = A^{-1} = i\gamma^2 = C\gamma^0 \qquad (10.10)$$

Using 10.7 and 10.10 we get

$$A\gamma^{\mu *} = -\gamma^{\mu} A \Rightarrow -C\gamma^0((\gamma^{\mu})^{+})^T = \gamma^{\mu} C\gamma^0 \Rightarrow -C\gamma^0(\gamma^0\gamma^{\mu}\gamma^0)^T = \gamma^{\mu} C\gamma^0 \Rightarrow$$

$$-C\gamma^0\gamma^0(\gamma^{\mu})^T\gamma^0 = \gamma^{\mu} C\gamma^0 \Rightarrow \quad -C(\gamma^{\mu})^T = \gamma^{\mu} C \Rightarrow \quad -C(\gamma^{\mu})^T C^{-1} = \gamma^{\mu} \Rightarrow$$

$$-\gamma^{\mu} = C(\gamma^{\mu})^T C^{-1}$$

The following two exercises are left for homework.

Exercise: Show that the Pauli matrices satisfy $\sigma^2\vec{\sigma}^* = -\vec{\sigma}\sigma^2$ (10.11)

Exercise: Show that in the Pauli-Dirac representation, matrix C satisfies

$$C = -C^{-1} = -C^{+} = -C^{T} \qquad (10.12)$$

Next we will apply what we have learned and explicitly construct the charge conjugate spinor of the positive energy solution $\Psi^{(1)}(x)$ of the Dirac equation and we will show that

$$\Psi_c^{(1)}(x) = [\Psi^{(1)}(x)]_c = i\gamma^2[u^{(1)}(\vec{p})e^{-ipx}]^* = u^{(4)}(-\vec{p})e^{ipx} = v^{(1)}(\vec{p})e^{ipx}$$

So we start by taking the charge conjugate of $\Psi^{(1)}(x)$ which represents an electron with positive helicity.

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$$\Psi_c^{(1)} = i\begin{pmatrix} 0 & i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}\sqrt{E+M}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}^*\cdot\vec{p}}{E+M} \end{pmatrix}\chi^1 e^{i\,p\,x} \Rightarrow$$

$$\Psi_c^{(1)} = \sqrt{E+M}\begin{pmatrix} i\sigma^2\dfrac{\vec{\sigma}^*\cdot\vec{p}}{E+M} \\ -i\,\sigma^2 \end{pmatrix}\chi^1 e^{i\,p\,x} \Rightarrow$$

$$\Psi_c^{(1)} = \sqrt{E+M}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+M}(-i\sigma^2) \\ -i\,\sigma^2 \end{pmatrix}\chi^1 e^{i\,p\,x} \Rightarrow$$

$$\Psi_c^{(1)} = \sqrt{E+M}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+M} \\ 1 \end{pmatrix}(-i\sigma^2)\,\chi^1 e^{i\,p\,x} \Rightarrow$$

$$\Psi_c^{(1)} = \sqrt{E+M}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+M} \\ 1 \end{pmatrix}\chi^2 e^{i\,p\,x}$$

Hence, we have shown that

$$\Psi_c^{(1)} = i\gamma^2[\Psi^{(1)}]^* = u^{(4)}(-\vec{p})e^{i\,p\,x} = v^{(1)}(\vec{p})e^{i\,p\,x} = \sqrt{E+M}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+M} \\ 1 \end{pmatrix}\chi^2 e^{i\,p\,x}$$

which we recognize it to be a positive energy solution. Furthermore, as we already know $u^{(4)}(\vec{p})$ is a spinor with negative helicity. Therefore, $u^{(4)}(-\vec{p})$ must have positive helicity. **This means that $v^{(1)}(\vec{p})$ describes a positron with positive helicity.** So charge conjugation transformed an electron with positive helicity to a positron which also has positive helicity. Hence, **the charge conjugation operation does not alter helicity**.

Again, the same result can be obtained from the negative energy solution

$$\Psi^{(4)} = \sqrt{|E|+M}\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E-M}\\ 1\end{pmatrix}\chi^2 e^{i(-p)x} = \sqrt{|E|+M}\begin{pmatrix}\dfrac{\vec{\sigma}\cdot(-\vec{p})}{|E|+M}\\ 1\end{pmatrix}\chi^2 e^{i(-p)x}$$

by substituting $p^{\mu}\rightarrow -p^{\mu}$ and is consistent with what we had guessed in the previous section.

The same way one can show that

$$\Psi_c^{(2)} = i\gamma^2[\Psi^{(2)}]^* = -u^{(3)}(-\vec{p})e^{ipx} = v^{(2)}(\vec{p})e^{ipx} = \sqrt{E+M}\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+M}\\ 1\end{pmatrix}\chi^1 e^{ipx}$$

which can also be obtained from $\Psi^{(3)}$ by substituting $p^{\mu}\rightarrow -p^{\mu}$.

Next, we will study the helicity of the two positron spinors $v^1(\vec{p}),\ v^2(\vec{p})$ and prove our previous statement, namely that the first describes a right-handed positron and the second a left-handed positron. We know that

$$\vec{\Sigma}\cdot\hat{p}\ \ u^3(\vec{p}) = +u^3(\vec{p})$$

and

$$\vec{\Sigma}\cdot\hat{p}\ \ u^4(\vec{p}) = -u^4(\vec{p})$$

By substituting $\vec{p}\rightarrow -\vec{p}$ one gets

$$-\vec{\Sigma}\cdot\hat{p}\ \ u^3(-\vec{p}) = +u^3(-\vec{p})\ \Rightarrow\ \vec{\Sigma}\cdot\hat{p}\ \ v^2(\vec{p}) = -v^2(\vec{p})$$

and

$$-\vec{\Sigma}\cdot\hat{p}\ \ u^4(-\vec{p}) = -u^4(-\vec{p})\ \Rightarrow\ \vec{\Sigma}\cdot\hat{p}\ \ v^1(\vec{p}) = +v^1(\vec{p})$$

Indeed,

$$\Psi_c^{(1)} = v^{(1)}(\vec{p})e^{ipx} = \sqrt{E+M}\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+M}\\ 1\end{pmatrix}\chi^2 e^{ipx}$$

represents a positron with positive helicity and

$$\Psi_c^{(2)} = v^{(2)}(\vec{p})e^{i\,p\,x} = \sqrt{E+M}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+M} \\ 1 \end{pmatrix}\chi^1 e^{i\,p\,x}$$

represents a positron with negative helicity.

Although throughout this discussion we refered to electrons and positrons, these results apply to any spin ½ fermion. Charge conjugation operation changes always particle to antiparticle states but leaves helicity unchanged (see Fig. 3). It would change for example a negative helicity neutrino, ν_L , which exists in nature to a positive helicity anti-neutrino, $\bar{\nu}_R$ which has never been detected. This means that the interaction which produces the neutrinos, the weak interaction, violates charge conjugation symmetry. This will be discussed more extensively in a later lecture about the Weak Interactions.

Shown in Figure 4 (left) is Dirac and in Figure 4 (right) is Anderson operating his cloud chamber which has been inserted in the magnet which provided the magnetic field for bending the particle trajectories.

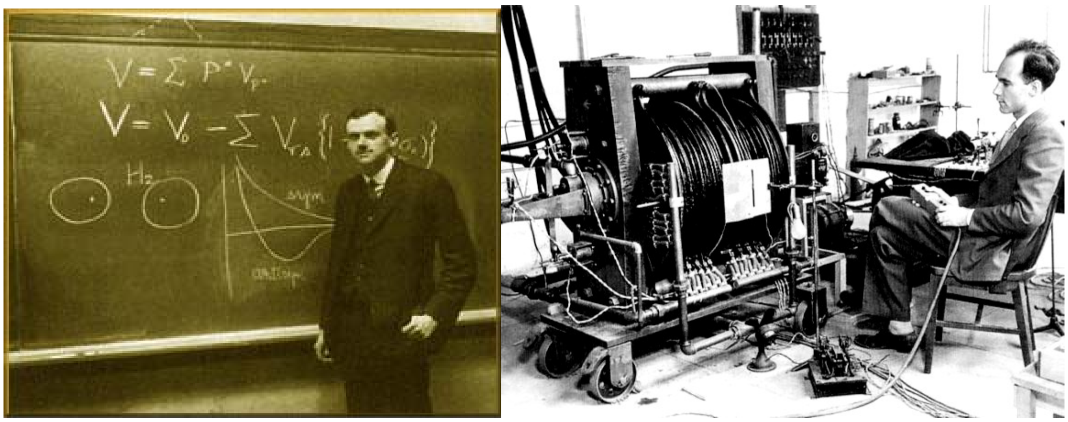

Figure 4: *Dirac (left) and Anderson (right) with his cloud chamber at CALTECH.*