Particle Physics Homework Assignment 6

Prof. Costas Foudas, October 2021

Problem 1: Show that: $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b}+i\vec{\sigma}(\vec{a}\times\vec{b})$

Problem 2:

1. Solve the Dirac equation $[\vec{a}\cdot\vec{p}+\beta\, m]\Psi = E\Psi$ in the particle rest frame using the Weyl representation.
2. Compute the result of the chirality operators $\dfrac{(1\pm\gamma_5)}{2}$ when they are acting on the solutions of the Dirac equation expressed in the Weyl representation.

Problem 3: Positive energy solutions of the Dirac Equation correspond to the 4-vectror current: $J^{\mu} = 2p^{\mu} = 2(E;\vec{p});\quad E>0$. Show that negative energy solutions correspond to the current $J^{\mu} = -2(E;\vec{p}) = 2(|E|;-\vec{p}) = -2p^{\mu};\quad E<0$.

Problem 4: 1. Show that the helicity operator commutes with the Hamiltonian:

$$[\vec{\Sigma}\cdot\hat{p}, H] = 0$$

2. Show explicitly that the solutions of the Dirac equation are eigenvectors of the helicity operator:

$$\vec{\Sigma}\cdot\hat{p}\ \Psi = \pm\Psi$$