Particle Physics Homework Assignment 4

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Problem 1: Show the the momentum, p of a particle moving in a circular trajectory of radius, R , in a magnetic field, B , is given by:

$$p = 0.3\,BR$$

where the momentum is given in **GeV/c** the magnetic field is in Tesla and the radius in meters.

Problem 2: .A particle of mass M and 4-momentum P decays into two particles of masses m_1 and m_2 .

(1) Show that the energy of the 1st particle in the frame of M is:

$$E_1 = \frac{M^2+m_1^2-m_2^2}{2M}$$

(2) Show that the kinetic energy T_i of the ith particle in the frame of M is:

$$T_i = \Delta M\left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right)$$

where $\Delta M = M - m_1 - m_2$ is the mass excess or Q value of the process.

(3) A lambda particle, Λ , is a neutral baryon of mass $M = 1115\ MeV$ that decays into a proton of mass $m_1 = 938\ MeV$ and pion of mass $m_2 = 140\ MeV$. Compute the kinetic energies of the proton and the pion in the lambda rest frame.

Problem 3: In a collision process a particle of mass m at rest in the laboratory, is struck by a particle of mass, M, momentum, $\vec{P}_{LAB}$, and total energy, E_{LAB}.

(1) Show that: $W^2 = M^2 + m^2 + 2mE_{LAB}$ where W is the total energy at the center of mass system of the two particles.

(2) Show that the momentum before the collision at the center of mass frame given by:

$$\vec{p}^{\,*} = \frac{m\vec{P}_{LAB}}{W}$$

(3) Show that the center of mass system of the two particles is boosted by:

$$\vec{\beta}_{cms} = \frac{\vec{P}_{LAB}}{m+E_{LAB}} \text{ and } \gamma = \frac{m+E_{LAB}}{W}$$

Problem 4: In an elastic scattering process the incident particle of mass M imparts energy to the stationary target of mass m. The energy ΔE lost by the incident particle appears as recoil kinetic energy of the energy of the target.

(1) Show that the kinetic energy transferred to the target particle at the lab frame is given by:

$$\Delta E = \frac{m P_{LAB}^2}{W^2}\left(1-\cos\theta^*\right)$$

where θ^* is the angle of the outgoing particles with respect the direction of the boost at the CM frame.

(2) Show that if $M \gg m$ the maximum kinetic energy of the recoil particle at the lab frame is:

$$\Delta E_{max} = 2\gamma^2\beta^2 m$$

where β, γ are characteristic to the incident particle.

Problem 5: A narrow pencil beam of singly charged particles of very high momentum p, traveling along the x-axis traverses a slab of material s radiation lengths in thickness. If ionization loss can be neglected, calculate the rms lateral spread of the beam in the y-direction, as it emerges from the slab. Use the formula you derive to compute the rms lateral spread of a beam of 10 GeV/c muons in traversing a 100 m pipe filled with (a) air (b) helium at NTP. The particle data book shows that the radiation length in air $X_0^{AIR} = 36.66\, g/cm^2$ and the density of air is $\rho^{AIR} = 1.293\, g/l$ where in Helium they are $X_0^{He} = 94.32\, g/cm^2$ and $\rho^{AIR} = 0.1786\, g/l$

Problem 6: A sampling electromagnetic calorimeter uses 1.5 mm thick iron plates as an the absorber spaced by 2.0 mm layers of liquid Argon (originally developed by Willis and Radeka). The MIP values for Argon and Iron are $1.519\, MeV\, g^{-1}\, cm^2$ and $1.451\, MeV\, g^{-1}\, cm^2$ respectively. The density of Argon is $\rho^{Ar} = 1.396\, g/cm^3$ and that of iron is $\rho^{Fe} = 7.870\, g/cm^3$. Iron has atomic number $Z^{Fe} = 26$, an average mass number of $A^{Fe} = 55.85$ and $X_0^{Fe} = 1.76\, cm$.

a) Compute the sampling fraction of this calorimeter. Assume that after the electromagnetic shower is formed all charged particles behave as minimum ionizing particles and compute the energy losses using the Bethe-Bloch equation.
b) Compute the critical energy for the absorber of the calorimeter.
c) Estimate the coefficient of the stochastic term of the calorimeter resolution if the energy cut off correction is $F = 0.869$.
d) Compute the number of sampling elements (pairs of absorber and active material) required to contain at least 95% of showers coming from electrons of energy ***30 GeV***.
e) Suppose that electrons illuminate the calorimeter. As electron strike the front face of the calorimeter one has to take in to account edge effects. That is energy miss-measurement due to electrons whose shower escapes transversely from the sides of the calorimeter. Assuming that the impact point of the electrons at the front face is known (for example via some tracking device placed in front of the calorimeter) how far does this impact point need to be from the edge of the calorimeter so that the shower is also 95% contained also in the transverse direction ?