Particle Physics Homework Assignment 3

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Problem 1: The HERA accelerator, which operated at the DESY laboratory in Hamburg Germany in the period between 1992 and 2007, collided 27.5 GeV electrons with 920 GeV protons.

I. Compute the centre of mass energy (total available energy at the electron proton centre of mass) assuming that the angle between the proton and the electron beam momenta is 180^0 (head on collision).

II. Compute the boost, $\vec{\beta}_{CM}$, of the electron-proton centre of mass frame relative to the laboratory frame.

III. What should be the energy of an electron beam colliding with protons at rest if the centre of mass energy were to be the same with HERA ? This type of experiment is called *fixed target experiment* to distinguish it with the previous which is called a *collider experiment.*

Problem 2 : Deduce an expression for the energy of γ-rays from the decay of the neutral pion, $\pi^0 \rightarrow \gamma\gamma$, in terms of the mass **m**, energy **E**, velocity βc of the pion and the angle of emission θ^* of the photon in the pion rest frame. Because the pions have zero spin the angular distribution is isotropic at the pion rest frame. Show that the γ-ray energy spectrum in the laboratory frame will be flat extending from $E(1+\beta)/2$ to $E(1-\beta)/2$. For relativistic pions, find an expression for the disparity D of the γ-rays and show that for D>3 one observes half the decays and for D>7 one quarter of them. (Perkins p33, 4th edition). The disparity D is defined as the ratio of the energy of the most energetic photon divided by the energy of the least energetic photon.

Problem 3: **A** high energy electron collides with an atomic electron which can be considered at rest. What is the threshold (the minimum kinetic energy of the incident electron) for producing and electron positron pair ?

Problem 4: An experiment which measured parity violation in weak interactions as well as the magnetic moment and the Lande' g factor of the $\boldsymbol{\mu^+}$, used a $\boldsymbol{\mu^+}$ beam which was produced from the decay in flight of a $\boldsymbol{\pi^+}$ beam ($\boldsymbol{\pi^+ \rightarrow \mu^+ \nu_\mu}$). The kinetic energy of the $\boldsymbol{\pi^+}$ beam was **85 MeV**. The $\boldsymbol{\pi^+}$ beam was produced by colliding a proton beam from an accelerator with a target and selecting the positively charged $\boldsymbol{\pi^+}$ from the negatively charged $\boldsymbol{\pi^-}$ using a magnetic field. Positive pions of a certain direction and momentum were selected by passing the $\boldsymbol{\pi^+}$ beam through an appropriately positioned concrete block with a straight hole through it.

Questions regarding the beam:

I. Compute the decay length of the pions in the laboratory frame. The lifetime of the $\boldsymbol{\pi^+}$ is **26 ns** and the $\boldsymbol{\pi^+}$ mass is about **140 MeV**. The incident beam to the experiment was a mixture of 10% $\boldsymbol{\mu^+}$ and 90% $\boldsymbol{\pi^+}$. How far was the experiment from the point where the pions were produced ?

II. Compute the pion mean free path in carbon assuming that the pion carbon cross section at the relevant energy is **10 mb**. This cross section includes all strong interaction processes that contribute. The carbon density is **ρ = 2.265 g/cm^3**.

III. Compute the muon and neutrino energies in the rest frame of the pion. Assume that the neutrino is massless and that the mass of the muon is $\boldsymbol{m_\mu = 106\,MeV}$.

IV. Compute the maximum and minimum energy of the muon in the lab frame.

V. The authors of the paper placed a carbon block, approximately **20 cm** long, infront of a tileted carbon target. Justify the need for this block given that pions and muons of this energy lose in carbon approximately **4.5 MeV/cm** due to ionization. Energy loss via ionization is the topic of the next lecture.

Questions Regarding the Apparatus:

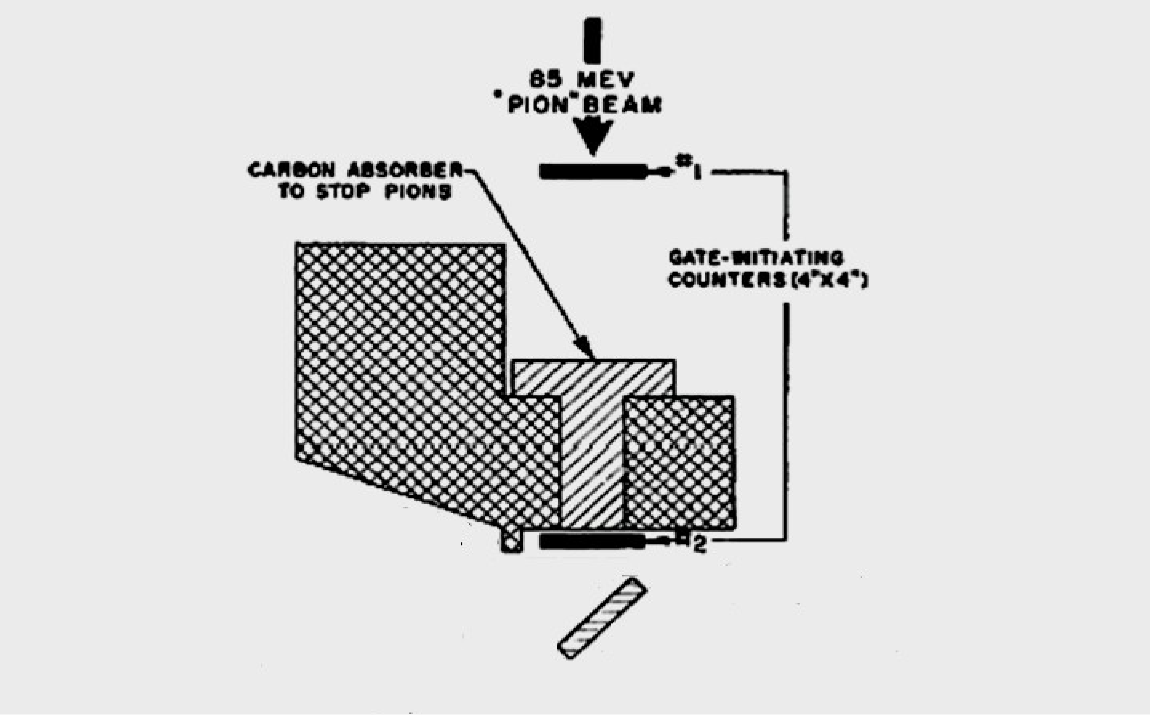

Figure 1: Part of the apparatus of the Garwin-Lederman-Weinrich experiment Phys. Rev.,105:1415-1417, 1957. The 85 MeV pion beam at Columbia University NEVIS labs is shown entering from the top. The carbon absorber length has been chosen so that only muons exit the carbon block whilst the pions stop in the block. Hence a signal coincidence from the counters #1 and #2 indicates that a muon has gone through. If the carbon length is chosen appropriately most muons will likely stop on the tilted carbon target below.

The experimental apparatus is shown in Fig. 1. A carbon block has been used to separate the pions from the muons. Actually the carbon block was introduced to for two reasons: (1) to stop the pions (2) to slow down the muons in such a way so that when they exit the carbon block they have very little kinetic energy left and thus they stop at the carbon target shown tilted at the bottom of Fig. 4. There they decay giving one electron and two neutrinos and the properties of the decay electron can be measured.

Two counters #1 and #2 have been placed at the path of the beam before and after the carbon block. Counters are devices made of liquid or plastic scintillator. When a charged particle goes through a counter it excites the scintillator material and produces light which can be collected and converted by photo-tubes to charge. Detection of this charge using electronic circuits provides information which indicates that a particle has gone through the scintillator material as well as allows a measurement of the time that the particle went through.

The signals from Counters #1 and #2 in this experiment are used in coincidence to provide an experimental trigger. This means that if both counters detected that a particle went though at the same time[1] (coincidence trigger) then that meant that a muon went through the carbon and exited from the other side albeit with very little kinetic energy so that it most likely stopped on the target. If the trigger coincidence condition was fulfilled then detectors around the target were activated appropriately and recorded the data from the muon decays.

VI. Compute the maximum possible angle of the decay muons from pions in the Laboratory frame. Use the results from (V) and explain how does the carbon block constrain this angle. Do the angles of the muons which are capable to penetrate the block extend up to this angle ?

VII. Given that this experiment does not record data unless if *there is a trigger,* compute and discuss how does the distance between the two counters affects the muon energy spectrum detected in this experiment. Assume that the beam enters at the center of the fist counter.

Hence, this part of the apparatus has been designed to collect muons which stop at the target (for sometime since the muon lives only 2.16 μsec) to study their properties. The rest of this experiment as well as the results of it will be discussed later in the course when we discuss parity violation in weak interactions and and about the muon magnetic moment.

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[1] The time difference introduced by the muon travelling between the two counters is very small and the experiment is not sensitive to it.

Problem 5: When the muon was originally discovered back in 1937 (see next lecture), people thought that this was the Yukawa particle. The Yukawa particle was considered then to be the mediator of the strong interaction. This incorrect interpretation of the nature of the muon was due to the fact that the muon mass (106 MeV) was not very different from the expected mass of the particle predicted by Yukawa. Later on it turned out that the Yukawa particle was the pion (140 MeV) which was discovered in 1947 at Bristol.

This problem[2] relates to the calculations done by Tomonaga and Araki who predicted that negative muons as they slow down in matter would be more likely to be captured by the nuclei rather than decay and the question of course is if the strong interaction is responsible for the muons which get 'swallowed' in the nucleus.

(a) Show that a negative muon captured in an S-state by a nucleus of charge Ze and mass A will spend a fraction $f \simeq 0.25\,A(Z/137)^3$ of its time inside the nuclear matter and that in time t it will travel a total distance $fct(Z/137)$ in the nuclear matter. The hydrogen atom ground state wave function can be used in these calculations with modifications to account for the fact that the muon mass is of the order of 200 times larger than the electron mass: $\Psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{\alpha_0}\right)^{3/2} e^{-\frac{Zr}{\alpha_0}}$ where $\alpha_0 = \frac{\hbar^2}{M_R e^2}$ and $M_R = \frac{A\,m_p m_\mu}{A\,m_p + m_\mu}$ is the reduced mass of the proton muon system. The proton and muon masses are $m_p = 938\,MeV$ and $m_\mu = 106\,MeV$.

(b) The law of radioactive decay of free muons is $dN/dt = -\Gamma_d N$, where $\Gamma = 1/\tau$ is the decay constant (width) and the lifetime is $\tau = 2.16\,\mu sec$. For a negative muon captured in an atom Z the decay constant is $\Gamma_{TOT} = \Gamma_d + \Gamma_c$, where Γ_c is the width for nuclear capture i.e. the probability per unit time of nuclear capture. For aluminium $(Z=13, A=27)$ the mean lifetime of negative muons is $\tau = 0.88\,\mu sec$. Calculate Γ_c and using he expression for f in (a), compute the interaction mean free path Λ for a muon in nuclear matter.

(c) From the magnitude of Λ estimate the magnitude of the coupling constant of the interaction that caused the nuclear capture $\mu^- + p \rightarrow n + \nu$ given that the strong interaction coupling constant is α_S and corresponds to a mean free path of $1\,fm$.

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[2] This is a modified version of problem 1.9 in p34 in Perkins 4th edition.

Conversi, Pancini and Piccioni[3] did experiments in Rome in the 40s to test Tomonaga's and Araki's hypothesis and found that positive muons traversing different materials always decay rather than being captured (not surprising). They also found that negative muons undergo nuclear capture in iron rather than decay as predicted by Tomonaga and Araki[4]. Compare the mean free path for a muon to be captured by an Aluminum nucleus with the typical mean free path of a strong interaction reaction and draw conclusions as to wheather the muon could be the mediator of the strong interaction.

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[3] Conversi, Pancini and Piccioni, Phys. Rev., 71. No 3, 1 Feb. 1947.

[4] They also found that negative muons do not get captured in Carbon and they thought that this contradicts Tomonaga's and Araki's prediction. However as it will become apparent this is due to the lower Z of the Carbon nucleus (Z=6) relative to Iron (Z=26).