Particle Physics Homework Assignment 2

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Problem 1: Show that $g_{\mu\nu} g^{\mu\nu} = 4$.

Problem 2: Show explicitly that $\Lambda^{\mu}{}_{\alpha} \Lambda_{\mu}{}^{\beta} = \delta_{\alpha}{}^{\beta}$. Use a Lorentz boost in the x-direction ($\vec{\beta} = \frac{v}{c} \hat{x}_0$) in the place of $\Lambda^{\mu}{}_{\nu}$.

Problem 3: Show that the expression $T^{\alpha\beta} x_{\alpha} y_{\beta}$ is a Lorentz invariant provided that $T^{\alpha\beta}$ transforms as a rank-2 tensor and x_{α}, y_{β} transform as covariant vectors.

Problem 4: Show that the 4-derivatives $\frac{\partial}{\partial x^{\mu}}$ and $\frac{\partial}{\partial x_{\mu}}$ transform as Lorentz covariant and contravariant vectors respectively.

Problem 5:

1) Write down the definition of a parity transformation.
2) Consider two Lorentz 4-vectors: X^{μ} and Y^{μ} . X^{μ} transforms as a polar vector, and Y^{μ} as an axial vector . How do they transform under parity ?
3) Which of the following Lorentz invariant quantities is invariant under parity and which is not:

$$(a)\ X^{\mu} X_{\mu} \quad (b)\ Y^{\mu} Y_{\mu} \quad (c)\ (X^{\mu} - Y^{\mu}) \cdot (X_{\mu} - Y_{\mu})$$

Problem 6:

1) Using Maxwell's equation in three dimensions show that the Electric Field, $\vec{E}$, is a vector and the magnetic field, $\vec{B}$, an axial vector.
2) As one can see, Maxwell's equations are not completely symmetric because although they include an electric charge density, ρ_e , and an electric current density $\vec{J}_e$, the equivalent magnetic quantities, ρ_m , $\vec{J}_m$, are absent indicating that there are no magnetic monopols. Introduce magnetic monopols and write down the completely symmetric Maxwell equations. Show that ρ_m must be a pseudoscalar and $\vec{J}_m$ an axial vector.