Particle Physics Homework Assignment 11

Prof. Costas Foudas 10/01/24

Problem 1: In homework assignment 10 we have shown that the CP transformation of a negative helicity and massless neutrino results to positive helicity antineutrino which is described by

$$\Psi_{CP}(x) = -v^{(2)}(-\vec{p}; m=0)e^{+ip^0 x^0 - i(-\vec{p})\cdot\vec{x}}$$

Apply a Time Reversal transformation on $\Psi_{CP}(x)$ to derive the TCP transformed spinor $\Psi_{TCP}(x)$.

Problem 2: Consider a negative energy electron coupled to an electromagnetic field. The electron is described by the Dirac equation

$$\left[\vec{\alpha}\cdot(-i\vec{\nabla} - e\vec{A}(x)) + \beta m + e\Phi(x)\right]\Psi(x) = -E\,\Psi(x) \quad \text{where} \quad E>0$$

1. Show, by requiring that this equation is invariant under TCP, that electromagnetic field transforms under TCP as $A^{\mu}_{TCP}(-x) = -A^{\mu}(x)$.

2. The TPC transformed electron corresponds to a positive energy solution.

Problem 3: Show that

a) $$F^{\mu\nu}\tilde{F}_{\mu\nu} = \vec{E}\cdot\vec{B}$$

b) this term violates both Parity and Time Reversal symmetries.

$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ is the Maxwell tensor and $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ its dual. $\vec{E}, \vec{B}$ are the electric and magnetic fields respectively.