Particle Physics Homework Assignment 10

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Problem 1: Show that $\sigma^2 \vec{\sigma}^* = -\vec{\sigma}\sigma^2$

Problem 2: Show that in the Pauli Dirac representation the matrix C satisfies

$$C = -C^{-1} = -C^{+} = -C^{T}$$

Problem 3: Show that

$$\Psi_c = C\bar{\Psi}^T \quad \text{and} \quad \bar{\Psi}_c = -\Psi^T C^{-1}$$

Problem 4: As shown in Homework Assignment 9 the spinor

$$\Psi(x) = \sqrt{E}\begin{pmatrix} 1 \\ \vec{\sigma}\cdot\hat{p} \end{pmatrix}\chi^2 e^{-ipx}$$

has negative helicity and can describe a neutrino with negative helicity which has been detected in nature. Show that the charge conjugate of this spinor represents an anti-neutrino with negative helicity which has not been detected in nature. This means that the interaction which is responsible for the production of neutrinos violates the charge conjugation symmetry.

Problem 5: Use the charge conjugate spinor of a neutrino with negative helicity from the previous problem

$$\Psi_C(x) = -\sqrt{E}\begin{pmatrix} \vec{\sigma}\cdot\hat{p} \\ 1 \end{pmatrix}\chi^1 e^{+ipx}$$

which as we have seen has negative helicity and calculate its parity inverted spinor Ψ_{PC} .

Problem 6: Consider the Majorana representation of the Dirac matrices which is given by

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \quad \gamma^1 = i\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \quad \gamma^3 = -i\begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}$$

Show that in this representation $\Psi_c = \Psi^*$. In this representation one can define a spinor $\chi = \Psi + \Psi_c$. Show that χ, provided that it represents a neutral particle, is also a solution of the Dirac equation which is real and satisfies $\chi = \chi_c$. In other words it represents a particle which is identical to its antiparticle.

Problem 7: Show that the particle and antiparticle spinors can be expressed as

(a) $$u^{(s)}(\vec{p}, m) = \frac{\gamma^{\mu} p_{\mu} + m}{\sqrt{E+m}} \times u^{(s)}(0, m)$$

(b) $$v^{(s)}(\vec{p}, m) = \frac{-\gamma^{\mu} p_{\mu} + m}{\sqrt{E+m}} \times v^{(s)}(0, m)$$

where

$$u^{(1)}(0,m) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u^{(2)}(0,m) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

$$v^{(1)}(0,m) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad v^{(2)}(0,m) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Problem 8: Show that

$$(\gamma^{\mu} p_{\mu} + m)\gamma^{0}(\gamma^{\mu} p_{\mu} + m) = 2E(\gamma^{\mu} p_{\mu} + m)$$

Problem 9: Show that

(a) $$\sum_{\alpha=1}^{2} u^{(\alpha)}(\vec{p}, m) \otimes \bar{u}^{(\alpha)}(\vec{p}, m) = \gamma^{\mu} p_{\mu} + m$$

(b) $$\sum_{\alpha=1}^{2} v^{(\alpha)}(\vec{p}, m) \otimes \bar{v}^{(\alpha)}(\vec{p}, m) = \gamma^{\mu} p_{\mu} - m$$